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A GENERAL INFINITE HORIZON MPC FORMULATION FOR STABLE, INTEGRATING AND UNSTABLE SYSTEMS

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RESUMO – *This work shows a general state space model formulation for dealing with stable, integrating and unstable systems. Also, the development of the infinite horizon model predictive controller (IHMPC) based on this model structure is addressed. Finally, we provide simulation results of the application of the proposed controller to an example system with repeated stable, integrating and unstable modes.*

1. INTRODUCTION

Among several infinite horizon model predictive control (IHMPC) techniques, formulations that are suitable for industrial applications have been proposed for systems with stable (Odloak, 2014), integrating (Santoro and Odloak, 2012) and unstable poles (Martins and Odloak, 2016). These controllers are based on state space models that are obtained from partial fraction expansion of the analytical step response of the system. The construction of this type of model is not straightforward and requires a previous knowledge about the system poles in order to determine the model structure. In addition, the usage of different internal models often leads to distinct IHMPC formulations, which is not desirable if one aims to obtain a general industrial package.

In this brief study, we present a state space model formulation whose construction is straightforward and suitable for the IHMPC implementation (Sencio and Odloak, 2018). We also provide the formulation of the constrained IHMPC, which was applied to an simulation example system with stable, integrating and unstable poles with double multiplicity.

2. THE IHMPC FORMULATION

Consider a system that is completely decomposed into stable, integrating and unstable subsystems (Nagar and Singh, 2004) and whose state space model is given below, in which $\tilde{A}_{st} \in \mathbb{R}^{n_{st} \times n_{st}}$, $\tilde{A}_{in} \in \mathbb{R}^{n_{in} \times n_{in}}$, $\tilde{A}_{un} \in \mathbb{R}^{n_{un} \times n_{un}}$ are diagonal matrices (or block diagonal for repeated poles) whose main diagonals are formed by n_{st} stable, n_{in} integrating and n_{un} unstable poles of the system, respectively, and $\tilde{x}_{st}(k) \in \mathbb{R}^{n_{st}}$, $\tilde{x}_{in}(k) \in \mathbb{R}^{n_{in}}$, $\tilde{x}_{un}(k) \in \mathbb{R}^{n_{un}}$, $\tilde{B}_{st} \in \mathbb{R}^{n_{st} \times n_u}$, $\tilde{B}_{in} \in \mathbb{R}^{n_{in} \times n_u}$, $\tilde{B}_{un} \in \mathbb{R}^{n_{un} \times n_u}$, $\tilde{C}_{st} \in \mathbb{R}^{n_y \times n_{st}}$, $\tilde{C}_{in} \in \mathbb{R}^{n_y \times n_{in}}$, $\tilde{C}_{un} \in \mathbb{R}^{n_y \times n_{un}}$ and n_u and n_y are the number of inputs and outputs, respectively.

$$\begin{bmatrix} \tilde{x}_{st}(k+1) \\ \tilde{x}_{in}(k+1) \\ \tilde{x}_{un}(k+1) \end{bmatrix} = \begin{bmatrix} \tilde{A}_{st} & 0 & 0 \\ 0 & \tilde{A}_{in} & 0 \\ 0 & 0 & \tilde{A}_{un} \end{bmatrix} \begin{bmatrix} \tilde{x}_{st}(k) \\ \tilde{x}_{in}(k) \\ \tilde{x}_{un}(k) \end{bmatrix} + \begin{bmatrix} \tilde{B}_{st} \\ \tilde{B}_{in} \\ \tilde{B}_{un} \end{bmatrix} u(k) \tag{1}$$

$$y(k) = \begin{bmatrix} \tilde{C}_{st} & \tilde{C}_{in} & \tilde{C}_{un} \end{bmatrix} \begin{bmatrix} \tilde{x}_{st}(k) \\ \tilde{x}_{in}(k) \\ \tilde{x}_{un}(k) \end{bmatrix} \tag{2}$$

After some algebraic manipulations described in (Sencio and Odloak, 2018), the following state space model is obtained, in which $x_\Delta(k)$ corresponds to the integrating states related to the incremental form of inputs and $x_{st}(k)$, $x_{in}(k)$ and $x_{un}(k)$ stand for the states related to stable, integrating and unstable modes of the original system.

