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Fuzzy set theory for parametric design: A case study of non-standard architectural practice in China

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Abstract

This paper introduces the *fuzzy set theory* to parametric architectural design and presents it as a strategy which architects can adopt to control a project's complexity during the stage of design development. We discuss how the *fuzzy set theory*'s 'vagueness' allows architects to delay their decision makings, especially when they are facing implementing situations where it is difficult to provide additional information needed for complex construction. In this study, we first introduce a metric for project complexity proposed by William Mitchell, who uses the notion of *design content* and *construction* content. Followed this we will explain the *fuzzy set theory* and its rationale for parametric designs.

Keywords: Fuzzy set theory; Parametric design; Non-standard façade; Local affordances; China.

INTRODUCTION

In many developing parts of the world, architects are often confronted with a challenging construction context typified by a less-developed production environment and poor craftsmanship on-site. However, in today's post-digital era, with the cost of computation having dropped dramatically, easily available digital design tools allow architects to develop non-standard forms that provide more opportunities of adaptation to a specific context. The architect's design capacity has thus been significantly amplified, including his/her easy access to computational modelling and direct digital fabrication. Hence, a gap presents itself between possible digital design explorations and their practical materialised solutions when design complexity created by the architect goes beyond the production capacities offered by projectspecific implementing resources. In other words, to materialise at full-scale in the real-world what has been visualised on a computer screen or prototyped with advanced fabrication tools in a lab environment, the architect needs to control a project's complexity so that its realisation process caters to the construction affordances.

Today's digital trend may lead to a shift of an architect's authorial role in project workflow (Carpo, 2011). He/she no longer solely contributes to the design annotations that are normally finished prior to construction. Given advantages provided by digital tools, architects nowadays are more capable than ever to respond to design alternations, since they can create versatile design options not only at the beginning of a project: ideas can also be changed later in the process. In this paper, we discuss what happens when a nonstandard design is met with a less-developed construction affordance and present a strategy for controlling a project's complexity by maintaining vagueness in design. Vagueness aims at coping with unclarified construction restrictions and allowing a designer to alter design later in a project phase. The *fuzzy* set theory, which was firstly introduced by Zadeh in 1965, is defined as the rationale for an architect to defer their design decisions, via parametric means, for better implementing feasibility. Koutamanis (2007) pointed out that fuzzy parametric modelling provides methods and techniques for qualifying and quantifying imprecise and uncertain information. Here, in order to illustrate the application of fuzzy set theory in accommodating real-world constraints, we study the case of the Tianyi Lake Children's Experience Pavilion, designed by Shenzhen-based architectural practice iDEA, and focus on its design-to-build process leading to its non-standard perforated facade. The gap is discussed between the desired facade pattern, which was intuitively generated, and the panel fabrication method provided by the façade manufacturer who joined the team later in the process. Three major aspects are discussed in the case study: 1) the fuzzification of design intent and requirements; 2) the altering of the design by manipulating a parametric model; 3) the defuzzification process to arrive at a final design statement.

METRIC FOR PROJECT COMPLEXITY

It is challenging to cross-compare the complexity of built projects from different times and regions, especially when the current digital trend has generally lowered the threshold for architects to design relatively complex shapes regardless of their industrial and construction environments. In academic lab environments, digital architects can directly fabricate complex forms by means of CNC machinery. However, this digital design-to-build scenario may not be applicable to many real-world construction sites typified by a lack of advanced fabrication tools and a need to rely on manual craftsmanship, time, and funds to realise a complex idea, often requiring additional time or financial resources. Also, a digital paradigm shift may not occur if the typical administrative workflow of common building practice remains unchanged. In China, for example, architects are normally treated as a professional drafter in a conventional 'design-bid-build' system, and their influence on a project materialisation is limited (Jiang, 2005).

Thus, the complexity of a project concerns both its designed intricacies and its specific construction context. William Mitchell (2005) introduced the concept of design content and construction content to evaluate the complexity of architectural practices. Design content, as he defined, refers to the needed steps or information that define a shape in a computer. Likewise, construction content means the sequence of operations that brings what has been designed to reality, such as the steps that are taken during fabrication and assembly processes. Today's digital tools provide architects, in general, the equal capacity to add design content to specific design systems. However, as Mitchell mentioned, the needed additional construction content to realise a designed object and the capacity of providing such construction content relates to a project's specific implementing environment and supply chain.

