

Architecture by numbers

An interdisciplinary approach towards computational design and architectural geometry

Marco Hemmerling

Cologne University of Applied Sciences, Germany

marco.hemmerling@th-koeln.de

Abstract

Architecture has always relied on mathematics to achieve proportioned aesthetics, structural performance, and reasonable construction. Computational tools have now given architects the means to design and build spatial concepts that would have been inconceivable even ten years ago. Against this background the paper discusses an educational approach that focuses on the early integration of computational principles, regarding the definition of geometry as well as material and fabrication parameters to inform the architectural design. Three case studies illustrate the interdisciplinary approach, conceived and carried out jointly by the Department of Architecture and the Department of Mathematics.

Keywords: Curriculum; Architectural Geometry; Architecture and Mathematics; Computational Design and Fabrication.

Introduction

The broad application of computational design and construction technologies has widely changed the use and perception of computer software in architecture. Thus, we have seen a shift from mere drawing-tools towards mighty parametric design methods. These tools have allowed many architects to form the conception and de-sign of very complex architectural projects. Hence, many formally intricate buildings have been designed and built over the past years and many of these projects have undergone massive so-called post-rationalization-processes - i.e. methods of (mostly) intelligent geometrical simplification, like the costly triangulation of double-curved surfaces. From today's point of view, such processes, however extremely elaborate in themselves, appear to be a bit anachronistic. They seem like attempts of after-computerization of actually post-modernist design approaches.

However, Patrik Schumacher describes the validity of parametric design as an important movement within the history of architecture when he states: "Contemporary avant-garde architecture is addressing the demand for an increased level of articulated complexity by means of retooling its methods on the basis of parametric de-sign systems" (Schumacher, 2010). Indeed, much of the attention in the architectural discourse of recent years has been relegated to the field of aesthetics and high levels of interest in the generation and articulation of complex geometric configurations. Nevertheless, beyond geometry, complexity involves much more invisible layers of interaction as well as interdependencies and non-linear responses. Furthermore, complexity in architecture is often confused with complicated structures. In fact, complexity does not require complicated systems and intricate rules to unfold its potential. On the contrary, complications lead to multiplicative chains of unanticipated effects rather than generating a practical and robust system. In other words,

redundancy and overcompensation - as characteristics of a so-called performative system - can solitarily be achieved by focusing on the relational qualities and traceable interdependencies, thus, the conditions of a system.

Managing complexity, negotiating multiple aspects of a design problem and generating various solutions through computational processes enable the architect to-day to develop a relational and adaptable architecture of interdependencies. Bernard Tschumi's statement "Architecture is not about conditions of design, but about the design of conditions" (Tschumi, 1995) can be seen as a programmatic testimonial or this reference system of architectural design.

Architecture and Mathematics

Architecture has always relied on mathematics (Pottmann et al., 2007) to achieve proportioned aesthetics, structural performance, and reasonable construction. Computational tools have now given architects the means to design and build spatial concepts that would have been inconceivable even ten years ago. Against this background the academic project, "Architecture by Numbers" focused on the early integration of optimization parameters regarding structural performance, physical properties, and material specification as well as aspects of fabrication to inform the architectural design. The curriculum has been conceived and carried out jointly by the Department of Architecture and the Department of Mathematics at the Politecnico di Milano. The course set-up was based on the experience of previous collaborations between the two departments (Hemmerling et al., 2016). Starting from the historic context of Architecture and Mathematics (Vitruvius) and the theoretical background (both Architecture and Mathematics) presented at the beginning of the course the module was conceived as an interconnected cycle of Computation, Design and Fabrication. Hence, the

process was driven by non-linear, but alternating methods and divers application of tools. As a common ground for the operative part the software application Rhinoceros (3D-Modelling) was chosen with the extensions of Grasshopper (Visual Programming) and Python (Programming Language) in order to connect architectural design and mathematics in a manageable framework for the students.

