APPLICATION OF AN OPTIMAL MPC TUNING STRATEGY IN CONTROL OF A NONLINEAR REACTRO SYSTEM

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ABSTRACT – This paper concerns the use of a particle swarm optimization-based MPC tuning method so as to compare the performances between the conventional MPC and an infinite horizon MPC, when both are applied to a reactor system. More specifically, the tuning method is carried out on a simulated CSTR system using linearized models along with process/model mismatch, and so the optimal tuning parameters are also applied to the CSTR nonlinear model. The simulated results show that the infinite horizon MPC remains stable in all simulated scenarios, whereas the conventional MPC destabilizes when the nonlinear system is required to be controlled.

1. INTRODUCTION

Considerable progress has been made in using advanced control strategies toward process industries, so that better economic and operational performances are achieved. Within this context, model predictive control (MPC) appears to be the most suitable tool, owing to its ability to handle multivariable dynamics and constraints of the process as well as to communicate with economic optimization layer (García et al., 1989; Maciejowski, 2000). On the other hand, the dynamic behavior of MPC is sensitive to its values of tuning parameters, which should be chosen in order to ensure better performance, robustness and stability of the closed-loop system. In this sense, methods to tune MPC parameters still remain an open issue.

As a general rule, the MPC tuning strategies can be divided into two categories (Garriga and Soroush, 2010): the ad hoc methods, which provide guidelines to determine the controller parameters by explicit expressions or bounds based on approximation/simulation or parameters of the process dynamic; and the self tuning methods, which compute the set of tuning parameters through optimization algorithms. The ad hoc tuning methods, like Lee and Yu (1994) and Shridhar and Cooper (1998a;b), can be a practical and efficacious way of estimating the MPC tuning parameters, even though they are not necessarily efficient. Still, the auto tuning methods, like Lee et al. (2008) and Susuki et al. (2008), provide optimal solutions for the controller parameters on the basis of a given desired performance criterion. Even so, there are few methods that explicitly deal with the model uncertainty in their problem formulation; more recently Nery Júnior et al. (2014) developed a robust MPC tuning strategy using a Particle Swarm Optimization (PSO) technique.

From the preceding discussion, this paper aims to compare the performance of two model predictive control strategies, namely conventional MPC and infinite horizon MPC

(IHMPC), when both are tuned through the method proposed by Nery Júnior et al. (2014). The focus of application of these control strategies are concerned with the control of a nonlinear CSTR system.

2. MODEL PREDICTIVE CONTROL

MPC algorithms rely upon the simulated response of the process to be controlled, and then the process model is a key element for successful implementations of this advanced control strategy. The resulting MPC control law computes, at each time step, the control actions (manipulated variables) obtained through the minimization of the difference between the output reference and the predicted behavior of the controlled outputs.

Although the models used in MPC algorithms to forecast the future behavior of the controlled process may be linear or nonlinear, the vast majority of industrial applications in the refining and petrochemical industries has employed linear MPC (Darby and Nikolaou, 2012), and so it is considered in this work. In particular, the model formulation adopted here is one based on the analytical expression of the step response model corresponding to the system transfer function, proposed by Odloak (2004). This model representation is summarized as follows. Let us consider a system with n_u inputs and n_y outputs, assuming also that the poles relating to any input u_j to any output y_i are non-repeated and stable, then the space-state model takes the following form

$$\left\{ \begin{aligned} \begin{bmatrix} \mathbf{x}^s(k+1) \\ \mathbf{x}^d(k+1) \end{bmatrix} &= \underbrace{\begin{bmatrix} \mathbf{I}_{n_y} & 0 \\ 0 & \mathbf{F} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \mathbf{x}^s(k) \\ \mathbf{x}^d(k) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{D}^0 \\ \mathbf{D}^d\,\mathbf{F}\,\mathbf{N} \end{bmatrix}}_{\mathbf{B}} \Delta\mathbf{u}(k) \\ \mathbf{y}(k) &= \underbrace{\begin{bmatrix} \mathbf{I}_{n_y} & \mathbf{\Psi} \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} \mathbf{x}^s(k) \\ \mathbf{x}^d(k) \end{bmatrix} \end{aligned} \right. , \tag{1}$$

where $\mathbf{x}^s$ and $\mathbf{x}^d$ are the system states, the former represents the integrating states obtained by the incremental form of inputs and corresponds to the predicted output steady-state, while the latter stands for the stable states. The more detailed rules to obtain the matrices $\mathbf{D}^0$, $\mathbf{D}^d$, $\mathbf{F}$, $\mathbf{N}$, $\mathbf{\Psi}$ and $\mathbf{I}_{n_y}$ are defined in the original work of Odloak (2004).

