Design Spaces of Structurally Pre-evaluated Funicular Shapes

Marcelo Álvarez [1], Marcelo Bernal, Ph.D. [2]

[1] Universidad Técnica Federico Santa María, Chile
marcelo.alvarez.13@sansano.usm.cl
[2] Perkins&Will, USA
Marcelo.Bernal@perkinswill.com

Abstract. In this paper, we develop a structural pre-evaluation and optimization technique for vault-like shapes. This implementation focuses on exploring design spaces in early design stages. The proposed technique approaches the problem of reducing the flexibility of the design space while advancing to later design stages for vault-like shapes. We start with a custom design space based on design intent. Then, we define a sampling criteria to study a reduced number of candidates. Later, the optimization process focuses on minimizing structural deformation values through shape manipulation. Results show a notorious enhancement for maximum deflection and displacement of the structure. Generally speaking, the shape optimization pattern is consistent with how vault-like shape works. All solutions reduce their span and boundary area while increasing the maximum height. Also, reaching maximum deformation values that are ten times better than the admissible final values on average.

Keywords: Funicular shape, Structural optimization, Design space, Early-design stage, Particle-spring system.

1 Introduction

Funicular shapes are very efficient, lightweight, robust, and bending-free structures (Adriaenssens et al., 2014). Nevertheless, structural optimization processes can enhance their overall performance even further. Several form-finding techniques allow shape definition from a structural perspective, such as graphic statics diagram subdivision (Akbarzadeh et al., 2014), thrust network analysis (Rippmann & Block, 2013), and shell structures topology design (Vuèiæ et al., 2018). These approaches focus on generating funicular structures by manipulating axial diagrams. Regardless of these methods having structural principles embedded in their internal processes, they tend to unlink the resulting shape from the initial diagram generation when applying structural optimization methods at later stages.

On the other hand, a pure shape-oriented modeling technique (Álvarez et al., 2020) does not necessarily generate optimized funicular structures. Such modeling technique assumes a proper structural behavior relying on a Particle-spring system (Kilian & Ochsendorf, 2005) funicular properties of shell structures (Figure 1). This study focuses on the enhancement of a given design space of vault-like shapes through structural optimization. This process leads to the concept of pre-evaluated funicular shapes, where the implementation of a form manipulation algorithm can modify an input shape to find an optimal structural solution. Also, the pre-evaluation process suits design space exploration (DSE) since generative design methods set up multiple sample solutions that this approach modifies within the early-stage scope.

Generative design methods based on parametric modeling technology are capable of generating multiple solutions. These methods overcome the limitations of other iterative or analog processes by enhancing the speed, accuracy, and complexity of the design catalog of alternatives (Chaszar & Joyce, 2016). Fuchkina et al. (2018) stated two problems regarding DSE. The first problem is related to the comparison criteria between a single design and another alternative seeking a superior solution. The second, how to implement such a comparison with a catalog of solutions. The authors conclude the need of reducing the design space before a systematic DSE. Brown & Mueller (2018) also point out the difficulty of the formulation of a parametric design space that does not limit the design flexibility. The challenge of preserving flexibility increases by including structural shape pre-evaluation.

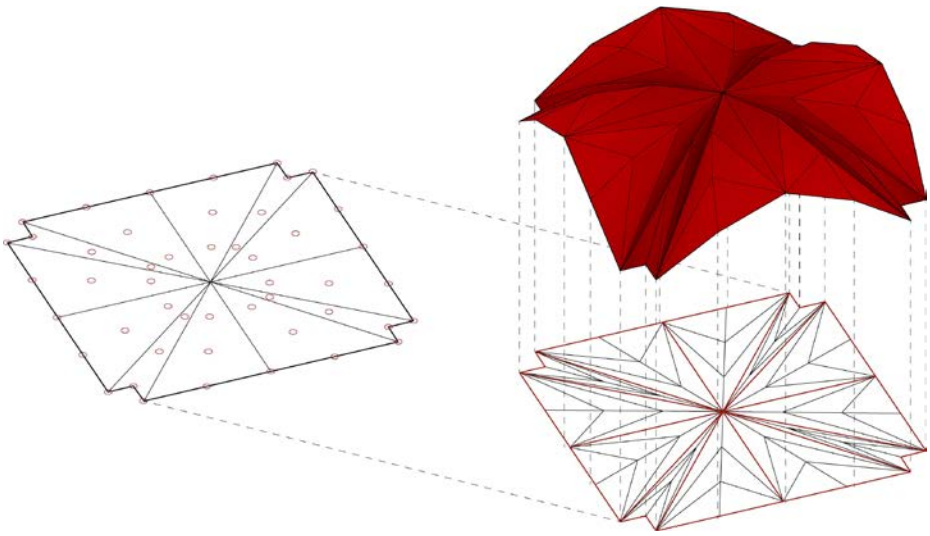

Figure 1. Shape-oriented modeling technique based on topological manipulation. From left to right: edge-vertex topology map, Delaunay triangulation map and 3D shape.