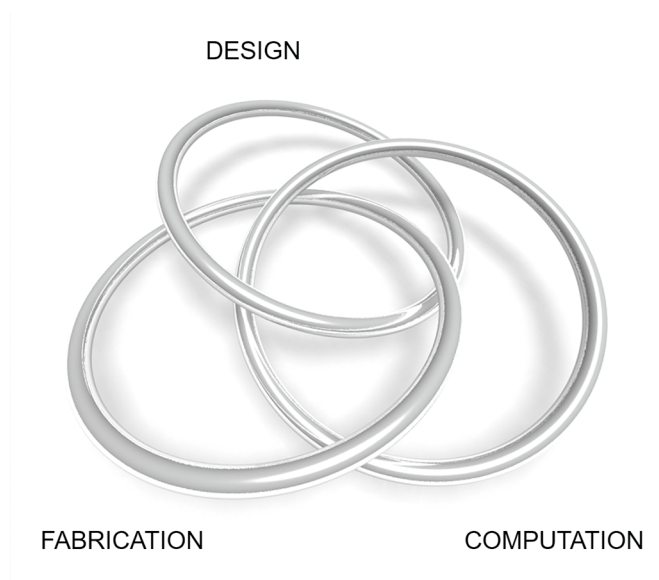


Figure 1: Conceptual diagram of the non-linear and interdisciplinary approach towards architectural education.

The research focused on an in-depth understanding of geometric principles and their mathematical definitions as a starting point for the development of individual architectural projects. Next to the interdisciplinary exchange between Architecture and Mathematics, the intercultural set-up of the design teams (originating from 12 different nationalities) proved to be both challenging and inspiring throughout the process.

The emphasis of the course was put on research-based design-strategies that aimed to unfold hidden complexities of rather simple geometric definitions. 3D-geometries are in first place mathematical objects and as such, they are a sequence of mathematical functions and relations used to describe a set of volumes and surfaces that constitute their separating boundaries. CAD applications are built using these mathematical concepts but generally, do not unveil them. In order to apply these principles in a comprehensive way, they have to be taught and understood by using the language of Mathematics. Moreover, the constraints that a geometrical shape must satisfy in order to be fabricated conveniently are described as well by mathematical equations. Hence, Mathematics plays a major role for the operability within the design and building process.

Taking these aspects into account, the three-folded teaching approach combined the definition and discussion of mathematical principles for the generation of spatial geometry and their conceptual, structural, and functional potential from

an architectural point of view as well as the potential and constraints for the fabrication of prototypes throughout the process. The considerations exposed above lead to the awareness of the opportunity for Architects to build a familiarity with the basic concepts of Computational Geometry, in particular with the representation and approximation of curves, surfaces, and volumes, and showed the necessity to build up a firm knowledge of the language appropriate for the discussion of such concepts.

The clear conception of a computation-process, whose rules lead to certain formal and structural consequences, is the necessary first step towards an architecture that is both structurally interesting and systematically coherent. Thus, in this seminar, the students attempted to develop strong and fresh architectural projects from fairly simple methods of parametrical definition of space-defining geometries. And while doing so a conceptual, historical, theoretical, and technical framework was built around the projects to take each design beyond the mere development of geometry and the application of computational tools.

Case Studies – (Un) folding Space

The following three projects are chosen as representative case studies for the interdisciplinary approach. The process was organized in a bottom-up strategy, starting with a general research about developable surfaces and foldable structures by the students.

Developable surfaces and folding strategies have been explored in architecture as a method to generate complex spatial and structural concepts from simple planar surfaces (Huffman, 1976 and Lang, 2011). The folding mechanism of a rigid material may seem as trivial as folding a sheet of paper. The striking elegance of models folded from paper appear more than just visually interesting. A simple crease along a line or a curve dictates a determined movement for both faces. The basic origami folds include valley and mountain folds, pleats, reverse folds, squash folds, and sinks. The number of basic folds is small, but they can be combined in a variety of ways to make intricate designs. Moreover, they inherit positive structural effects and allow for the creation of multiple pieces from one folded sheet, which may save material, fabrication time, and building costs. Hence, the ability to create a structural form from thin, flat materials have made folded structures an ideal candidate for lightweight deployable structures in architecture and engineering.

