

Optimizing Exhibition Spaces

A Multi-Objective Approach

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Nowadays, there is a widespread awareness towards environmental issues. This is already visible in architecture by the increasing number of analysis tools that evaluate different performance criteria. However, the application of these tools is usually restricted to the final design stages, conditioning the implementation of design changes. Performance-Based Design (PBD) is an approach that addresses this limitation. Through PBD, architects integrate analysis tools since early design stages to make informed decisions regarding the performance of their designs. Since the success of PBD highly depends on the number of evaluations that can be performed, these approaches usually end up benefiting from Parametric Models (PMs), which facilitate the generation of a wide range of design variations, by simply changing the values of the parameters. Furthermore, in order to more efficiently achieve a PBD approach, architects can take advantage of the combination between PMs, analysis tools, and optimization processes. In this paper, we explore this combination to optimize an exhibition space regarding its daylight performance and the material cost of the new elements intended for that space.

Keywords: Environmental Design, Algorithmic Design and Analysis, Performance-Based Design, Multi-Objective Optimization, Daylight Optimization

INTRODUCTION

Nowadays, more than ever, there is an increasing awareness regarding the impact of human activities on the planet. Consequentially, the interest in sustainability is also raising. In the architectural field, this interest is visible in the paradigm shift towards a more environmentally-friendly design process based on building performance (Kolarevic 2005). To support this paradigm shift, new and more accurate tools

have been developed to help architects evaluate different performance aspects and, thus, making more informed decisions regarding their designs (Oxman and Oxman 2014).

Nevertheless, architects usually do not take advantage of the full potential of these tools, as they only use them at final design stages, where the building shape is already defined and design changes are more difficult and tedious to do (Méndez Echenagu-

cia et al. 2015; Anderson 2014). This results in either small-scale design changes, which have a low impact in the overall design's performance, frequently resulting in a building that will rely more on active systems post-construction, or larger-scale ones, which are more difficult and time-consuming to implement and may possibly alter the architect's design intent. Therefore, to successfully create buildings with better environmental performance, architects must integrate performance-oriented studies in early design stages, when space, geometry, and proportion are still being explored (Anderson 2014).

Performance-Based Design (PBD) is an approach to design that addresses this problem. In PBD, architects consider performance as a guiding design principle and, thus, use analysis tools from an early design stage. As a result, decisions regarding building performance are more informed, ensuring that the final building design is an improved version of the architect's initial design intent.

In order to use PDB, it is necessary to evaluate alternative designs. To help generate these designs, a recent design approach was proposed: Algorithmic Design (AD), which can be defined as the formulation of designs through an algorithmic description. AD allows architects to build a program that generates the model, instead of manipulating the model's geometry directly in a modeling tool. This means that through the use of AD, architects have the possibility to create Parametric Models (PMs) by defining the relations between the different elements of their models and assigning parameters to it. This facilitates the generation of different design variations by simply changing the values given to the parameters (Woodbury 2010). Moreover, as the success of PBD highly depends on the number of design evaluations performed, the use of PMs becomes quite relevant in this context, since it allows for the easy generation of a wide range of design variations to be then evaluated in terms of their performance.

Still, despite their availability, existing analysis tools usually require a specific analytical model, i.e., a simplification of the building's model containing only

the information required by the analysis tool. Unfortunately, the process of transferring from a CAD or BIM model to an analytical one is not trivial. Furthermore, more often than not, portability between tools cannot be achieved, forcing the architect to rebuild the model in a format suitable to the analysis tool being used (Castelo Branco and Leitão 2017). Moreover, even in the cases where portability can be achieved, either information losses occur during the conversion process or the analysis tool cannot deal with the complexity of the model's geometry (Moon et al. 2011). To overcome these limitations, we can follow a design strategy that integrates AD tools with analysis tools, where the analytical model is generated without requiring additional work from the architect.

ALGORITHMIC DESIGN AND ANALYSIS

Algorithmic Design and Analysis (ADA) (Aguar et al. 2017) is an approach that takes advantage of the AD paradigm to automate the generation of analytical models. As a result, when a change is made in the algorithmic description of the design, the corresponding analytical models are updated concurrently.

Since this approach supports the automatic generation of analytical models in the format required by the respective analysis tool, it minimizes the information losses that typically occur during the process of converting models between CAD/BIM and analysis tools. In addition, the resulting analytical models contain only the elements and details that are necessary for the simulation tool. For example, in the case of a lighting analysis using RADIANCE (Ward 1994), a wall in the generic model is simply represented by a set of surfaces.