Mitchell proposed an equation to measure a project's complexity:

complexity = added design content / added construction content

This equation covers an architect's design decisions in relation to a construction context. The *added design content* means the extra information that a designer needs to put into a design system in addition to its preencoded contents or contents he/she has previously defined. The design of a non-standard geometry requires more *added design contents* than a regular one since the designer needs to consider each detail individually. The *added construction content*, also, is a relative definition to a referencing starting point. For example, the assembly of pre-fabricated building components requires less *construction content* than the in-situ fabrication of raw materials.

In general, this metric evaluates how many design decisions or indications an architect needs to make in order to allow a project's construction to proceed. In many developing regions where the starting point for construction is a low-tech, high-touch construction reality, architects have to add more *design contents* to elaborate on their intentions since there are no advanced digital means available to pre-fabricate complex sub-systems automatically in a computer-numerically-controlled fashion. Similarly, when facing a 'design-bid-build' contract setup in which the industrial possibilities are unclear during early-stage design, architects may encounter difficulties to define a project's complexities so that its materialisation is feasible to a specific context.

In this paper, we present a theoretical strategy that allows an architect to defer making decisions of a project's complexity for a better adaptation to its local context. By means of parametric tools, a designer may transform construction uncertainties to the vagueness of a design shape. This vagueness can be stored inside a modelling system using parametric algorithms through which a designer does not need to finalise decision making until acknowledging a project-specific capacity of adding *construction content*. The *fuzzy set theory* is introduced as the rationale behind this theoretical strategy. Its operations, which include 1) input fuzzification, 2) transformation from input to output fuzzy sets, and 3) output defuzzification, are the major steps discussed of an architect's parametric design.

FUZZY SET THEORY IN PARAMETRIC DESIGN

Fuzzy set theory is used to describe a class of objects with a continuum of grades of membership (Zadeh, 1965). explains how conventional **Zadeh** knowledgerepresentation techniques are not well-suited for describing common-sense knowledge generated by humans, as these knowledge are usually lacking crisp denotations. Architecture is a multi-disciplinary science involving many dimensions. A design should be a collection of knowledge from all disciplines. Gathering all this information makes a design task ill-structured, which is generally not convenient to be dealt with through conventional analytical computation and design methods borrowed from other disciplines (Ciftcioglu and Durmisevic, 2001). Thus, fuzzy logic can play an important role in transforming qualitative design requirements to quantitative construction content.

Mitchell (2005) refers to the materialisation of a design project as the process of converting a state description, which represents the desired end condition, into the implementation of a process description. Precise computational design and fabrication tools seem to allow digital architects of today to design and make at the same time (Carpo, 2011). However, real-world constraints always separate an architect from his/her control over the realisation of an end product. In addition, information of design requirements can be omitted during a state-toprocess translation due to causes such as miscommunications among disciplines or an overly complex design idea that falls beyond the constructability provided by a local construction environment. In a parametric design, the role of fuzzy logic is to seek a neutral territory between an ideal digital design-build mode, which can, for example, be found within a lab environment, and a segregated state-to-process workflow that involves different project parties and stakeholders. The mathematical rationale behind the fuzzy set theory supports its applications of parametric algorithms. Together, an architect can use a parametric model to represent non-linear functions of arbitrary complexity with a desired degree of accuracy, leading to maintaining the partial vagueness of a design without jeopardising the holistic project development.

THE MATHEMATICAL RATIONALE OF FUZZY SET THEORY

Fuzzy set theory contributes to set-based design. With the set-based design method, a designer creates a large design space allowing each project party to independently define sets of design variables and solutions. The boundary of these sets is gradually narrowed until the design trade-offs are more completely understood. Some key advantages of set-based design include its allowance of greater parallelism in the process and its search for optimal solutions by developing sets of solutions simultaneously (Ward et al. 1995). A parametric model is a tool for set-based design. Today's digital architect can use scripts to easily generate infinite design content (or a set of design solutions) in response to particular design requirements. In early-stage design, a set of concepts may contain *design contents* that are incompatible with a project-specific construction context. Thus, a fuzzy set provides the tolerance for such imprecise information as well as the representation of infinite design solutions, and it eventually achieves one precise result.

