A COMPARISON OF PERFORMANCE AND IMPLEMENTATION CHARACTERISTICS OF NMPC FORMULATIONS

G. A. A. GONÇALVES[1], E. L. ALVARISTO[2], G. C. SILVA[2], M. B. SOUZA Jr.[2], A. R. SECCHI[1]

[1]Universidade Federal do Rio de Janeiro, COPPE, Programa de Engenharia Química
[2]Universidade Federal do Rio de Janeiro, Escola de Química, Programa de Pós Graduação em Tecnologia de Processos Químicos e Bioquímicos

ABSTRACT – Three nonlinear model predictive control (NMPC) strategies are compared on the control of the isothermal CSTR (continuous stirred tank reactor) with van der Vusse kinetics, which is largely employed in control studies. This reactor exhibits sign change of the process static gain and nonminimum phase dynamic behavior. The first strategy considers a NMPC coupled with a state estimator. The second one uses neural networks as the internal NMPC multivariable model. In the last one, a proposed approach for the adaptation of the linear MPC (model predictive control) to nonlinear systems is employed in order to generate predictions through successive local linearizations around steady states. The results show that the NMPC with state estimation stabilized the system at the expense of higher computational cost. The strategy based on neural networks demanded a shorter time for the calculation of the control actions, allowing the use of a shorter sampling time. The adaptive MPC stabilizes the nonlinear system around points which are unstable under linear MPC control, demanding less computational effort than the NMPC with a state estimator.

1. INTRODUCTION

If chemical processes are operated within limited ranges where nonlinearities are not relevant, satisfactory regulatory control may be obtained. However, process changes, such as changes in the characteristics of the feed, may drive the operation to regions of pronounced nonlinearity. In these regions, the regulatory control may not be able to perform accordingly, as it may be faced with conditions for which it was not designed. For instance, processes that operate normally under linear predictive control may suffer changes in the process gains as well as in the trajectories predicted by the controller leading to significant difficulties.

Process losses due to those adversities are relevant. Therefore, adequately treating of the nonlinearities in chemical processes is an important subject that has been being studied by some authors (Qin & Badgwell, 2000; Duraiski, 2001; Glavicet al. 2002; Cervantes et al., 2002; Benamor et al., 2004; De Oliveira & Camponogara, 2010; Manenti, 2011). Several methodologies have been developed aiming the implementation of NMPC strategies with viable execution time and optimal behavior. Neural network based NMPCs are among the strategies that tried to reduce the computational time through the use of models that do not need to be numerically integrated. This kind of model is able to provide future response predictions using known (past and current) process and (past) controller data. Akpan & Hassapis (2011) and Salahshoor et al. (2013) have recently employed this strategy to design nonlinear controllers. Another line of work employs successive linearizations in the approximation of a nonlinear model, as performed by Duraiski (2001). Finally, the nonlinear optimization problem of the MPC can be solved using classic optimization algorithms such as interior-point algorithm and sequential quadratic programming, as in the works of Rawlings & Mayne (2009) and Lopez-Negrete et al. (2012).

This work compares the strategies based on neural networks, resolution of nonlinear predictive control through the interior-point algorithm employing sequential methodology, and the proposed successive linearizations. The algorithms are compared in terms of complexity for the resolution of the problem and attained performance. The predictive control problem is formulated and the employed algorithms are described in next section. Then, the comparisons are performed and the results discussed in the following sections.

2. FORMULATION AND DIFFERENT APPROACHES FOR THE RESOLUTION OF THE NMPC PROBLEM

The NMPC problem (Problem P1) consists on the minimization of the following objective function:

$$V(\boldsymbol{x},\boldsymbol{u},k) = \sum_{i=k}^{k+N-1} \ell\big(\boldsymbol{x}(i),\boldsymbol{u}(i)\big) + F(\boldsymbol{x}(k+N)) \tag{1}$$

subject to the following constraints ($i = k,\ldots,k+N-1$):

$$\boldsymbol{x}(i+1) = f(\boldsymbol{x}(i),\boldsymbol{u}(i)) \tag{2}$$

$$\boldsymbol{y}(i) = h(\boldsymbol{x}(i)) \tag{3}$$

$$\boldsymbol{u}(i) \in \mathbb{U} \tag{4}$$

$$\boldsymbol{x}(i) \in \mathbb{X} \tag{5}$$

$$\boldsymbol{x}(k+N) \in X_f \subset \mathbb{X} \tag{6}$$

In this formulation, the control actions are applied using the receding horizon approach, where only the first action, of the N calculated ones, is applied. Therefore, given the sequence calculated at the current time:

$$\boldsymbol{u}^0(x) = \{\boldsymbol{u}^0(0;\boldsymbol{x}),\boldsymbol{u}^0(1;\boldsymbol{x}),\ldots,\boldsymbol{u}^0(N-1;\boldsymbol{x})\} \tag{7}$$