Moreover, the ADA approach also simplifies and speeds up the interaction between the architect and the analysis tool, encouraging architects to understand the impacts of their design choices on the building's performance and, accordingly, implement performance-oriented changes early in the design process.

Finally, ADA allows the architect to define the

analysis features directly in the algorithmic description, for example, specifying the placement of the light sensors in a lighting simulation, hence automating and, therefore, facilitating the execution of multiple analysis (Aguai et al. 2017). Having both the generation and the evaluation process automated unlocks the possibility to also implement automated optimization processes. To this end, a wide range of optimization algorithms can be integrated into the ADA approach to automatically seek for optimal design solutions (Belém and Leitão 2018).

MULTI-OBJECTIVE OPTIMIZATION

In the architectural context, architects rarely aim at improving a single performance aspect (or objective) of their designs. On the contrary, they typically ought to address multiple objectives simultaneously, e.g., lighting comfort vs thermal comfort or energy consumption vs thermal comfort, among others (Khazaii 2016; Nguyen et al. 2014). In this view, one of the main goals of applying Multi-Objective Optimization (MOO) process in a PBD approach is to find a way to reconcile all the existing objectives in a creative and effective way (Kolarevic 2005).

However, problems involving more than one objective are difficult to optimize, particularly when the objectives conflict with each other. Considering this, a possible strategy is to simplify the multi-objective problem by combining the different objectives into a single weighted function and, then, solving the optimization problem as being single-objective (Nguyen et al. 2014). Another possible solution is to use a Pareto Optimization approach. Instead of producing a single optimal solution, the Pareto Optimization approach produces a set of optimal solutions, in which it is impossible to improve one of the objectives without worsening the others. In these cases, a trade-off between the performance goals must be made, being the final decision in the hands of the architect, the one responsible for choosing the solution that best suits his design intents.

In Pareto Optimization, solutions can be classified as dominated or non-dominated. The domi-

nated solutions are the non-optimal solutions, from which it is possible to improve one objective without worsening the others, i.e., there is always a better solution that dominates it. Conversely, non-dominated solutions correspond to the solutions from which it is impossible to improve one objective without deteriorating others (Khazaii 2016; Wortmann 2017). The set of all non-dominated solutions is known as the Pareto Front. Depending on the number of optimization objectives to evaluate, this front is represented differently. As an example, in a bi-objective optimization problem the Pareto Front often corresponds to a line, whereas in a tri-objective problem, it is usually represented by a surface. Unfortunately, when we have more than three objectives, it becomes very difficult to represent the Pareto Front.

In figure 1, we can see three different stages of a MOO problem plot involving two objectives. One intended to be minimized, which corresponds to the costs and is represented in the X-axis, and other intended to be maximized, which corresponds to the daylight conditions and is represented in the Y-axis. Figures 1.a and 1.b illustrate the optimal solutions for each objective. However due to the fact that the optimal solution for the cost is the worst for the daylight performance, and vice versa, the result of a Pareto-based approach is a set of optimal solutions representing the trade-offs between the two objectives (figure 1.c).

METHODOLOGY

In this section, we propose a PBD methodology, that combines the ADA approach with an optimization algorithm.

In order to optimize a design, it is necessary to generate different design variations to be then evaluated regarding their performance. To this end, the ADA approach can be used to automatically generate and evaluate a wide range of design solutions by simply changing the values given to the parameters. Additionally, we can further benefit from the ADA approach by combining it with optimization algorithms in the search for optimal solutions. The latter attempt

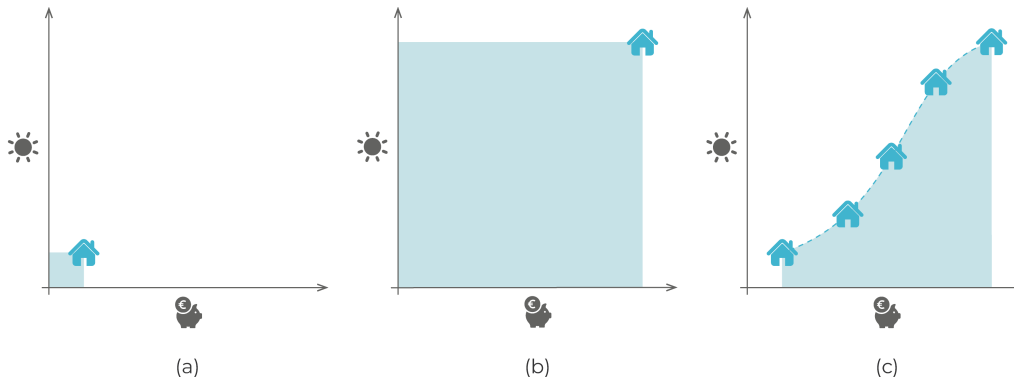


Figure 1
MOO problem with two objectives, where (a) represents the best solution for the cost, (b) represents the best solution for the daylight performance, (c) illustrates trade-offs between the two objectives, resulting in a set of optimal solutions.