Structural optimization problems in architectural design link multiple disciplines to obtain an optimal structure or shape. Structural optimization is a synthesis of disciplines: structural and sensitivity analysis, CAD, and mathematical implementations (Ramm et al., 1993). The term optimization seeks excellence in design while the optimization methods implement different approaches to achieve such as excellence (Cohn, 1994). Ramm et al. suggest that the term optimization is misleading since it would lead to a single optimal solution. Also, Firl (2010) defines the goal of structural optimization: to formulate an evolution process that improves specific structural properties.

Considering optimization goals as part of a design process, approaches such as evolutionary algorithms applied over a set of solutions could enable structural performance environments to study an early design space. Evolutionary algorithms may refer to many techniques such as genetic algorithms, evolution strategies, evolutionary programming, and genetic programming. Genetic algorithms are *parallel searching methods* because they search for solutions using the whole design space instead of a single potential solution. These algorithms use a fitness function to score suitable solutions. A highly-scored solution is more optimal for a given problem, and solutions with higher scores have more chances of being selected in the next generation. Solvers for genetic algorithms will mostly output optimal solutions according to the negotiation between quality and time.

This study approaches the specific problem of design alternatives reduction when exploring design spaces based on structural behavior. This reduction limits its properties since shapes are already defined, and these processes tend to consider material properties over geometry. The main goal is to formulate a parametric model capable of producing more suitable solutions while preserving the design space flexibility. We implement a parametric optimization model to improve vault-like shapes at a geometrical level based on the structural behavior component. To test the proposed method's flexibility, we define an "Open-air" Theater architectural context to work as an external requirement. The external requirements are part of the fitness function definition for optimization. Structurally optimizing these solutions only under a geometrical and Particle-spring system manipulation would strengthen the link between an early design space and the later stages.

Regarding the potential limitation in flexibility for form-finding techniques that generate early design spaces aiming to further usability, this study formulates the following question: How to include structural optimization methods into early DSE modifying only the modeling environment parameters? This approach defines a parametric design space model that mutates single solutions into more suitable structural shapes preserving the initial topology. Designers can quickly modify and evaluate them to enhance performance. The proposed pre-evaluation and optimization technique for vault-like shapes can preserve or improve its flexibility based on the objective structural behavior while exploring the design space.

2 Methodology

This implementation works as a three-step process: generation, selection, and optimization. The generation process outputs a design space, supported by a custom Design of Experiments for sampling and reducing the size of this design space (Bernal et al., 2020). The selection step is based on designers' preferences within the pool of admissible alternatives. Finally, the optimization process modifies the solutions under an evolutionary structural optimization process based on genetic algorithms. This implementation grants modified vault-like shapes based on optimized structural behavior, in other words, minimizing deflection and displacement. Deflection is understood as the angle or the distance degree of displacement under a load. For this study, a value over l/300 implies that the deformation exceeds the admissible behavior so it could lead to a not feasible structure. Displacement stands for Nodal Displacement, referring to the distance between the initial and final vertex movement under a load. Figure 2 illustrates the methodology workflow diagram.

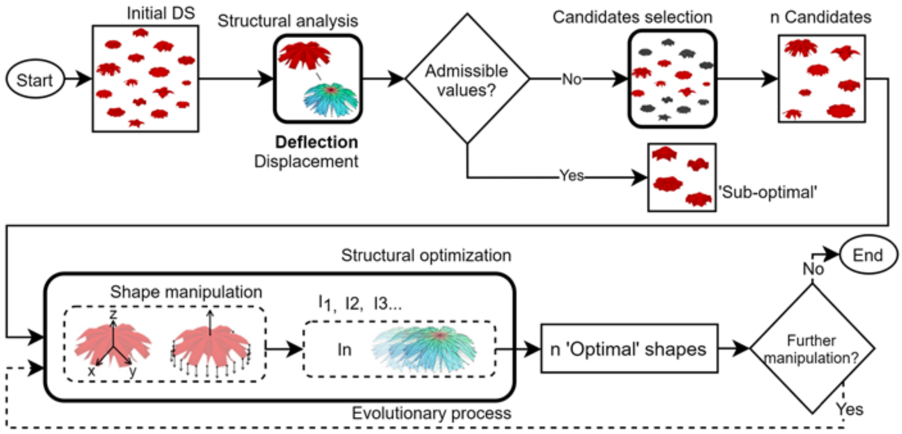

Figure 2. Methodology workflow diagram for the implementation of pre-evaluation.