The basic formulation of MPC (denoted here as conventional MPC), for the model description defined in Equation 1, is based on the solution of the following optimization problem:

Problem 1:

$$\min_{\Delta \mathbf{u}_k} V(k) = \sum_{j=1}^{p} \|\mathbf{y}(k+j|k) - \mathbf{y}_{\text{sp}}\|^2_{\mathbf{Q}} + \sum_{j=0}^{m-1} \|\Delta \mathbf{u}(k+j|k)\|^2_{\mathbf{R}}\,, \tag{2}$$

subject to :

$$\Delta \mathbf{u}(k+j) \in \mathbb{U}, \tag{3}$$

$$\mathbb{U} = \begin{cases} -\Delta \mathbf{u}_{\max} \leq \Delta \mathbf{u}(k+j) \leq \Delta \mathbf{u}_{\max} \\ \Delta \mathbf{u}(k+j|k) = 0, \ \forall j \geq m \\ \mathbf{u}_{\min} \leq \mathbf{u}(k-1) + \sum_{i=0}^{j} \mathbf{u}(k+i|k) \leq \mathbf{u}_{\max}. \end{cases} \tag{4}$$

where p and m are the prediction and control horizons, respectively; $\mathbf{Q}$ and $\mathbf{R}$ are assumed as diagonal positive definite matrices; $\mathbf{y}_{\text{sp}}$ is the set-point vector; $\Delta \mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$ is the input move vector.

It is worth mentioning that Problem 1, combined to the state-space model in the incremental form of the inputs, provides an offset free MPC, thus allowing the elimination of the target calculation layer (other optimization problem) that is frequently adopted to prevent offset in MPC implementations (see e.g. González et al. (2008)).

The disadvantage of the conventional MPC lies in fact that its control law does not guarantee a priori the closed-loop system stability. Indeed, the literature concerning the development of nominally stabilizing MPC algorithms is vast; the interested reader on this subject is referred to the recent work by Christofides et al. (2013) and the references therein. Despite this large amount of research works, the more popular approach to obtain a stable MPC consists in adopting an infinite prediction horizon, proposed in the seminal paper of Rawlings and Muske (1993). In this way, the IHMPC formulation results from the solution to the following optimization problem (Odloak, 2004):

Problem 2:

$$\min_{\Delta \mathbf{u}_k, \boldsymbol{\delta}_{y,k}} V(k) = \sum_{j=0}^{m} \|\mathbf{y}(k+j|k) - \mathbf{y}_{\text{sp}} - \boldsymbol{\delta}_{y,k}\|^2_{\mathbf{Q}} + \left\|\mathbf{x}^d(k+m|k)\right\|^2_{\bar{\mathbf{Q}}} + \sum_{j=0}^{m-1} \|\Delta \mathbf{u}(k+j|k)\|^2_{\mathbf{R}} + \|\boldsymbol{\delta}_{y,k}\|^2_{\mathbf{S}_y}\,, \tag{5}$$

subject to Equations 3 and 4, and

$$\mathbf{x}^s(k+m|k) - \mathbf{y}_{sp} - \boldsymbol{\delta}_{y,k} = 0, \tag{6}$$

where $\boldsymbol{\delta}_{y,k}$ is a vector of slack variables introduced into the control problem in order to enlarge the feasible region of the controller; $\mathbf{S}_y$ is assumed to be a diagonal positive definite weighting matrix associated with the slack vector, and the terminal weighting matrix $\bar{\mathbf{Q}}$ is calculated by the Lyapunov equation (Equation 7) of the system defined in Equation 1, or

$$\bar{\mathbf{Q}} - \mathbf{F}^\top \bar{\mathbf{Q}} \mathbf{F} = (\boldsymbol{\Psi} \mathbf{F})^\top \mathbf{Q} (\boldsymbol{\Psi} \mathbf{F})\,. \tag{7}$$

3. TUNING METHOD

It is well-known that the parameters of a MPC controller, such as prediction and control horizons (m, p) and weighting matrices ($\mathbf{Q}$, $\mathbf{R}$), play an important role in the dynamic behavior of the closed-loop system. Therefore, these parameters should be chosen (or tuned) in such a way to obtain robustness, stability and better control performance. In most practical cases, the values related with these parameters are determined in an empirical manner, which can be functional but may not yield optimum results. In this sense, optimal tuning methods of MPC have recently become a demanding issue.