to find optimal design solutions through the suggestion of values for the design's parameters.

Figure 2 represents the proposed methodology, which is specially tailored for addressing MOO problems.

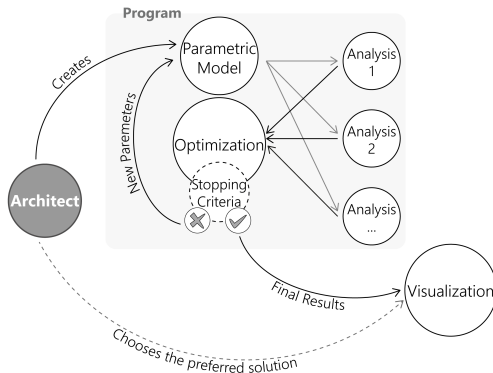


Figure 2
Proposed workflow.

After creating the PM, the architect defines the design space he is comfortable with and establishes his goals for the optimization process, i.e., the performance objectives to be evaluated. Then, he chooses a MOO algorithm. In general, these algorithms start by selecting random solutions from the design space to be evaluated. Then, based on the collected information about the problem, they try to improve the results at each design iteration, continuing this pro-

cess until the stopping criteria defined by the architect is satisfied. In the end, the solutions produced by the optimization process can be visualized in either a spreadsheet or a graph depending on his preferences. Furthermore, the architect can visualize the optimal solutions, from which he then chooses the one that best suits his goals. In a MOO problem, the final decision is always the architect's responsibility.

CASE STUDY

To evaluate the proposed methodology, we developed a real case study: the Black Pavilion at Pimenta Palace, in Lisbon.

The space of the Black Pavilion upon which we focus is destined to receive temporary art exhibitions and has a rectangular shape, being its only current daylight source a glazed curtain-wall, occupying half of the south façade and the entire east façade. The architects' intention for this space was to place a rectangular skylight in the opposite side of the curtain-wall, as a way to balance the daylight entering the space, while taking into account its material costs. To evaluate the daylight performance, we used the lighting analysis tool RADIANCE. Regarding the material cost, it was calculated through an analytical function defined by us, that considers the area of the skylight elements and their material cost per m^2 .

In order to apply the proposed methodology, we first needed to have a PM of the project. Given that

the architects had only provided us with a 3D model of the pavilion in Revit, our first step was to create the corresponding PM using an AD approach. Although all the building's elements were designed in a parametric way, in the end, only the skylight was considered parametric, since it is the one whose size affects the building daylight performance. Moreover, considering that we are dealing with a building intended for art exhibitions, its interior space should have diffused sunlight, instead of direct sunlight, because diffused light is softer and does not cast harsh shadows. To this end, we decided to use a translucent material for both the curtain-walls and the skylight. However, the building industry does not have standardized metrics for this kind of materials. To overcome this problem, we defined a function for translucent materials (Jacobs 2014). This function creates a definition for these materials based only in the total transmission (diffuse and specular), assuming that the material is a perfect Lambertian diffuser, i.e., the specular transmittance is 0.

Daylight Requirements

To evaluate daylight performance, we considered the Spatial Useful Daylight Illuminance (sUDI) metric, which measures the percentage of area between a defined range of illuminances, during at least 50% of the annual occupied time (Nabil and Mardaljevic 2005; Santos et al. 2018). However, given that we were working on an exhibition space, which requires particular light requirements, instead of using the predefined sUDI ranges, i.e., from 300 lux to 3000 lux, we decided to adopt a range that was more appropriate to that purpose.

According to CIE (2014), and considering the material classification for our space as low sensitivity, it should ideally have a constant light of 200 lux. However, it is almost impossible to guarantee these values by only using natural light, reason why constraining the sUDI to a very small range is not realistic. To overcome this problem, we decided to delimit the sUDI between 0 lux and 220 lux. This way, we guarantee that the space does not have too much daylight,

which could damage the art pieces. Finally, we stipulated our goal for the daylight optimization: to obtain the maximum sUDI possible, i.e., the maximum percentage of area between the range of 0 lux to 220 lux, during at least 50% of the annual occupied time.