Zadeh (1965) has described the *fuzzy set theory* as the counterparty to the *classic set* (crisp set) *theory*. Mathematically speaking, a classic set only allows two conditions: membership or no membership, meaning the relation of a variable *x* to the given set *A* can be described as: if $x \in A$ then $\mu_A(x)=1$, if not, then $\mu_A(x)=0$ (Figure 1 left); In comparison, a fuzzy set allows partial membership: the value of $\mu_A(x)$ may stay arbitrary as long as it within a domain of [0,1] (Figure 1 right). Partial membership, thus, describes infinite *design content* created by an architect using parametric means. For all members in a fuzzy set, their construction feasibilities remain uncertain until the *process description*. In this fashion, different project parties can collaborate and develop a fuzzy set based on its domain boundaries.



Figure 1: comparison of classical set theory and fuzzy set theory. Source: Authors.

Here, we discuss two advantages of applying *fuzzy set theory* in parametric design: 1) the creation of infinite *design content* to represent an idea, and 2) keeping process vagueness so that a *state description* can be delayed for better construction feasibility.

Regarding the first point, Oguntade and Gero (1983) have discussed architectural design complexity and its contradiction relating to the perpetual imprecision of human experience. Conceptual design, either on a sketch paper or a computer screen, defines a language with which an architect expresses his values. This linguistic representation contains imprecise messages that cannot be fully defined by numerical entities, especially during an early communication among architects and others. Thus, fuzziness is needed to describe a cognitive structure of human thinking and language. In a parametric environment, can this linguistic fuzziness not only be visualised with a series of geometrics but an architect can regulate rules that each project party should follow.

The second point relates to design flexibility and decision making. Koutamanis (2007) has argued that, for an architect, the crispness of a computer-based model may impede a flexible form design whose advantages of precision and information accuracy are based on a determined design decision. Parametric modelling, in contrast, allows an architect to easily make design changes. However, in real-world practice, changes usually take place in the later phases in response to construction affordances. Thus, communicating with fuzzy sets can help to increase a certain degree of design flexibility while, at the same time, allowing for a deferral of the end condition.

APPLICATION IN PARAMETRIC DESIGN

Entities in a parametric environment are numerically related. Designers are able to add *design content* to describe an idea without the consequence of altering components individually. This convenience can easily cause impractical results. Thus, the precise definition of a fuzzy set domain is necessary when a designer aims to describe undetermined design values. These values may relate to construction constraints and design requirements which emerged from early discussions between architects and other parties.

Creating set boundaries helps to regulate the degree of design freedom. These domains describe a fuzziness level (Koutamanis referred as autonomy) of a design's entities. Within defined tolerances, entities may change form, organisation, and relations. In building practice, entities that are restricted by rules such as building codes are *hard shapes* (Koutamanis, 2007). Therefore, there is limited space to alter their shapes. For example, load-bearing and vertical circulation systems are generally difficult to change in form due to their functionalities. Comparatively, entities such as decoration components are *soft shapes*, which allow a higher level of fuzziness in design, therefore softer.

There are three general steps to operate with the *fuzzy set theory*. These include: 1) generating input fuzzy set, 2) transforming input fuzzy set to output fuzzy set, and 3) extracting precise outcome from output fuzzy set. The first step is called *fuzzification*, which maps precise input values to a set of fuzzy members and also defines the boundary of this set. Then a *fuzzy inference engine* (Cox, 1992), following specific rules, transforms input sets to output sets while maintaining previously defined boundaries. The last step is *defuzzification*, which returns all arbitrary values to a precise result. In mathematics, this last step may adopt techniques such as the use of a centroid or a maximum, but in social science, this step can be subjective to the operator.