$$x(k+1) = Ax(k) + B\Delta u(k) \tag{3}$$

$$y(k) = Cx(k) \tag{4}$$

with

$$x(k) = \begin{bmatrix} x_\Delta(k) \\ x_{st}(k) \\ x_{in}(k) \\ x_{un}(k) \end{bmatrix}, \; A = \begin{bmatrix} I_{n_y} & 0 & C_{in}A_{in} & 0 \\ 0 & A_{st} & 0 & 0 \\ 0 & 0 & A_{in} & 0 \\ 0 & 0 & 0 & A_{un} \end{bmatrix}, \; B = \begin{bmatrix} B_\Delta \\ B_{st} \\ B_{in} \\ B_{un} \end{bmatrix}, \; C = \begin{bmatrix} I_{n_y} & C_{st} & 0 & C_{un} \end{bmatrix},$$

$$A_{st} = \tilde{A}_{st},\; A_{in} = \tilde{A}_{in},\; A_{un} = \tilde{A}_{un},\; C_{st} = \tilde{C}_{st},\; C_{in} = \tilde{C}_{in},\; C_{un} = \tilde{C}_{un},$$

$$B_\Delta = \tilde{C}_{st}\left(I - \tilde{A}_{st}\right)^{-1}\tilde{B}_{st} + \tilde{C}_{in}\tilde{B}_{in} + \tilde{C}_{un}\left(I - \tilde{A}_{un}\right)^{-1}\tilde{B}_{un},$$

$$B_{st} = -\left(I - \tilde{A}_{st}\right)^{-1}\tilde{A}_{st}\tilde{B}_{st},\; B_{in} = \tilde{B}_{in},\; B_{un} = -\left(I - \tilde{A}_{un}\right)^{-1}\tilde{A}_{un}\tilde{B}_{un}.$$

Note that the matrices to be inverted are non-singular since $\tilde{A}_{st}$ and $\tilde{A}_{un}$ are diagonal or block diagonal. Now, let us define the following quadratic cost function on which the IHMPC is based:

$$V(k) = \sum_{j=0}^{\infty} \|y(k+j|k) - y_{sp,k}\|_Q^2 + \sum_{j=0}^{\infty} \|\Delta u(k+j|k)\|_R^2 \tag{5}$$

where $Q \in \mathbb{R}^{n_y \times n_y}$ and $R \in \mathbb{R}^{n_u \times n_u}$ are positive definite weighting matrices, $y(k+j|k)$ is the output prediction at time step $k+j$ computed at time step k, $y_{sp,k} \in \mathbb{R}^{n_y}$ is the output setpoint at time step k, $\Delta u(k+j|k)$ is the vector of input moves with $\Delta u(k+j|k) = 0$ for $j \geq m$, in which m is the control horizon. Thus, in order to prevent the infinite summation in 5 to be unbounded, we must set to zero the states $x_{in}(k+m)$, $x_{un}(k+m)$ as well as the difference between $x_\Delta(k+m)$ and y_{sp} at the end of the control horizon. However, depending on the control horizon length, the terminal constraints may not be satisfied and have to be softened so that the control problem is always feasible. Then, the following constraint may be considered:

$$N\left(A^m x(k) + W\Delta u_k\right) - \gamma_k + \delta_k = 0 \tag{6}$$

in which $W = \begin{bmatrix} A^{m-1}B & \cdots & AB & B \end{bmatrix}$, $\Delta u_k = \begin{bmatrix} \Delta u(k|k)^T & \cdots & \Delta u(k+m-1|k)^T \end{bmatrix}^T$,
$N = \begin{bmatrix} I_{n_y} & 0 & 0 & 0 \\ 0 & 0 & I_{n_{in}} & 0 \\ 0 & 0 & 0 & I_{n_{un}} \end{bmatrix}$, $\gamma_k = \begin{bmatrix} y_{sp,k} \\ 0_{n_{in}} \\ 0_{n_{un}} \end{bmatrix}$, $\delta_k = \begin{bmatrix} \delta_{\Delta,k} \\ \delta_{in,k} \\ \delta_{un,k} \end{bmatrix}$, $\delta_{\Delta,k} \in \mathbb{R}^{n_y}$, $\delta_{in,k} \in \mathbb{R}^{n_{in}}$, $\delta_{un,k} \in \mathbb{R}^{n_{un}}$.

Finally, the controller can be defined as solution of the problem stated below:

Problem P1

$$\begin{aligned}\min_{\Delta u_k,\, \delta_k} \; V_1(k) = & \sum_{j=0}^{m} \left\| y(k+j|k) - y_{sp,k} + CA^{j-m}N^T\delta_k \right\|_Q^2 \\ & + \sum_{j=0}^{m-1} \|\Delta u(k+j|k)\|_R^2 + \|x_{st}(k+m|k)\|_{\bar{Q}}^2 + \|\delta_k\|_S^2 \end{aligned} \tag{7}$$

subject to 3, 4, 6 and

$$\Delta u_{min} \leq \Delta u(k+j|k) \leq \Delta u_{max}, \qquad j = 0, \ldots, m-1 \tag{8}$$

$$u_{min} \leq u(k-1) + \sum_{i=0}^{j} \Delta u(k+i|k) \leq u_{max}, \qquad j = 0, \ldots, m-1 \tag{9}$$

$$\Delta u(k+j|k) = 0, \qquad j \geq m \tag{10}$$

in which $\bar{Q} \in \mathbb{R}^{n_{st} \times n_{st}}$ is the solution to the following Lyapunov equation

$$\bar{Q} - A_{st}^T \bar{Q} A_{st} = A_{st}^T C_{st}^T Q C_{st} A_{st} \tag{11}$$