Current Conditions

Working on a real case study helps us not only to evaluate our methodology, but it also proves how real projects can benefit from the ADA approach. Therefore, the first stage was to measure the original conditions of the exhibition room, i.e., without the skylights and considering the glazed curtain-walls. After specifying the design parameters that produced the design variation closest to the current building, we evaluated it using the RADIANCE analysis tool, thus receiving as a result the corresponding sUDI value, which was 70%. Even though this sUDI percentage represented an acceptable value according to the objectives defined for this space, we observed situations of intolerable glare. Based on these results, it was possible to conclude that there was room for improvements, which could have a substantial impact on the overall daylight performance of the exhibition room.

EVALUATION

We proposed a methodology for a MOO approach towards a PBD. This approach requires the definition of (1) a PM, (2) the variables (representing the design parameters) and the corresponding constraints (e.g., upper and lower bounds of the accepted variable's values), (3) the optimization objectives, and (4) the optimization algorithm.

Over the next sections, we describe the process that led to the definition of the variables constraints, a process known as Sensitivity Analysis (SA), the objectives and the relations between them, and the chosen optimization algorithm. We end up by presenting and discussing the results obtained.

Sensitivity Analysis

Prior to the application of the proposed methodology, we first decided to perform SAs to understand the impact of the different parameters on the daylight performance of our case study.

SA is a process through which the architect can measure the influence each parameter has in the overall building performance, based on the results of multiple simulations. This allows him to identify trends between the different variables and the simulation results, therefore, discovering unexpected outcomes regarding the influence of each variable. Despite not being a standard procedure in an architectural optimization process, SA can be an extremely valuable resource in helping designers understand *how* and *how much* the design parameters affect the objective function (Castillo et al. 2008).

By performing a SA prior to the MOO, we could define with more precision the constraints for each variable, which helped us decrease the time and number of evaluations needed to find the optimal (or near optimal) solutions. The variables considered in our case study were the width, length, and height of the skylight, as well as the translucent material applied to both the skylight and the curtain-wall. All the materials tested were translucent panels of different total transmissions.

In the next paragraphs, we present the variables and the corresponding constraints that we adopted for the MOO, based on the SAs results. Regarding the skylight height, we noticed that higher values had a negative impact on the sUDI values. For this reason, we fixed the skylight height at 1.5 m, which corresponded to the minimum possible value acceptable for this space. Concerning the width and length variables, we observed that skylights with bigger areas resulted in higher values of sUDI. Therefore, the ranges set for the width and length variables were from 1.5 m to 4 m and from 6.5 m to 17.5 m, correspondingly.

Finally, regarding the materials of both the skylight and curtain-wall, we tested four different translucent panels with the following total transmis-

sions: 15%, 25%, 45%, and 65%. Results demonstrated that the 25% Translucent Panel and the 45% Translucent Panel positively influenced the sUDI value of the solutions evaluated. Therefore, we decided to test these two materials during the MOO, as well as the one in between them, i.e., the 35% Translucent Panel.

Optimization Setup

We decided to evaluate our methodology using only two objectives: (1) maximize the sUDI, by considering the range between 0 lux and 220 lux, and (2) minimize the material cost of the skylight. From the results of the SA, we realized that these objectives were contradictory. This means that the least expensive skylights are those that produce the worst sUDI percentages, whereas the most expensive ones produce the best sUDI percentages. Given the existing conflict between the objectives, we expected to obtain a set of optimal solutions, i.e., the non-dominated solutions, instead of a single optimal solution.

One other important aspect of any optimization process is the optimization algorithm to use. In this work, we chose a MOO genetic algorithm named NSGA-II (Deb et al. 2002). This algorithm starts by generating an initial set of solutions, called population, from which, at each iteration of the algorithm, a new and improved population is created. The early populations are frequently more diverse, whereas the final ones are generally more similar to the fittest individuals. In our case, for one run of the algorithm, we specified the population size as 10 and the number of iterations as 20, generating a total of 200 candidate solutions for each run. We decided to run the algorithm three times to assure diversity in the obtained results. Nonetheless, in the context of a real project, where architects usually have to deal with very tight deadlines, they can opt to make a single run of the algorithm and increase the population size and/or the number of iterations.