The implemented control action is given by:

$$\boldsymbol{u}(k) = \boldsymbol{u}^0(0;\boldsymbol{x}) \tag{8}$$

In order to guarantee the stability of the MPC controller, the term $F(\boldsymbol{x}(k+N))$, called terminal cost function, and the terminal state set, X_f, of Equation (6), are of fundamental importance (Mayne et al., 2000). Keerthi & Gilbert (1988) stated that a terminal state equality constraint guarantees stability if the optimization problem is feasible at the first instant. Later, Jadbabaie & Hauser (2001) showed that the use of a terminal state equality constraint is not required.They demonstrate that the stability of the receding horizon control can be achieved using a terminal cost, which can be any general non-negative function. Moreover, the implementation of the terminal constraint can increase the computational burden of the optimization problem, possibly demanding longer computational times for a given tolerance than the sampling period (Allgöwer et al., 2004). Limon et al. (2006) showed that weighting the terminal cost enlarges the domain of attraction of the MPC, proving also that, for any state of the system that can reach X_f, there is a weight so that the state will go inside the attraction region of the controller.

2.1. Direct resolution using sequential method

In the sequential method (Biegler & Hughes, 1985), two computational routines are employed separately, one for the integration of the system and another for its optimization, being the communication between them made through the trajectories provided by the numerical integration algorithm. In this strategy, only the control actions are discretized, being the trajectories of the system always feasible. In the present work, the interior-point algorithm of the MATLAB optimization toolbox (Mathworks, 2008) was employed for optimization. In this approach, the more computationally expensive step is the system integration, requiring efficient integration routines. The DASSLC (Secchi, 2012) routines were used here with that purpose.

2.2. Resolution based on adaptive MPC

Considering a process with nonlinear characteristics, whose state-space model can be described by Equation (2), the adaptive approach proposed in the present work starts by finding a reference steady-state for the vector of manipulated variables at each sampling time, that is, considering that $\boldsymbol{u}$ will be kept constant for any time higher than the sampling time, the analytical or numerical steady state solution of the problem $f\big(\boldsymbol{x}(k), \boldsymbol{u}(k)\big) = \boldsymbol{0}$ is determined. Then, the model described by Equation (2) is linearized around this reference state, resulting in a linear state space model:

$$\boldsymbol{\Delta}\mathbf{x}(\mathrm{k}+1) = \boldsymbol{A}(k)\boldsymbol{\Delta x}(k) + \boldsymbol{B}(k)\boldsymbol{\Delta u}(k) \tag{9}$$

$$\boldsymbol{\Delta y}(k) = \boldsymbol{C}(k)\boldsymbol{\Delta x}(k) \tag{10}$$

where $\boldsymbol{\Delta}$ indicates a vector of deviation variables from the reference state. Briefly, at each sampling time, the vector of manipulated variables is updated by the optimization and the matrices of the state space model are recalculated, in order to provide one-step ahead predictions with smaller error when compared to the process measurements. This procedure is repeated to generate predictions over the prediction horizon. As the dimension of the decision variables vector is equal to the control horizon, the last control action is kept constant until the prediction horizon is reached. This way, based on the prediction provided by Equation (9), it is possible to obtain the optimal control actions by solving P1. In this approach, the nonlinear model, Equation (2), is replaced by Equations (9) and (10), so that the integration of the nonlinear equation system is unnecessary.

2.2. Resolution using a neural network model

The controller proposed here uses previously fitted neural networks to predict the future behavior of the controlled variable from past and present data of process and manipulated variables. The patterns for fitting and validation of the neural networks were generated by open loop simulations, where the inputs were randomly varied. Multilayer *perceptrons* (MLP) were used. Linear input and output layers and a hidden layer with hyperbolic tangent activation function were considered. Equation (11) presents the calculated output of a neuron *j* of a hidden layer *k*.

$$s_{\boldsymbol{j},\boldsymbol{k}} = f\left(\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}_{\boldsymbol{k}-\mathbf{1}}} w_{\boldsymbol{j}\boldsymbol{i}} s_{\boldsymbol{i},\boldsymbol{k}-\mathbf{1}} + \theta_{\boldsymbol{j},\boldsymbol{k}}\right) \tag{11}$$

$$f(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})} \tag{12}$$

where the parameters *w* and θ are respectively the interlayer weights and neuron biases.

