PRESSURE DROP ANALYSIS IN COOLING WATER NETWORKS

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ABSTRACT – Cooling water systems are the most common method for the rejection of waste heat in chemical processes. Conventional re-circulating cooling water systems have a network of coolers in a parallel configuration, demanding both large cooling water circulation and cooling towers. Although reuse of cooling water reduces the amount of water that is necessary in the system and, hence, increases the cooling tower performance and capacity, it may significantly increase the pressure drop due to series-parallel arrangements. This paper develops a methodology for the pressure drop calculation based on the Graph Theory. From the adjacency matrix of cooler network, it uses the topological sorting and critical path algorithms in order to evaluate the pressure drop in a network with coolers in series-parallel arrangements. Applying this methodology to a cooler network with cooling water reuse, it is possible to analyse the impact of increased pressure drop when compared to a network with no reuse.

1. INTRODUCTION

Most chemical processes use cooling water systems as a method for the rejection of waste heat to the atmosphere. These systems basically consist of cooling towers in association with a heat-exchanger network. Cooling water is used in the heat-exchangers to remove the heat from the process and reject it through evaporation in a cooling tower.

The heat-exchanger networks are conventionally designed with the coolers in a parallel configuration. Since it is not possible to reuse cooling water in this configuration, it usually requires not only substantial amount of re-circulating water, but also large cooling towers. Some researches have been made to apply process integration techniques using a series-parallel configuration. Wang and Smith (1994) introduced a methodology to target the maximum cooling water reuse based on pinch analysis, reducing the cooling water requirement through heat-exchangers in series-parallel arrangement. Later, also applying pinch analysis, Kim and Smith (2001) studied a method to improve the cooling towers capacity in de-bottlenecking situations. Recent studies have used mathematical programming to achieve optimum designs of cooling water networks (Panjeshahi et al. (2009), Gololo and Majozi (2012)).

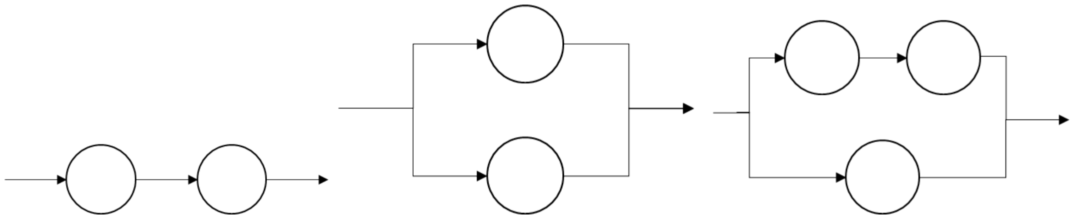


Figure 1 – Heat-exchangers arrangement types

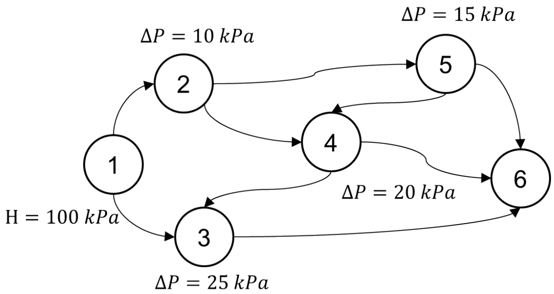
Although cooling water reuse can reduce both cooling water and cooling tower requirements, the pressure drop in the heat-exchanger network can significantly increase when compared to the parallel design. Some researches have used a methodology consisted in mixed-integer non-linear programming (MINLP) to evaluate the pressure drop in cooler networks (Kim and Smith (2003), Gololo and Majozi (2012)). Besides using optimization software, this method requires linearisation and Reformulation-Relaxation techniques in order to get starting points for solving the MINLP model (Gololo and Majozi, 2012).

This paper introduces a simple methodology based on the Graph Theory in order to evaluate the pressure drop in a heat-exchanger network. From the network adjacency matrix and pressure drop in each equipment, it presents how to apply the topological sorting and critical path algorithms to evaluate the pressure drop in a network represented by graphs. In addiction, it does not require optimization software and can be implemented in most programming language, such as C, Fortran or VBA.