Results and Discussion

At the end of the three runs, we combined all the results obtained and eliminated repeated solutions, i.e.,

the solutions whose variables had the same values. We obtained a total of 473 solutions being that 22 were non-dominated solutions. We decided to visualize these results on a scatter plot where we represented the Pareto front, as well as the dominated and non-dominated solutions. Through the visualization of the scatter plot, we were able to (1) quickly identify the best solutions found by the algorithm, (2) analyze the diversity of all the evaluated solutions, (3) measure the extent of the algorithm search, and, finally, (4) understand the convergence of the algorithm.

Moreover, in order to better understand how the skylights of different solutions affect the exhibition space, we produced two rendered images for three solutions along the Pareto front, one during the Summer solstice and the other during the Winter solstice, both at 12 pm. Figure 3 represents the correspondence between the Pareto front solutions and the rendered images.

As mentioned previously, we decided to use three translucent materials with different total transmissions. However, none of the non-dominated solutions found by the optimization algorithm used the 45% Translucent Panel and only three used the 35% Translucent Panel.

Regarding the other two variables, the range decided for the width (between 1.5 m and 4.0 m) proved to be adequate as the width of the non-dominated solutions varied between 1.5 m and 3.8 m. Nevertheless, concerning the length variable, the algorithm found non-dominated solutions only for values lower than 8.9 m and the maximum value set for the length range was 17.5 m. On the one hand, this may mean that there are no non-dominated solutions with a skylight length higher than 8.9 m, being the range given to the algorithm too extensive. On the other hand, this may denote that the optimization algorithm did not analyze enough solutions with a length higher than this one, therefore not finding any non-dominated solutions with a skylight length above 8.9 m. Due to the complex nature of an architectural MOO problem, it is almost impossible to know for sure which are the optimal solutions. The

only way to be certain that there are no additional non-dominated solutions than those found by NSGA-II in this situation, is to test a wider range of solutions with the same algorithm or with different algorithms, and then compare the results with those obtained here. In an ideal situation, the architect would initially test multiple optimization algorithms with a small sample of the design space and, then, choose for the main optimization process the one with the best performance (Belém and Leitão 2018).

Concerning the first non-dominated dot represented in the plot, it corresponds to two different solutions with the same cost and sUDI. These solutions correspond to the design variations that had the best cost (3 723.75 €) but the worse sUDI percentage (45%). Note that these solutions only differ in the material, i.e., one uses the 25% Translucent Panel, whereas the other uses the 35% Translucent Panel. Regarding the other variables, both solutions have a skylight with 1.5 x 6.5 m.

In what concerns the other extremity of the Pareto front, i.e., the solution with the best sUDI percentage (86%) and the worst material cost (9 304.70 €), it corresponds to a skylight of 3.8 x 8.9 m that uses the 25% Translucent Panel.

Besides evaluating the proposed methodology in a real case study, we also set out to improve the daylighting conditions of the exhibition space, which currently presents a sUDI value of 70%. This means that not all the non-dominated solutions found represent a solution with an improved daylight performance, being that their sUDI values ranged from 44% to 86%. On the one hand, it is important to note that the solution with the best sUDI value may not necessarily be the best solution, as it also represents the solution with the most expensive skylight. On the other hand, since the goal was to improve the daylight performance of the exhibition area, the architects would certainly not be interested in choosing a solution with a sUDI percentage lower or equal to 70%. In short, the architects must select the solution that appraises them the most within the set of solutions with an improved sUDI value, while taking