The form-finding process started with the research of basic folding principles, both in computational and physical modelling. The individual projects were developed in a bottom-up approach that allowed for the integration of findings throughout the design process. The projects developed from a basic understanding of the underlying mathematical principles towards the application of a computational design strategy that allowed for the generation of variations within the given geometric typology. Digital and physical models were developed in parallel to investigate and evaluate the properties, aesthetics, and structural performance of the different concepts. Against this background, the teaching

concept focused much more on the findings through-out the research process than on the formal results.

Dot & Line

The *Dot & Line* project started out with a research on translational and rotational surfaces. Therefore, a straight line was related to a central point by applying various sequences of rotational and translational operations. The resulting 3D models were examined against the constraint of developable surfaces in order to be able to produce an accordingly physical model of the resulting geometry from a flat sheet of paper. The finally chosen circular loop surface was not only a developable surface but also proved to be foldable without cutting an edge of the entire pattern. Hence, the kinetic property is inscribed in the continuity of the surface in such a way that it can adapt to different heights without changing its central diameter. In order to secure this feature in the computational parametric model, the Pythagoras theorem was applied to the rotation of the generating lines around the central point. After testing the principle with a series of paper models a final mock-up was CNC-milled from ACM-boards (aluminum composite material, diameter: 2,00m) as a proof of the concept.

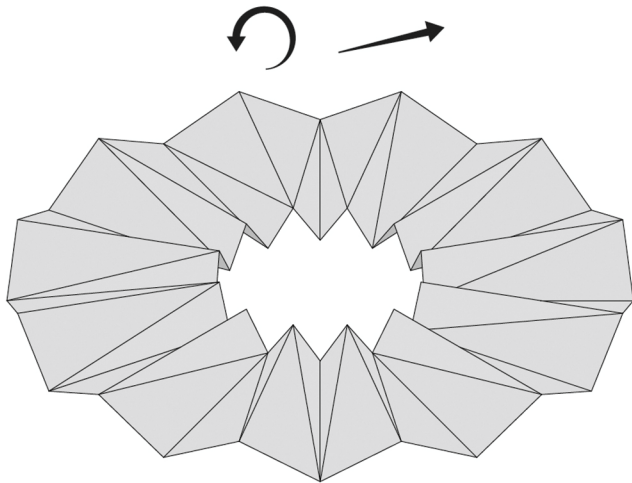


Figure 2: Dot & Line: Computational design model.

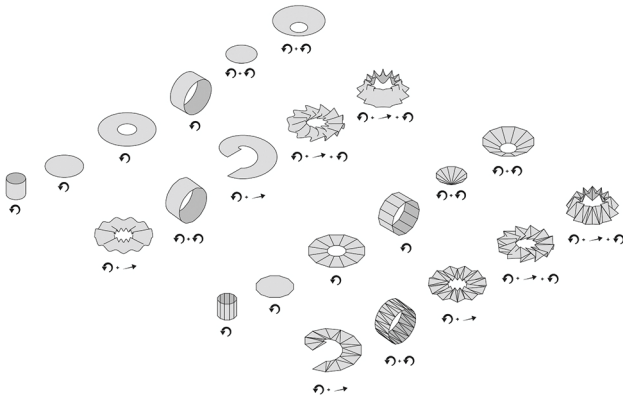


Figure 3: Dot & Line: Variations of the design principle.

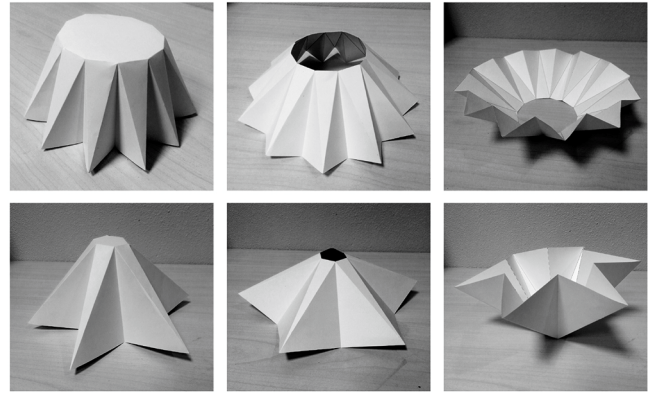


Figure 4: Dot & Line: Physical prototypes from cardboard.