For finite horizon predictions, the neural networks are recursively employed to obtain *N* predictions. This way, the predicted output vector is used in the calculation of the optimal control actions by solving P1.

3. RESULTS AND DISCUSSION

The isothermal CSTR with van der Vusse kinetics was used as case study. This system is largely studied due to its nonlinear characteristics and is described by:

$$\frac{dC_a}{dt} = \frac{F}{V}\left(C_{a_{in}} - C_a\right) - k_1 C_a - k_3 C_a^2 \tag{13}$$

$$\frac{dC_b}{dt} = \frac{F}{V}(-C_b) + k_1 C_a - k_2 C_b \tag{14}$$

where C_a and C_b are output molar concentrations of reactant A and product B, respectively; $C_{a_{in}}$ is the input molar concentrations of reactant A; F is the volumetric flowrate; *V*, the volume; and k_i, the reaction rate constants. The parameters of the model are presented in Trierweiler (1997). The steady state profile of the component B concentration (C_b), for varying flowrate (F), is presented in Figure (1), where the change in the sign of the static gain can be observed.

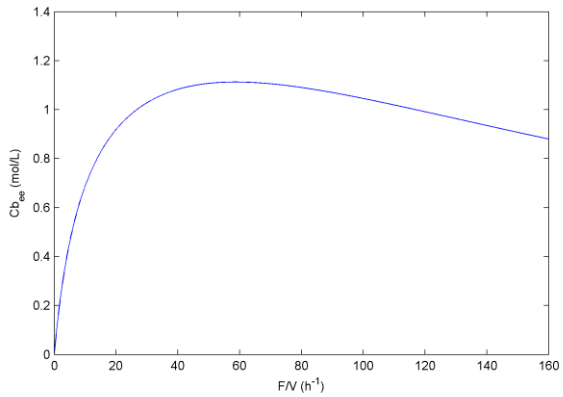

Figure1–Steady state (SS) concentration of component B vs. flowrate.

The objective function is given by Equation (15). A SISO (single input, single output) problem was assumed. The weighting parameters in the deviation of the variables were carefully tuned. A sampling period of 0.008 h, a predictive horizon of 25 and a control horizon of 5 were chosen.

$$\ell(\boldsymbol{x},\boldsymbol{u}) = \left(\boldsymbol{x}-\boldsymbol{x}_{sp}\right)^T\left(\boldsymbol{x}-\boldsymbol{x}_{sp}\right) + 5\times 10^{-4}\Delta\boldsymbol{u}^T\Delta\boldsymbol{u} + 10^4\left(x(N)-x_{sp}(N)\right)^2 \tag{15}$$

Two tests, with three setpoint (SP) changes each, were conducted. In the first one, shown in Figure 2, the system was initially at a SS to the left side of the maximum C_b concentration in Figure 1, when the setpoint was sequentially changed first to a reachable value, then to the maximum concentration and finally to an unreachable value. A similar experiment was conducted in Figure 3, with the system initially at a SS to the right of the maximum concentration. Table 1 presents two indices for these tests: the integral of the squared error (ISE) and the manipulation effort, which is given by the summation of the variations of the manipulated variable weighted by the sampling period. The results show that the linear controller can lead the system to instability. On the other side, the controller based on successive linearizations managed to keep the system stable at the expenses of a relatively small computational burden, because it does not require the integration of the differential equations system, as it is also the case for the linear controller. The adaptive controller also had less control effort than the NMPC.

In Figures 2 and 3, it can be seen that the controller based on neural networks (composed of 4-6-1 neurons in the forward layers) presented results that approximated the ones of the controller that employed the full model based on the integration of the differential equations, however the presumed cost to design (and eventually update) the neural model can be expected to be higher than linearizing the system at each sampling time. For the experiments with unreachable setpoint, the adaptive MPC approximated the setpoint in a stable way, keeping a constant offset and guaranteeing the stability of the system. The linear MPC was unable to keep stability due to its linear nature. The benefits provided by the adaption of the internal model of the MPC are more evident when the nonlinearity is stronger as in the region of gain inversion. The stability under adaptive control, despite the offset, allows the process to be operated for longer periods of time. The offset presented by the adaptive approach can be minimized through a deeper investigation of the weighting parameters employed in the objective function.