2. METHODS

Since pressure drop is an intensive property, it can only be calculated for a heat-exchanger network when its configuration is fixed (Kim and Smith, 2003). There are basically two ways to arrange two heat-exchangers: in series (Figure 1a) or in parallel (Figure 1b). If the units are connected in series, the pressure drop of a flow is calculated as the summation of the pressure drop in the two equipments. For a parallel arrangement, the pressure drop is equivalent to the maximum value of the two. With more than two heat-exchangers, it is possible to arrange the equipments in series-parallel configuration, as illustrated in Figure 1c.

For conventional cooling-water networks, the heat-exchangers are arranged in parallel configuration and, hence, its pressure drop is easily calculated after evaluating the pressure drop for each equipment. However, for networks with cooling water reuse, it is possible to arrange some heat exchangers in series-parallel configurations and its pressure drop can become more complex to be evaluated. In this case, a graph representation



(a) Digraph Representation

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \quad P = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{pmatrix} 0 \\ 10 \\ 25 \\ 20 \\ 15 \\ 0 \end{pmatrix}$$

(b) Adjacency matrix and pressure drop array

Figure 2 – Heat-exchanger network example

comes as a simple and efficient tool to solve and analyse such a problem.

2.1. Graph representation

Graphs are very versatile models which can be used to analyse a wide range of practical problems in numerous areas, such as engineering, physical and biological sciences (Deo, 1974). They consist of circles (or dots) and connections which can have some physical or conceptual interpretations (Gross and Yellen, 2005). If the connections is directed, the direction is indicated by an arrow and the graph is called directed graph or digraph.

In order to evaluate the pressure drop in a heat-exchanger network, the problem can be represented by a digraph, where the circles and the arrows symbolise the equipments and the flow direction, respectively. Mathematically, the graph can be represented by an adjacency matrix, in which the entry $a_{i,j} = 1$ if there is an arrow from vertex i to vertex j and $a_{i,j} = 0$ if otherwise. Figure 2 illustrates a digraph of a heat-exchanger network (Figure 2a) and its respective adjacency matrix and pressure drop array (Figure 2b).

2.2. Topological Sorting Algorithm

According to Yunus and Cimbala (2006), since pressure is a point function, the pressure drop between two points must be the same independently of the path taken in a piping network. The same principle can be used for a heat-exchanger network. In a streamline of a closed-system, the energy head in a certain point can only be calculated taking into account other point whose energy head and head losses between these two points are well-known. Assuming water as an incompressible fluid and that there is no potential and kinetic energy variation, the energy head can also be simplified as the pressure in a given point.

The topological sorting algorithm is useful for defining the order the pressure must be evaluated in a network. It is defined as a permutation p of the vertices of a graph such that an edge (i, j) implies that i appears before j in p. In other words, assuming the vertices represent tasks and the arrows represent constraints that one task must be performed before another, the algorithm gives the sequence to perform all the tasks.

A limitation of this algorithm is that it can only be used for directed acyclic graphs because no vertex in a directed cycle can take precedence over all the rest (Pemmaraju and Skiena, 2003). However, a cycle in a heat-exchanger networks would only be possible if a pump is inserted into the network to recycle part of cooling water. As long as the aim of this methodology is to evaluate a pressure drop in a heat-exchanger network, it is not valid for networks with cooling water recycling pumps.

2.3. Critical Path Algorithm

The critical path (or longest path) algorithm is commonly applied for scheduling a set of project activities (PM, 2013). In this context, it calculates the longest path of planned activities, determining the shortest time possible to complete a project. Furthermore, it indicates the activities which are "critical" (i.e., makes the project longer if delayed) and "total float" (i.e., does not make the project longer if delayed).

Besides the management scheduling applications, this algorithm can be very useful for determining the pressure drop in networks as well. The tasks durations follow the same principle described for pressure drop in the beginning of Section 2. If two tasks can be performed at the same time (i.e., in parallel), the required time to accomplish both tasks is the longest task duration. In case a task must be done before other (i.e., in series), it is required the summation of the tasks duration to complete both ones. By analogy, applying the critical path algorithm for a heat-exchanger network, it will calculate the pressure drop for the network, instead of the project duration.