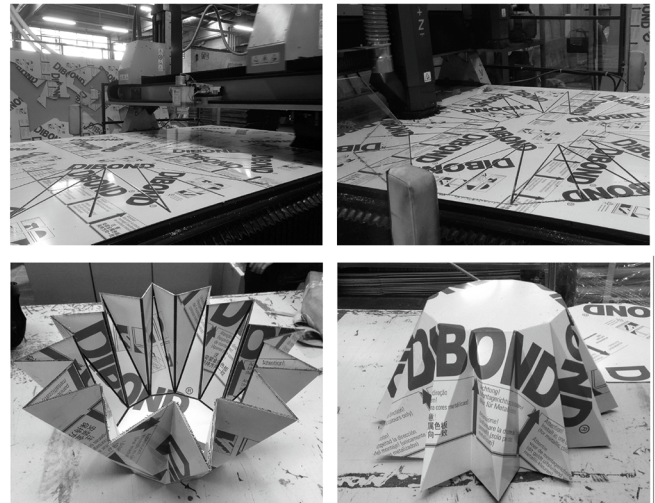


Figure 5: Dot & Line: Production of the physical prototype from ACM.

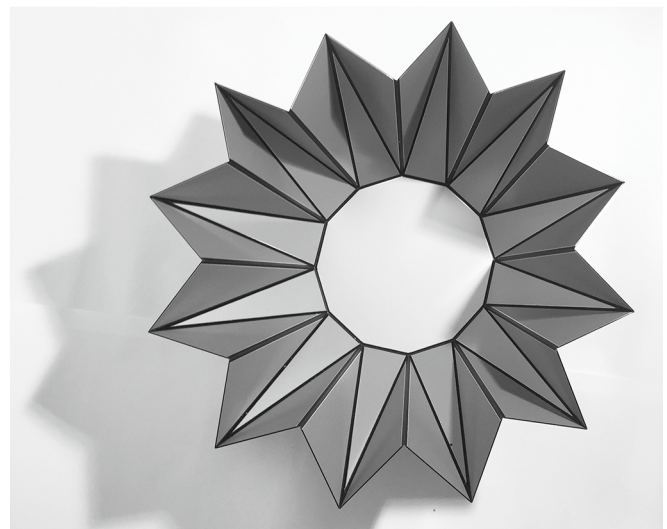


Figure 6: Dot & Line: Physical prototype from ACM.

Movement Trajectories

The interest in kinetic principles that can be found in folding was the driving force for the research of the project *Movement Trajectories*. Starting out again from a simple definition of the principle, the group developed a first computational model by defining a rotational movement around a fixed central point, which consequently resulted in a spherical surface. Gradually, the degrees of freedom and the number of points and axis's were increased in order to generate and control more complex movements. In a final transformation step, the folding control methods were adapted to simple mechanisms by defining two fixed base point, two points with determined trajectories and two points with trajectories defined by intersections with surfaces. As a result from this research, a series of simple mechanisms and systems with movable base points were computed, such as (semi)circular fold-able surfaces and umbrella structures.

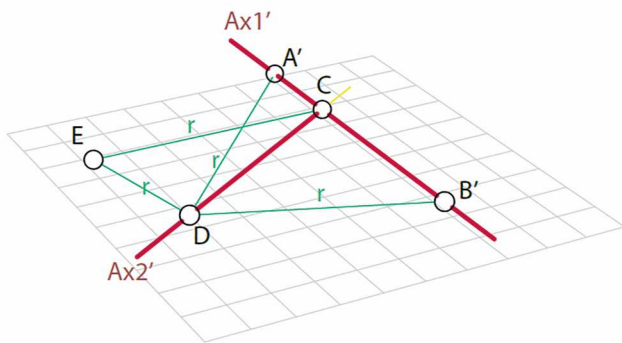


Figure 7: Movement Trajectories: Computational model for the movement of points A,B,C,D, E along the axis Ax1 and Ax2.