To operate with *fuzzy set theory*, the first step of generating an input fuzzy set can be to use a parametric model to sketch formal and functional requirements (Alexander, 1967) of a design idea. Koutamanis (2007) described this step as defining spatial tolerance for input

geometries and has come up with an equation to define this. He used variable F to present a vibrant design form, used *C* for the initial input geometries (which Koutamanis called a *canonical form*), and used I and O for both the inner and outer limit of a domain boundary. Hence, a fuzzy conceptual form can be described as F = (I, C, O). Due to the various factors from real-world practice that can influence domain boundaries. a design's conceptual fuzzy form should be an integration of all concerns. In the second step, the *fuzzy inference engine* is a mechanism through which an architect transforms design content to information for the sake of further development or direct fabrication. This is a step (transforming fuzzy inputs to fuzzy outputs) that bridges between state description with process description, yet maintains the vagueness defined in early steps. In this fashion, a direct relation between needed construction content and a vibrant design content can be established with parametric models. In the last step, the material fact of an architectural product requires a designer to reduce the fluctuation of design variability, eventually arriving at one precise state description for materialisation. The *defuzzification* process, in many architectural designs, is subjective. The end condition can be created based on a designer's preference as long as it obeys pre-defined rules.

Introducing *fuzzy set theory* to parametric design aims at a more feasible workflow for today's architects to control complexity. This can be achieved through maintaining design vagueness inside certain boundaries, altering and gradually clarifying uncertainties as a project proceeds, and eventually arriving at a design's *state description* which satisfies a project-specific context in its capacity of adding *construction content*. This ideology stands out, especially when an architect's design capacity offered by today's digital tools has significantly surpassed construction affordances in its developing context.

CASE STUDY — TIANYI LAKE CHILDREN'S EXPERIENCE PAVILION



Figure 2: Tianyi Lake Children's Experience Pavilion (photo © iDEA)

Tianyi Lake Children's Experience Pavilion (Figure 2) was designed by Shenzhen-based architectural practice *iDEA*. With a total area of 9,500 square meters, the building accommodates more than 6,000 children and 600 adults at the same time. The concept has been inspired by the local topography where the architect drew three reference curves for further form exploration. Multiple ellipses were created based on these curves. Parameters including

focus distance and location, radius, and arc length were the driving variables for generating the initial building footprint. The parametric model was created with the *Rhinoceros* modelling software, and its procedural modelling plug-in *Grasshopper*. The overall form design concerns both geometric and functional requirements such as experimental spaces for children, commercial uses, and service rooms. Direct parametric relations have been created in a building scale digital model so that any model manipulations affect the whole.

OVERALL GEOMETRIC OPTIMISATION

iDEA provided planning, design and management services. The entire design-to-build process was required to be accomplished within one year. Therefore, frictions appeared between the tight schedules and the desired building form. Besides, since the building is located in a remote new town where limited construction content was made available to the architect, an overall geometric simplification had to be carried out beforehand. Nonstandard components needed to be reduced in number for the sake of fabrication, causing the focus to shift on the façade pattern development.

Compared to the initial concept, the building's footprint was optimised from a combination of ellipses and arcs to using only arcs. In doing so, the intricacies were converted to regular geometries where a conventional mapping system was sufficient to implement concrete works. Also, for the sake of fabricating structural components, building envelopes, which used to consist of doubly curved surfaces, were reshaped and made of single-curved surfaces. Structural component segments were converted into 2D drawings so that the steel manufacturer could directly take the information provided by the architect and convert this into in-house shop drawings (Figure 3).



Figure 3: representing 3D structure with a 2D diagram (photo $\textcircled{\mbox{${\rm o}$}}$ iDEA)

The rationalisation effort of the overall building form enabled the architect to dedicate more time to exploring façade patterns. The design's parametric model was created in a staged manner, meaning the parametric relations are defined according to model scales. An overall geometry had been frozen at the point when parameter changes would no longer affect the whole. The building project was carried out under a 'design-bid-build' administrative system. Freezing an overall form in early stage was needed for the bid placements. In the following paragraph, we interpret the application of *fuzzy set theory* during the architect's parametric design process and discuss how the architect used digital means to reduce façade panel variations and finalise a feasible solution balancing desired patterns with fabrication techniques available at the time.