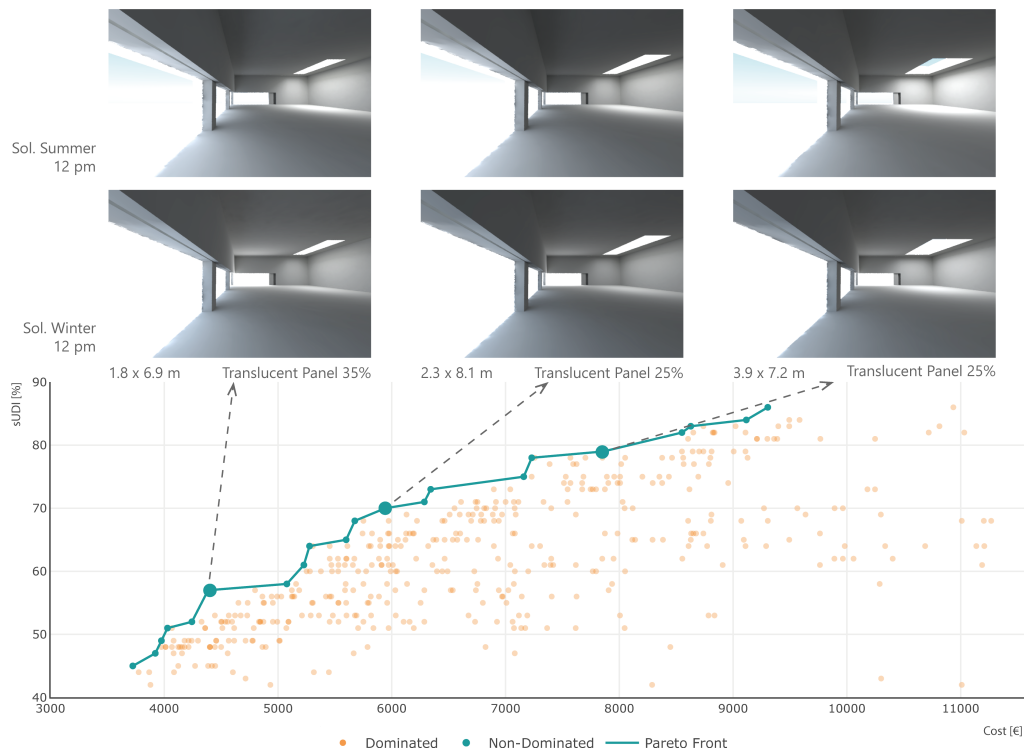


Figure 3
Render images of
the exhibition room
for different
solutions of the
Pareto front.

into account the trade-offs between daylight performance and cost.

CONCLUSIONS

In this paper, we focused our PBD research on a MOO problem combining both daylight performance and material cost. To this end, we proposed a PBD methodology that takes advantage of the ADA approach, combining it with optimization algorithms. Moreover, we also demonstrated how SA can provide valuable information about the relations between the different design parameters, thus promoting a more adequate definition of the problem's design space.

Given that MOO frequently unveils the trade-offs

that exist between the different objectives, the proposed methodology provides the architect with a set of optimal (or near optimal) solutions representing the optimal trade-offs, thus delegating to him the selection of the final solution.

Finally, the proposed methodology proved to be a powerful tool in helping architects make more informed decisions regarding different performance aspects, from the early design stages.

The research carried out in the course of this paper emphasized the importance of evaluating different building performance criteria during the architectural design process, as well as the usefulness of the feedback provided by the analysis tools to the decision-making process. It was also highlighted

that, in order to have a successful PBD approach, multiple evaluations of design variations must be done. Not only does this enable the collection of relevant information about the existing relations between the design parameters, but it also appraises their impacts on building performance, thus, reinforcing the usefulness of parametric modeling strategies for PBD approaches.

The use of the proposed methodology proved that architects can effortlessly evaluate different performance aspects according to the requirements of their projects. This is due to the automatic generation of analytical models provided by the ADA approach, from which we took advantage in our MOO approach. Moreover, in the end of each optimization process, the architect is presented with a list of all the evaluated solutions, from which it is then possible to extract all the optimal solutions. This allows architects to have a more in-depth look on the different trade-offs between the objectives and decide which solution best meets their requirements regarding aesthetical value and/or performance requirements.

In the future, we intend to test our methodology with different optimization algorithms. Considering the current variety of optimization algorithms and the new ones that are being proposed, and that different algorithms can perform better than others depending on the specific characteristics of each problem, it is therefore important to test different algorithms to identify those that best perform according to the optimization requirements (Belém and Leitão 2018). Similarly, it is also important to understand which algorithms deal best with single-objective optimization, which behave the best with MOO, or, even, which ones best support extensive design spaces.

Another interesting topic to address in our future research is the application of the methodology to problems with higher number of optimization objectives. In the context of this paper, we performed a MOO with two objectives. However, the MOO methodology we proposed is not restricted to only

two objectives and, in fact, it allows designers to add as many objectives as they need. This is an important advantage of the proposed methodology, since an architectural project usually has a wide range of conflicting requirements that need to be accomplished simultaneously. Nonetheless, it is noteworthy that when an optimization problem includes more than three objectives, not only the optimization process requires considerable computational resources, but also the visualization of the results becomes a very difficult task. Thus, we also plan to research visualization mechanism to improve upon these problems.

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