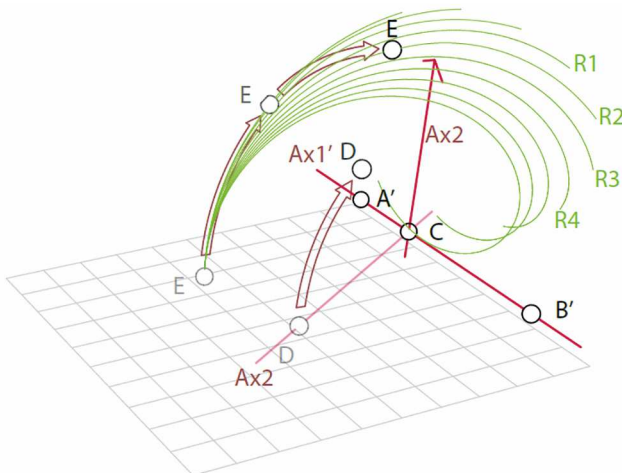


Figure 8: Movement Trajectories: Kinetic movement of points D and E along the axis Ax1 and Ax2.

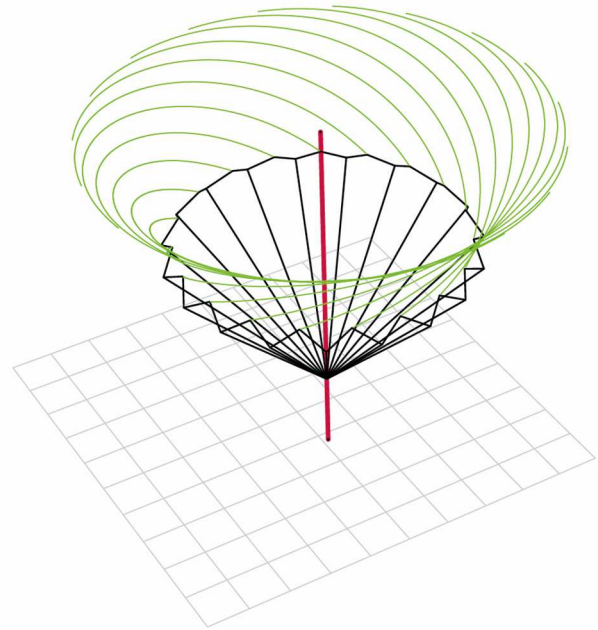


Figure 9: Movement Trajectories: Computational model of a kinetic, foldable umbrella structure following the principle introduced before.

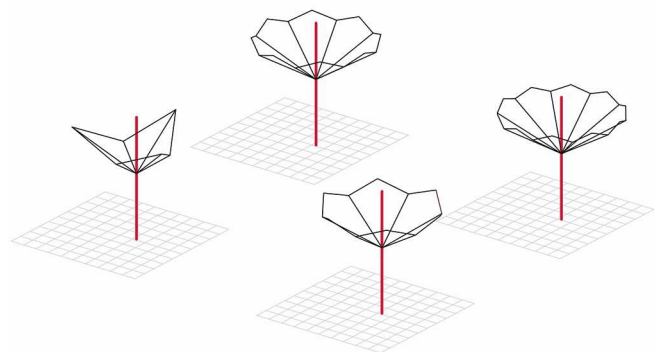


Figure 10: Movement Trajectories: Variation of folds for the umbrella structure by changing the resolution/number of elements.