A FUZZY SIMPLIFICATION PROCESS

The façade pattern was designed to imitate water ripples. The entire system includes three layers added outside of the concrete wall: 1) waterproof insulation, 2) translucent colour polycarbonate boards, 3) and perforated aluminium panels (Figure 4). Both PVC and aluminium panels were installed on a steel frame that defines the building envelope. Since the overall geometry had been fixed in the early stage, the architect's *exploration space*, his parametrically controlled design flexibility, focused on the building component scale. This flexibility included altering pattern styles, perforation methods and colouration. A parametric façade model was kept *active* during the entire panel development phase and was used for information output generation for direct machine fabrications.



Figure 4: façade system (photo © iDEA)

Fuzzy set theory can be interpreted from the façade's design-to-fabrication process. The first stage of the conceptual design was an intended *fuzzification* process. Input geometries (crisp inputs) included a base surface extracted from the overall building envelope and points defined as ripple centres. The visual effect was created from a series of concentric circles based on inputted centre points. The circles' performance was controlled by essential parameters including 1) centre point locations and 2) *gravity* of these points which would affect curve densities. The façade design intent was represented as an input fuzzy set (Figure 5). The architect was able to add as much *design content* as he needed to meet expected aesthetic requirements as there were no significant restrictions so far.

Transforming input fuzzy sets to output fuzzy sets during panel development was informed by material properties. The *fuzzy inference engine* was defined by a customised script embedded in the parametric model, which took previously generated curves as a guide for panel perforation. Critical variables for panel size and perforation types were defined in relation to desired ripple effects. The façade parametric model's direct parent-child relations among entities were kept active. Therefore, the façade component design was fully autonomous allowing maximum control flexibility. The bid-winning façade consultant and panel manufacturer joined the team after the release of the conceptual design. The latter's fabrication capacity and experience defined a range of expected construction content that could be added to project implementation. In this case, the fuzzy set domain was driven by two major factors: 1) perforation machining process, and 2) façade subdivision.

A stamping machine was used to pre-fabricate perforated panels. Therefore, non-standard variations impacted financial efficiency the most. A vivid ripple effect requires highly densified perforations and as many panel different types as possible. Hence, this was the moment when conflicts emerged between the desired *design content* and needed *added construction content*. The project complexity, even though simplifications had taken place beforehand, dramatically increased due to previously unknown fabrication affordances.



Figure 6: fuzzy set boundary (right), fuzzy members (middle), and potential crisp output (right). Source: Authors.

To arrive at a result that balanced formal requirement and implementing feasibility, design vagueness was needed in a back-and-forth negotiation process between the architect, consultant and manufacturer. This vagueness needed to be bound so that negotiations could be progressive and pragmatic and lead to a built project yetto-come. In this case, the first domain set was the panel dimensions which required accommodation of manual installation. A standard panel unit of 500mm in width and 1000mm in length was fixed so that contractors would be able to carry and install each unit with manually. Rest variables, such as reference curves, perforation method, and opening sizes remained flexible for later decision.

To increase financial efficiency, more domain boundaries needed to be defined to consistently smoothen a fluctuating *design content*. The next frozen variables were



Figure 5: ripple patterns (photo © iDEA)

5

the reference curves for panel perforations. What remained ambiguous was an output fuzzy set of panel openings. Then, the design's vagueness was further reduced by allowing flexibility of only six opening locations with two types, meaning the rest area would be filled with small openings. Still, flexible parameters included 1) grid distance of openings, 2) opening diameters (two values), and 3) gravity of reference curves. Data extracted from digital models were directly used for die fabrication. Therefore, fuzzy members of a panel unit might remain parametrically flexible until die production. In Figure 6, we diagrammatically described a panel's fuzzy set boundary (Figure 6 left), arbitrary partial memberships (Figure 6 middle), and expected crisp output (Figure 6 right).

Let's define the vagueness of a panel unit (Figure 6 middle) as F_p . The inner limit I_p represents the minimum radius for small openings, as whose value tends to be 0 or the highest precision that a die molding machine can get. The outer limit O_p is defined based on opening locations and equals to 1. Therefore, opening types belonging to domain [0,1] are valid. By selecting some members in a fuzzy set F_p which meet design and materialisation requirements, Figure 7 then shows the comparison of ripple effects among four different conditions of opening types: the top-left indicates openings of 75mm (large) and 10mm (small); the top-right shows openings of 65mm (large) and 20mm (small); the bottom-left contains openings of 55 mm (large) and 30mm (small); and the bottom-right is the effect of 45mm (large) and 40mm (small).