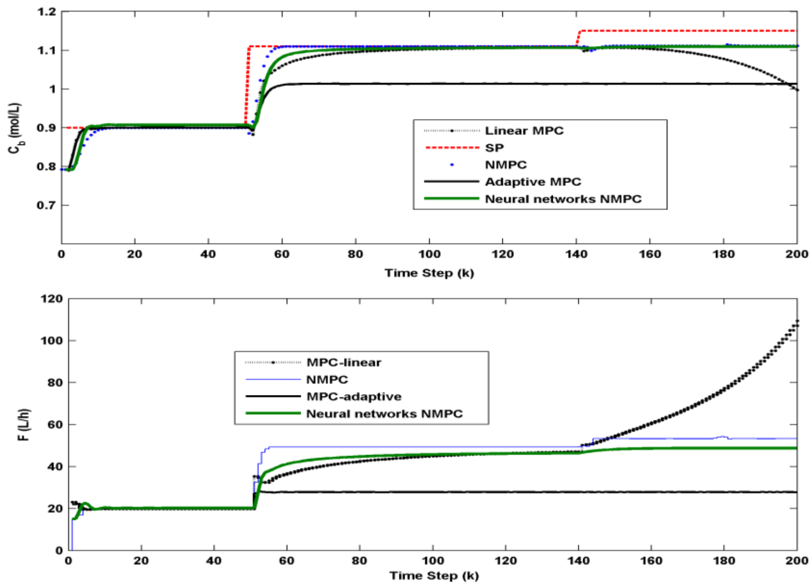

Figure 2 – Comparison between controllers for an initial steady-state to the left of the maximum.

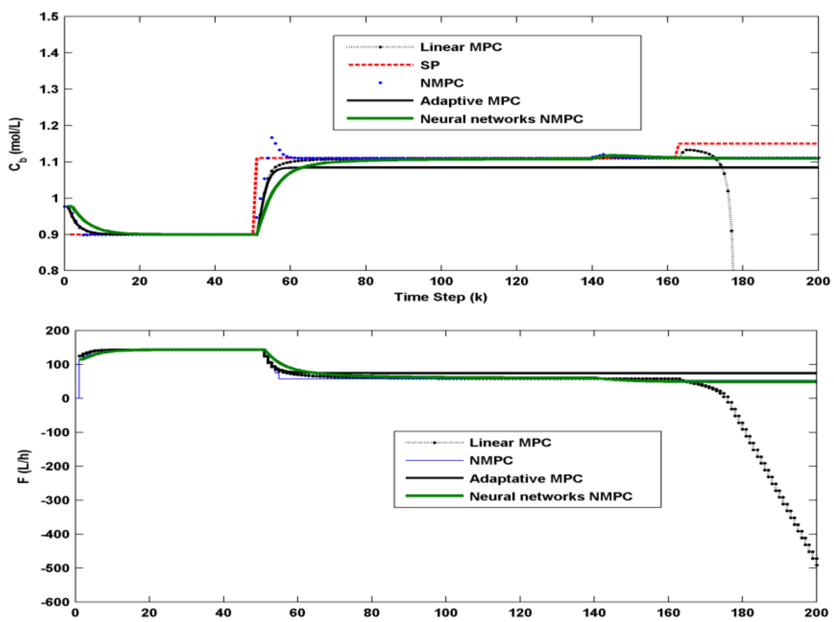

Figure 3–Comparison between controllers for an initial steady-state to the right of the maximum.

Table 1–Performance indices of the controllers

	Left side		Right side	
	ISE	Manipulation effort	ISE	Manipulation effort
Adaptive MPC				
Reachable SP	0.00024	0.41506	0.00008	1.28690
Cb_max SP	0.00783	0.45509	0.00123	7.35780
Unreachable SP	0.00906	0.00208	0.00130	0
NMPC				
Reachable SP	0.00049	1.88555	0.00013	109.330
Cb_max SP	0.00094	2.17909	0.00038	11.7840
Unreachable SP	0.00074	0.09733	0.00069	0.03120
Neural network NMPC				
Reachable SP	0.00039	0.25401	0.00018	0.66246
Cb_max SP	0.00140	0.91013	0.00116	4.12949
Unreachable SP	0.00083	0.00319	0.00072	0.05598
Linear MPC				
Reachable SP	0.00020	0.5518	0.00008	1.25000
Cb_max SP	0.00150	2.0073	1.25000	8.20000
Unreachable SP	unstable	unstable	unstable	unstable

4. CONCLUSIONS

Linear MPCs are currently largely used in the industry. However, their nonlinear counterparts still need improvements related to implementation and performance aspects in order to gain more applications. In this work, up-to-date NMPC algorithms were improved (in the case of the adaptive MPC) and compared. The superiority of the NMPC against the linear version was shown in regions of pronounced nonlinearity. It was also demonstrated that the NMPC approaches based on simplified models of the process (adaptive linear and neural networks) demand a smaller computational burden than the ones that depend on the integration of the full model, with similar performance.