Solid Transformations

A simple cube served as a starting point for the research *Solid Transformations*. In a first step, the computational design model was generated by dividing the edges of the cube by n points. The corresponding division points were then connected to generate a rotated square pattern on each face of the cube. The so defined squares were finally extruded and intersected to a unified solid by applying a Boolean operation. Based on this computational parametric model, a series of variation exploring the form generation principle was carried out. The research was concluded by unrolling the 3D surface to a 2D cutting pattern to produce a range of physical models in paper and finally from CNC-milled ACM boards in a bigger scale.

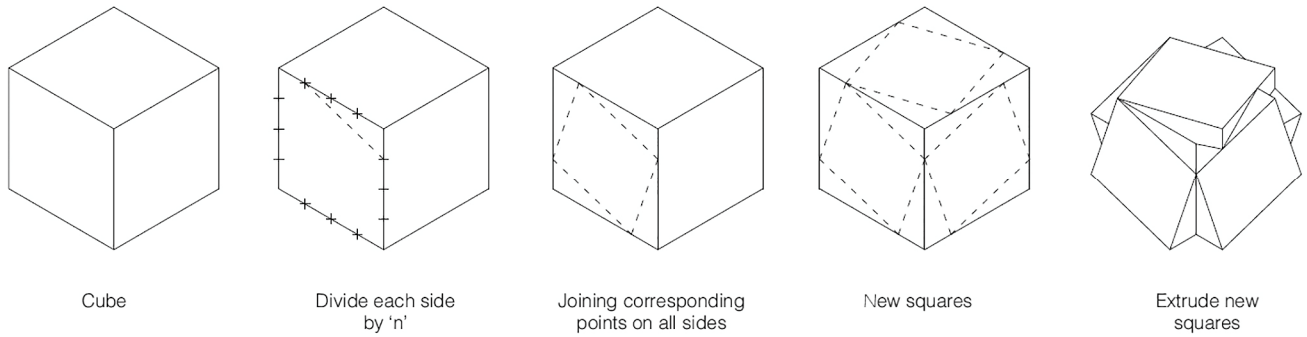


Figure 11: Solid Transformations: Geometric principle for the generation of the cube transformation.

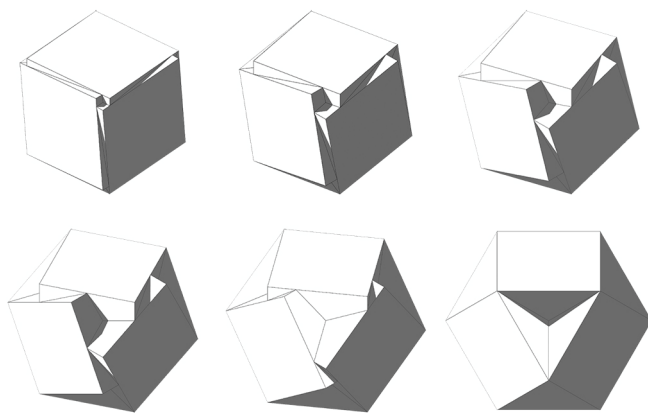


Figure 12: Solid Transformations: Variations of the cube geometry.

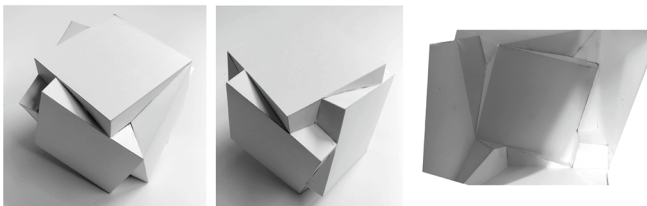


Figure 13: Solid Transformations: Physical prototypes from paper.



Figure 14: Solid Transformations: Assembly of the CNC-milled ACM model.