In Problem P1, the penalty matrix of slack variables is such that $S = diag\left(\begin{bmatrix} S_\Delta & S_{in} & S_{un} \end{bmatrix}\right)$ in which $S_\Delta \in \mathbb{R}^{n_y}$ is a positive semi-definite weighting matrix related to $\delta_{\Delta,k}$, while $S_{in} \in \mathbb{R}^{n_{in}}$ and $S_{un} \in \mathbb{R}^{n_{un}}$ are positive definite weighting matrices related to $\delta_{in,k}$ and $\delta_{un,k}$, respectively.

4. APPLICATION EXAMPLE

The proposed controller was applied to the following system:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{0.5(s+0.7)}{s^2(s+0.6)} & \frac{1}{s^2(s+0.6)} \\ \frac{1}{(s+0.5)^2(s-0.2)^2} & \frac{s-0.3}{(s+0.5)^2(s-0.2)^2} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} \tag{12}$$

with $u_{max} = [1\ 1]^T$, $u_{min} = [-1\ -1]^T$, $\Delta u_{max} = [1\ 1]^T$ and $\Delta u_{min} = -\Delta u_{max}$.
The controller tuning parameters are $m = 10$, $Q = diag([1\ 1])$, $R = diag([1\ 1]) \times 10^{-1}$, $S_\Delta = diag([1\ 1]) \times 10^3$ and $S_{in} = S_{un} = diag([1\ 1]) \times 10^5$. Figure 1 depicts the response of controlled and manipulated variables after setpoint changes. As observed, the manipulated variables briefly reached their limits after setpoint changes, which was properly dealt by the controller while the outputs were driven toward their desired values. Also, Figure 2 shows the non-increasing behavior of the controller cost function.

5. CONCLUSION

In this work, we proposed a state space model suitable for the IHMPC implementation as well as the controller formulation for systems with stable, integrating and unstable poles. Also, a simulation example showed the controller application to a system with pole multiplicity. The proposed state space model can be easily extended to consider time delay systems, while the controller may be adapted to deal with output control zones and input targets. Finally, a robust IHMPC with multi-model uncertainty will be addressed in future research.

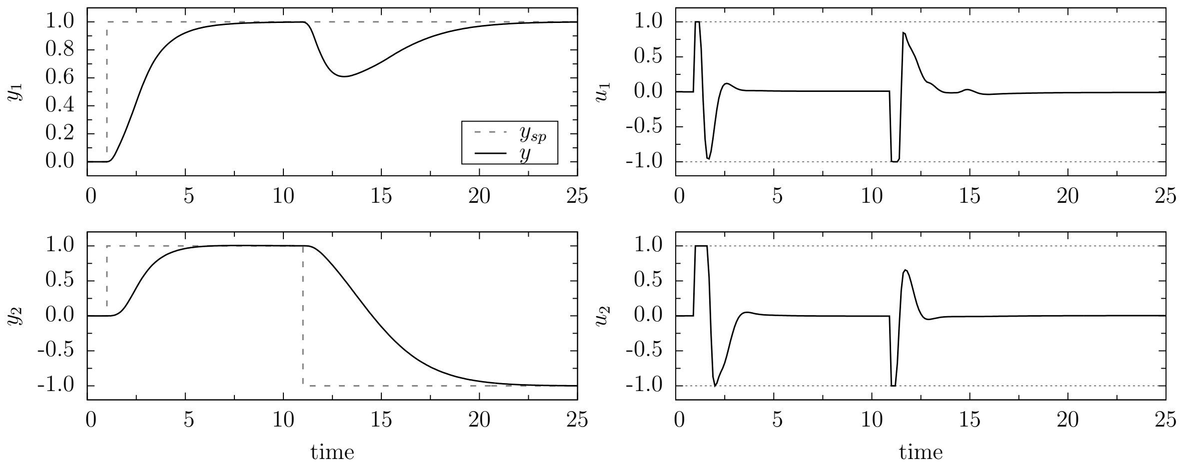

Figure 1: System response after setpoint changes.

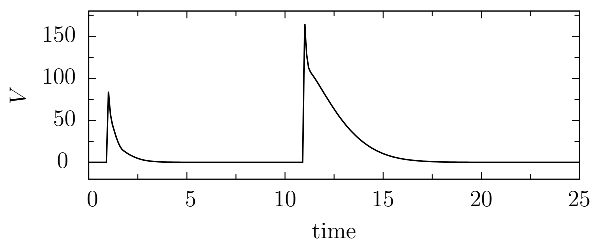

Figure 2: Control cost function.

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