The optimal MPC tuning problem consists in determining the set of parameters by minimizing a given criterion, which is associated with the response of the closed-loop simulation. The most used criteria in controller tuning problems are the performance indexes: Integral Absolute Error (IAE); Integral Time-weighted Absolute Error (ITAE), Integral Square Error (ISE) and Integral Time-weighted Square Error (ITSE). A modified ITSE index was used by Nery Júnior et al. (2014) to tune a finite horizon MPC controller, which will also be used here. It is described as follows:

Problem 3:

$$\min_{\boldsymbol{\Omega}} \Phi\left(\boldsymbol{\Omega}\right) = \sum_{k=0}^{n_{\text{sim}}} k \cdot T \cdot \left\| \mathbf{y}_m\left(k\right) - \mathbf{y}_{\text{sp}}\left(k\right) \right\|_{\boldsymbol{\Lambda}}^2 + \left\| \Delta\mathbf{u}\left(k\right) \right\|_{\boldsymbol{\Xi}}^2, \tag{8}$$

subject to Equations 3 and 4, and

$$\boldsymbol{\Omega}_{\min} \leq \boldsymbol{\Omega} \leq \boldsymbol{\Omega}_{\max}, \tag{9}$$

where: $\mathbf{y}_m$ is the measured output; $\Delta\mathbf{u}\left(k\right)$ is the solution of control action; n_{sim} is the total number of time steps; T is the sampling time; $\boldsymbol{\Lambda}$ and $\boldsymbol{\Xi}$ are weighting matrices and $\boldsymbol{\Omega} = [p,\ m,\ \mathbf{Q},\ \mathbf{R}]$ is set of parameters of the MPC controller.

Notice that Problem 3 is formulated as a non-convex mixed-integer nonlinear optimization problem, which is sensitive to changes in initial condition and much more challenging its solution. With the purpose of solving this kind of problem one has preferred meta-heuristic algorithms like PSO, e.g. Nery Júnior et al. (2014), and it is used in this work.

4. RESULTS

The case study considered here is a CSTR system described by the following nonlinear state-space model:

$$\begin{cases} \dfrac{dx_1}{dt} &= u_1 + u_2 - 0.2\sqrt{x_1} \\ \dfrac{dx_2}{dt} &= \left(C_{B_1} - x_2\right) \cdot \dfrac{u_1}{x_1} + \left(C_{B_2} - x_2\right) \cdot \dfrac{u_2}{x_1} - \dfrac{k_1 \cdot x_2}{\left(1 + k_2 \cdot x_2\right)^2} \\ y_1 &= x_1 \\ y_2 &= x_2, \end{cases} \tag{10}$$

in which x_1 is the liquid level inside the reactor (unit of length - u.l.); x_2 is reactant concentration at outlet of CSTR (unit of concentration - u.c.); u_i is the i-th reactant feed stream (unit of length divided by unit of time - u.l./u.t.); k_1 $\left(\text{u.t.}^{-1}\right)$ and k_2 $\left(\text{u.c.}^{-1}\right)$ are kinetic constants; C_{B_i} is the concentration of reactant at i-th feed stream.

A linearized form of Equation 10 can be obtained at the steady state $\mathbf{u}_{\text{ss}} = [0.1330,\ 1.1316]^{\top}$ u.l./u.t. and $\mathbf{x}_{\text{ss}} = [39.99 \text{ u.l.},\ \ 0.10 \text{ u.c.}]^{\top}$, and so substituting $C_{B_1} = 24.9$ u.c. and $C_{B_2} = 0.1$ u.c., it turns out to be:

$$\begin{cases} \mathbf{x}(t) &= \begin{bmatrix} -0.0158 & 0 \\ -0.0021 & \frac{0.2 \cdot k_1 \cdot k_2}{(0.1 \cdot k_2 + 1)^3} - \frac{k_1}{(0.1 \cdot k_2 + 1)^2} - 0.0316 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 1 \\ 0.62 & -9.95 \times 10^{-8} \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{x}(t). \end{cases} \tag{11}$$

In order to characterize the mismatch between the process model (plant) and the nominal model (used internally in the controller), it is assumed that the values of kinetic constants are uncertain. Thus, the plant is represented by Equation 11 with $k_1 = k_2 = 1.03$, whereas the nominal model is obtained using $k_1 = k_2 = 1.00$.

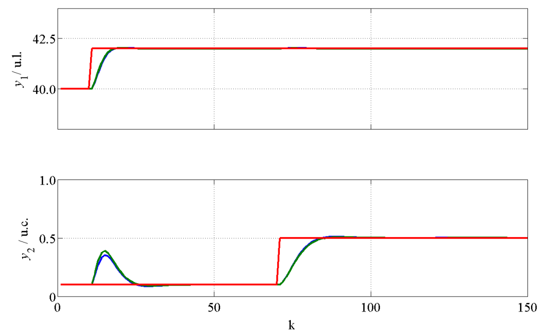
In the first step, both control strategies are applied to the linearized plant and nominal models of CSTR so as to optimally tune the parameters by Problem 3. The results are shown in Table 1.

Table 1 – Results obtained from the tuning method.