In this case, the design *defuzzification* process was a subjective decision made by the architect based on his

aesthetic evaluation of each option. Parametrically speaking, this included two steps: 1) finalise outstanding undefined parameters, and 2) further simplification for cost efficiency. The architect has chosen 65mm in diameter for large openings and 20mm for the small ones. For all panel units, the grid of openings was designed identically. The distribution of large and small openings was relevant to their distances to the closest reference curve. Thus, a total of 64 panel types were generated accordingly.

The second step of *defuzzification* aimed at further increasing a panel standardisation level for production efficiency. In Figure 8, the first row shows 7 panel categories, in total 64 variations, sorted by the number of large openings. In the first row, the total number of variations was reduced to 36 because of the panel's central symmetries. From the first row to the second, another customised script was added to the facade parametric model. Large openings were sorted based on distances between their centre points to the closest reference curve. By doing so, for panels with three large openings, the last point in a distance list was culled out and then turned into a small opening. Likewise, for all panels with four large openings, the first points of a distance list were converted to large ones. This altering was also a subjective decision made by the architect, hence, from the second to the third row, the final number of panel types was locked in 17.

FABRICATION AND CONSTRUCTION

Contractually speaking, a digital design-to-fabrication process would here not have been feasible due to the potential financial risk to parties like design architect who



Figure 8: panel simplification for financial efficiency. Source: Authors.

is not contractually responsible for component production. Among large-scale non-standard buildings designed by architects like *Zaha Architects*, *MAD Architects*, and *Coop Himmelb(I)au*, third-party consultants such as structural, geometry, and software engineers will dilute legal responsibilities by interpreting information for project implementers. In other words, these third-party consultancies are helping to alter a ratio of added design content to added construction content.

This was not the case in this project. In order to streamline a design-to-fabrication scenario, the architect had chosen to take the financial risk of directly generating construction related information himself. This included panel type tags, dimensions, and installation indications (install directions), as well as the outlines for producing panel dies. Likewise, the layout of colour translucent PVC plates was also directly extracted for the façade consultant. The entire fabrication-to-installation process lasted for 45 days (Figure 9 and 10).



Figure 9: façade installation (photo © iDEA)



Figure 10: finished façade (photo © iDEA)

DISCUSSION

Carpo (2011) argued that in the digital paradigm an architect's role and authorship shifts in the process. Parametric modelling may have changed design representations, but the gap between design problems and materialisation restrictions remains in many construction sites. This paper builds on the discussion of an architect's increasing affordance to explore design content. He/she may use precision, offered by computational tools, to create tolerances that allow for the inclusion of process uncertainties.

The *fuzzy set theory* in parametric design provides an opportunity for architects to alter *design content* in a later

project phase. Parametric fuzziness leads to a rational approach by adding design domains. These domains' boundaries are redefined along project developments. Through multi-disciplinary discourses of its potential and limits, the variable design content will eventually become precise data for industrial production. In the case of *Tianyi* Lake Children's Experience Pavilion, a fuzzy parametric process was set up to deal with its non-standardisation and materialisation conflicts when realising its perforated facade. A staged modelling strategy shattered the long ambiguous design process and kept the design flexibility in a rational sense by gradually concluding crisp outputs. In the facade's digital model, entities remained modifiable until the final industrial production. Their fuzzy forms facilitated local intelligence and autonomy (Koutamanis, 2007) in response to the balance between design problems and needed shapes. The project was successfully implemented within the given timeline and resources, and maximally fulfilled an original design intent. Fuzzy parametric modelling contributed to a decision deferral of the architect so that his role extended during the project delivery process.

CONCLUSION

A holistic digital trend of the architectural industry may fall short of expectations when a less-developed construction context is incompatible with an expanded design capacity of today's digital architects. Thus, in order to materialise non-standard design ideas, architects facing such conditions need to bypass barriers and alter design complexity for implementation matters. In this paper, we presented the *fuzzy set theory* for parametric design as an interpretation and a theoretical strategy for architects to maintain design flexibility within such contexts. With parametric tools, today's architects are more capable of managing a project's complexity so that a proposed *design content* can better fit in local construction affordance.

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7

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