This algorithm starts calculating the earliest start time for each task according to Equation 1 (Zhao and Tseng, 2003). This equation indicates that the earliest start time ES of an activity j is the maximum value of its predecessors ES_i added to its respectively duration time D_i.

$$ES_j = \max\{ES_i + D_i \| \; i \in P_j\} \tag{1}$$

Assigning the zero start value for the first activity, the earliest start time values are calculated successively. As soon as the last activity is calculated, a latest start time (LS) variable is created and it receives the same value of its calculated earliest start time. Then, a backward pass method is done following Equation 2 (Zhao and Tseng, 2003). This equation indicates that the latest start time value of a predecessor i is equal to the minimum value of its successors LS_j minus their respectively duration time D_i.

$$LS_i = \min\{LS_j - D_i \| \ j \in S_i\} \tag{2}$$

After calculating the ES and LS for every activity, the critical (or longest) path is determined as the path which contains activities with the same value for ES and LS (Vukmirovic et al., 2012). The project duration corresponds to their maximum value, i.e., the value of the final activity for either ES or LS.

2.4. Pressure drop application

For pressure drop evaluation, instead of managing duration of an activity, the critical path calculation deals with pressure drop. This way, the variables $ES(i)$, $LS(i)$ and $D(i)$ are not appropriate for this purpose. As long as the difference between the $ES(i)$ and $LS(i)$ represents a slack time that a task can have without delaying the project schedule, by analogy, it can be represented by a slack pressure that a heat-exchanger can receive in its inlet. Then, for pressure analysis, the $ES(i)$ and $LS(i)$ are more appropriate to be named as $P_{\text{in}}^{min}(i)$ and $P_{\text{in}}^{\max}(i)$ respectively. They represent the minimum and maximum pressure that a heat-exchanger i can receive in its inlet respectively. If both values are equal, it means that the heat-exchanger i has a inlet pressure well established and is critical for the whole network. Furthermore, the array $P_{\text{drop}}(i)$ contains the pressure drop for each heat-exchanger and is used rather than the duration array $D(i)$.

From an adjacency matrix of a heat-exchanger network, both algorithm must be applied to evaluate the pressure drop in a network. In an adjacency matrix, it is important to note that the row index represents the predecessor vertex and the column, the successor vertex. Thus, the entry $a_{i,j} = 1$ means that there is a connection from the vertex i (predecessor) to the vertex j (successor). This definition is very important during the critical path algorithm, since the minimum inlet pressure and maximum inlet pressure calculation involves the relationship between predecessor and successors vertices.

Before applying the critical path algorithm from a adjacency matrix, it is suitable to sort the rows and columns, i.e., the predecessors and successors, into the topological order. During the pressure drop evaluation, a successor vertex must be only considered after every predecessor vertices. Thus, both rows and columns of a adjacency matrix must be permuted to follow the topological order and a redefined adjacency matrix A^* is applied in the critical path algorithm.

After the adjacency matrix has been redefined, the critical path algorithm can be implemented in order to determine the critical path and the pressure drop of the heat-exchanger network. For this, two arrays of n vertices are created, the $P_{\text{in}}^{\max}(n)$ and the $P_{\text{in}}^{min}(n)$.

If a pump is considered in the source node, the initial value for $P_{\text{in}}^{\max}(i)$ must be equivalent to the pressure head that the pump can deliver to the network. This procedure must be taken since the pressure head is the maximum possible pressure in every heat-exchangers before considering their pressure drops.

The algorithm starts from the first row of the redefined matrix A^* until the last one, calculating the maximum inlet pressure for each vertex j ($P_{\text{in}}^{\max}(j)$) according to Equation 3. This equation calculates the maximum inlet pressure of a vertex j as the minimum of the subtraction between the maximum inlet pressure of its predecessor vertices i ($P_{\text{in}}^{\max}(i)$) and their respectively pressure drop ($P_{\text{drop}}^{*}(i)$).

$$P_{\text{in}}^{\max}(j) = \min\{P_{\text{in}}^{\max}(i) - a(i,j) \times P_{\text{drop}}^{*}(i) \| \; i \in P_{\text{in}}^{\max}(i)\} \quad (3)$$

As soon as the last row is evaluated, the $P_{\text{in}}^{min}(n)$ receives the $P_{\text{in}}^{\max}(n)$ value and the minimum pressure inlet of the vertex i ($P_{\text{in}}^{min}(i)$) is evaluated by a backward pass method, according to Equation 4. In contrast to Equation 3, this equation calculates the minimum inlet pressure of a vertex i ($P_{\text{in}}^{min}(i)$) as the summation of the minimum inlet pressure of its successor vertices j ($P_{\text{in}}^{\min}(j)$) with its pressure drop value ($P_{\text{drop}}^{*}(i)$).