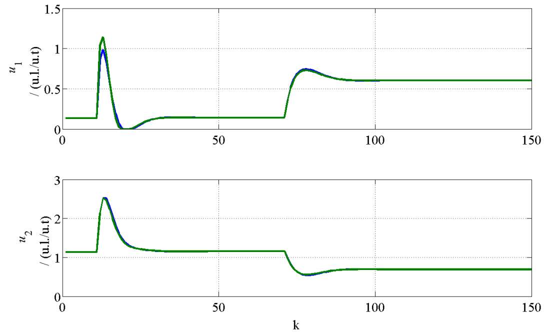
Parameters	conventional MPC	IHMPC
$\mathbf{R}$	diag ([0.76 0.80])	diag ([0.91 0.85])
$\mathbf{Q}$	diag ([0.68 0.46])	diag ([0.16 0.66])
m	3	5
p	9	∞
Processing time / h	1.94	4.90
Iterations	565	1000
Objective function	0.12	0.57

It is possible to note (Table 1) that the tuning method applied to the conventional MPC yields better numerical results of optimization than the IHMPC controller, because the PSO algorithm is solved with a smaller number of iteration, processing time and value of objective function. The closed-loop system behavior is presented in Figure 1. The simulated responses show (Figure 1) that there is not a clear superiority of any of the controllers, i.e. either the conventional MPC or IHMPC can effectively perform the system control.

In the second step, the performance of both controllers previously tuned is now evaluated for the nonlinear case of CSTR, and then the plant model will be that represented by Equation 10. The simulated responses of the controlled outputs and manipulated inputs are plotted in Figure 2. From this figure, it is clear the superior performance of IHMPC seeing that the system controlled by the conventional MPC becomes unstable in closed-loop while the infinite horizon controller outperforms it without major problems.

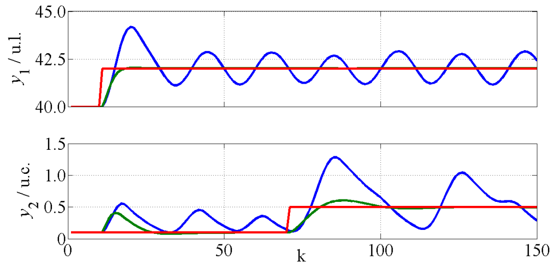


(a) Response of the controlled variables

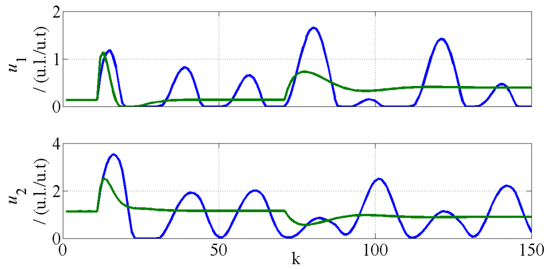


(b) Response of the manipulated variables

Figure 1 – Dynamics of the controlled outputs and manipulated inputs for the linear case. Conventional MPC (blue), IHMPC (green) and Set-point (red).



(a) Response of the controlled variables



(b) Response of the manipulated variables

Figure 2 – Dynamics of the controlled outputs and manipulated inputs for the nonlinear case. Conventional MPC (blue), IHMPC (green) and Set-point (red).

With the intention of quantifying the comparison of performance of the controllers, the ISE index (Equation 12) was computed for both the linear and nonlinear cases, whose values can be seen in Table 2.

$$\text{ISE}\left(\mathbf{y}\right) = \sum_{k=0}^{n_{\text{sim}}} \left(\mathbf{y}_{\text{sp}}(k) - \mathbf{y}(k)\right)^2 \qquad (12)$$

Table 2 – Results to the ISE calculated for each variable in each case.

Variable	Linear		Nonlinear	
	conventional MPC	IHMPC	conventional MPC	IHMPC
ISE(y_1)	9.15	8.68	81.01	8.68
ISE(y_2)	0.97	1.10	12.77	1.29

With regard to the first scenario (linear case), there is a little difference between the values of ISE, and this corroborates the similar performances obtained of both the controllers. Nonetheless, for the second scenario the ISE values evaluated to IHMPC are practically equal to the linear case, whereas the ISE values from the conventional

MPC are significantly different and larger than those corresponding to the first case, thus emphasizing its loss of stability and robustness.

5. CONCLUSION

In this paper was presented the results of application of a PSO-based MPC tuning method in an infinite horizon model predictive control. The performance from the IHMPC controller is compared with one corresponding to the conventional MPC, when a CSTR system is sought to be controlled for condition of plant/nominal model mismatch. The obtained results show that the conventional MPC gives a similar performance to IHMPC for the linear case, however, for the nonlinear case, it has become unstable.

On the other hand, it is remarkable that IHMPC maintains the closed-loop system stability in both simulated scenarios, which implies that it is more reliable than the conventional MPC in case of practical implementations.

For future works, this tuning method can be used in analyzing the coverage region of the parameters and its applicability in the robust control schemes.

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