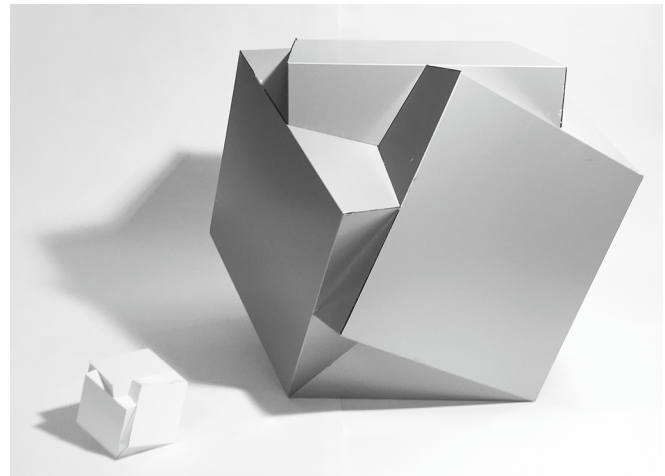


Figure 15: Solid Transformations: Physical models of the cube in paper and ACM.

Conclusion

The Master course connected complexity deliberately with a bottom-up design methodology to constitute an attainable learning base for the students. As a result, almost all of the projects reached a high level of complexity that started from a fairly simple analysis of a phenomenon or mathematical problem. In order to generate an accessibility to the chosen topic, the abstraction of the principle was an important first step (and a threshold for most of the teams as it appeared to be quite different from a standard design process). The definition of a mathematical model that represented the basic idea served as an essential starting point. Hence, the more profound the model was elaborated, the more potential it inherited for the further design process. Furthermore, the approach allowed for the successive development of a resilient system that is able to generate variations (as a base for design decision making) and to increase the level of complexity throughout the design process gradually.

The three case studies, described before, were conducted with different tool sets (*Dot & Line*: Rhinoceros+Grasshopper, *Movement Trajectories*: Rhinoceros+Grasshopper+Python, *Solid Transformations*: Rhinoceros) and achieved consequently different levels of information and complexity.

The most sophisticated computational project (*Movement Trajectories*) generated a complex geometric solution with a rich set-of tools, but didn't manage to integrate the fabrication and materialization within the design process in time. On the other hand the reduced computational tools of the *Solid Transformations* project allowed for a fairly easy translation into various physical prototypes.

Yet, the case studies also showed that the integration of parameters and requirements of functionality, efficiency, and aesthetics as well as structural performance shifts the focus from a purely formal design practice to an optimization process that relies on a systematically coherent approach. The understanding of the mathematical definition laid the solid foundation for the generation of a computational design concept and allowed for an accessible handling of complexity throughout the whole design to build process.

Acknowledgments

The interdisciplinary Master course "Architecture by numbers" was conducted at the Politecnico di Milano in 2016/17 by Prof. Marco Hemmerling (Architecture), Prof. Carlo de Falco (Mathematics) and Alessio Mazzucchi.

The case studies were carried out by the following students: Giancarlo Gastaldin, Diego Oberti, Kristiyan Topchiev, Dominika Bugajska, Krzysztof Zinger, Atapaka Mohana Krishna, Sara Hajifathaliha, Shabarish Sathyanarayan, Shreyaa Jaya.

References

- Hemmerling, M., Mazzucchi, A. (2016). Colonna Curva - A case study on curved folding for the production of architectural components, Proceedings of the 20th SIGraDI-Conference, Buenos Aires, Argentina.
- Huffman, D. A. (1976). Curvature and creases: a primer on paper. IEEE Trans. Computers C-25, 1010–1019.
- Lang, R.J. (2011). Origami Design Secrets: Mathematical Methods for an Ancient Art, Taylor & Francis Ltd.
- Pottmann, H., Asperl, A., Hofer, M., and Kilian, A. (2007). Architectural Geometry. Bentley Institute Press.
- Schumacher, P. (2010). The Autopoiesis of Architecture: A New Framework for Architecture, Volume 1, Wiley & Sons.
- Tschumi, B. (1995). Responding to the question of complexity, in: Complexity. Art, Architecture, Philosophy, Journal of Philosophy and the Visual Art, Andrew Benjamin (ed.), Nr. 6. London
- Vitruvius, (1st century B.C). De architectura libri decem
- www.rhinoceros3d.com
- www.grasshopper3d.com
- www.python.org