$$P_{\text{in}}^{\min}(i) = \max\{P_{\text{in}}^{\min}(j) + a(i,j) \times P_{\text{drop}}^{*}(i) \| \; i \in P_{\text{in}}^{\min}(i)\} \quad (4)$$

Finally, the critical path can be determined by the vertices whose $P_{\text{in}}^{\max}(n)$ and $P_{\text{in}}^{\min}(n)$ have the same value. In other words, if $P_{\text{in}}^{\max}(i)$ is equal to $P_{\text{in}}^{\min}(i)$, the inlet pressure of the heat exchanger in vertex i must be fixed at this value. If $P_{\text{in}}^{\max}(i)$ is different from $P_{\text{in}}^{\min}(i)$, the inlet pressure of the heat exchanger in the vertex i can vary between these two values. Furthermore, the pressure drop of the heat-exchanger network corresponds to the difference between the pressure in the source node (vertex 1) and in the sink node (vertex n).

3. CASE STUDY

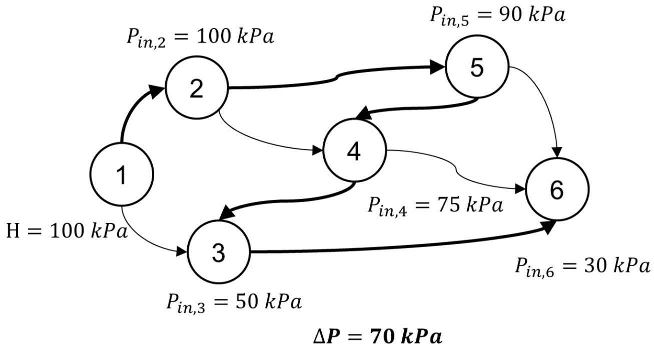
For illustration, the methodology is explained over a example of four heat-exchangers in a series-parallel configuration (Figure 2a). The network is already represented by a digraph, in which the vertex 1 (source node) represents a pump which drives the cooling water into the network with a energy head H; the vertex 6 represents the sink node or, in other words, the cooling water network outlet, and the other vertices represent the heat-exchangers. It is assumed that the pressure drop for each equipment was previously evaluated according to their project design specifications. The adjacency matrix for this digraph and the pressure drop array are shown in Figure 2b.

Applying the procedures described in the preceding Section 2 to this example, it is possible to determine the critical path and the pressure drop in this heat-exchanger network.

Although there can be more than one possible topological order in a network, applying the topological sorting algorithm for this network, only the sequence (1,2,5,4,3,6)

$$A^* = \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 4 \\ 3 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 5 & 4 & 3 & 6 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

(a) Redefined adjacency matrix



(b) Heat-exchangers results

Figure 3 – Case study

is acceptable. Analysing this sequence, it is important to note that the vertex 4, for example, would never come before the vertex 2 because this one is its predecessor. This way, during the pressure calculation, the pressure in the inlet of vertex 4 can only be evaluated after considering the vertex 1, 2 and 5 in this order.

Rearranging the rows and columns, following the topological order, the redefined adjacency matrix A^* is shown in Figure 3a. Applying Equation 3 and Equation 4 for the evaluation of maximum and minimum inlet pressure for each heat-exchanger results in both arrays $P_{in}^{min}(n)$ and $P_{in}^{max}(n)$. From them, the critical path and pressure drop of the heat-exchanger network are determined. The pressure drop is equal to the difference between the source and sink nodes pressure and the critical path follows the path where the $P_{in}^{min}(n)$ and $P_{in}^{max}(n)$ are equal.

4. CONCLUSION

In this paper, a methodology was described to evaluate the pressure drop in a heat-exchanger network. From a adjacency matrix and the pressure drop in each equipment, both pressure drop and critical path were determined through the topological sorting and critical path algorithms.

This methodology has demonstrated the advantage of evaluating the pressure drop of heat-exchanger networks in most programming language rather than using specific optimization programs. Furthermore, it can be used as an efficient tool to help in the evaluation of the impact of increased pressure drop for cooler networks with cooling water reuse.

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