5. REFERENCES

ALLGÖWER, F.; FINDEISEN, R.; NAGY, Z. K. Nonlinear model predictive control from theory to application. J Chin. Inst. Chem. Engrs, v. 35, n. 3, p. 299-315, 2004.

AKPAN, V. A.; HASSAPIS, G. D., Nonlinear model identification and adaptive model predictive control using neural networks. ISA transactions, v. 50, n. 2 , p. 177–94, 2011.

BENAMOR, S.; DOYLE, F. J.; MCFARLANE, R. Polymer grade transition control using advanced real-time optimization software. Journal of Process Control, v. 14, p. 349-364, 2004.

BIEGLER, L. T.; HUGHES, R. R., Feasible path optimization with sequential modular simulators, Computers & Chemical Engineering, v. 9, n. 4, p. 379–394,1985.

CERVANTES, A. M.; TONELLI, S.; BRANDOLIN, A.; BANDONI, J. A.; BIEGLER, L. T. Largescale dynamic optimization for grade transitions in a low density polyethylene plant. Computers and Chemical Engineering, v. 26, p. 227-237, 2002.

DE OLIVEIRA, L. B.; CAMPONOGARA, E. Multi-agent model predictive control of signaling split in urban traffic networks. Transportation Research Part C - Emerging Technologies, v. 18, p. 120-139, 2010.

DURAISKI, R. G. Controle Preditivo Não Linear Utilizando Linearizações ao Longo da Trajetória. Universidade Federal do Rio Grande do Sul. M.Sc. dissertation. 2001 (in Portuguese).

GLAVIC, P.; BUTINAR, B.; BIKIC, D. Optimal reactor systems for Van de Vusse reaction scheme with multicomponent feed. Computers & Chemical Engineering, v. 26, p. 1335-1343, 2002.

JADBABAIE, A.; HAUSER, J., On the stability of unconstrained receding horizon control with a general terminal cost, Decision and Control, 2001. Proceedings of the 40th IEEE Conference on , pp.4826,4831 vol.5, 2001

KEERTHI, S., GILBERT, E. Optimal infinite horizon feedback laws for a general class of constrained discrete-time systems: stability and moving horizon approximations, Journal of Optimization Theory and Applications, 57, p. 265-293, 1988.

LIMON,D.; ALAMO, T.; SALAS, F.; CAMACHO, E. F. On the stability of MPC without terminal constraint. IEEE Trans. Auto. Cont., v. 51, n. 5, p.832-836, 2006.

LOPEZ-NEGRETE, R. D'A.; BIEGLER, L. T.; KUMAR, A. Fast nonlinear model predictive control: Formulation and industrial process applications, Computers & Chemical Engineering, v.51, p.55-64, 2013.

MANENTI, F. Considerations on nonlinear model predictive control techniques.Computers and Chemical Engineering, v. 35, p. 2491-2509, 2011.

MATHWORKS, MATLAB, version 7.8.0 (R2008a). Natick, Massachusetts, TheMathWorksInc., 2008.

MAYNE, D. Q.; RAWLINGS, J. B.; RAO, C. V.; SCOKAERT, P. O. M. Constrained model predictive control : Stability and optimality, Automatica, v. 36, p. 789–814, 2000.

QIN, S. J.; BADGWELL, T. A.An Overview of Nonlinear Model Predictive Control Applications.Progress in Systems as ControlTheory, Switzerland, v. 26, p. 369-392, 2000.

RAWLINGS, J. B., MAYNE, D. Q. *Model Predictive control: Theory and Design.* Madison: Nob Hill Publishing, 2009.

SALAHSHOOR, K.; ZAKERI, S.; HAGHIGHAT SEFAT, M., Stabilization of gas-lift oil wells by a nonlinear model predictive control scheme based on adaptive neural network models, Engineering Applications of Artificial Intelligence, v. 26, n. 8 (set.), pp. 1902–1910, 2013.

SECCHI, A.R.. "DASSLC v 3.8: Differential-Algebraic System Solver in C". http://www.enq.ufrgs.br/enqlib/numeric, 2012.

TRIERWEILER, J. O. A Systematic Approach to Control Structure Design.D.Sc. Thesis,UniversitätDortmund. Germany. 1997.