2.1. Structural Analysis

Design space stands for all possible solutions a parametric algorithm can generate. These solutions are the result of the combination of user-defined parameters as a whole. This method uses a vault-like shape modeling algorithm as the initial design space to implement the early structural optimization process. This design space is a selection of 128 representative samples derived from a design of experiments from Álvarez et al. modeling technique previously described. Millipede, an analysis and optimization plugin for Grasshopper, supports the structural analysis for each solution, recording the maximum deflection and displacement values for the selection and optimization steps (Figure 3).

![decorative header]

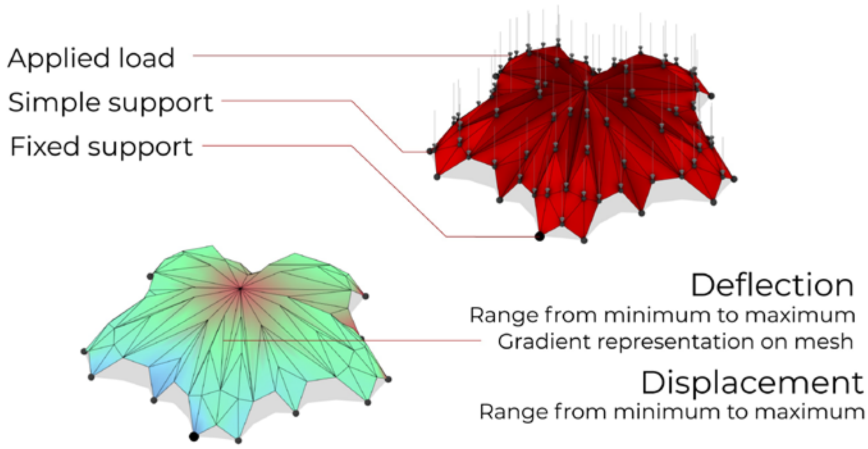

Figure 3. Structural analysis for maximum deflection and displacement values for solutions.

2.2. Candidates Selection

Since this implementation aims to enhance the structural behavior of vault-like shapes, this study focuses on the best and worst candidates not performing under the admissible value of l/300 for deflection (Figure 4). Since this study is a pre-evaluation of structural shapes, this implementation does not filter solutions under additional specific user criteria. To define the selected candidates, we focus on the best initial solutions not meeting the maximum admissible value. Also, the candidate set considers the worst initial solutions based on the maximum deflection value. This definition aims to test how further an almost viable vault-like shape can improve its performance. On the other hand, this definition also tries to enhance these shapes, far from performing under the admissible value, enough to be considered viable.

2.3. Structural Optimization

The evolutionary process for selected solutions relies on shape manipulation parameters under Kangaroo 2, a Particle-spring system (PSS) environment to preserve their funicular properties. Figure 5 shows the geometrical and PSS manipulation parameters from which the evolutionary solver Galapagos operates on. This process requires a fitness function to minimize the values of the deflection and displacement. The algorithm remaps the values within a 0 to 1 domain and adds both values to determine the fitness function. The evolutionary solver outputs an optimized shape for each solution with associated parameters to analyze the results.

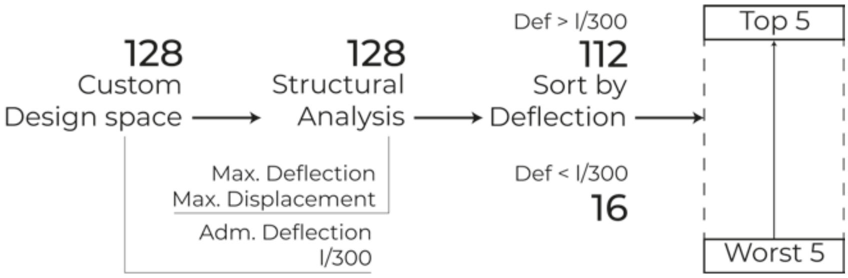

Figure 4. Filtration and Selection of candidates for evaluation.

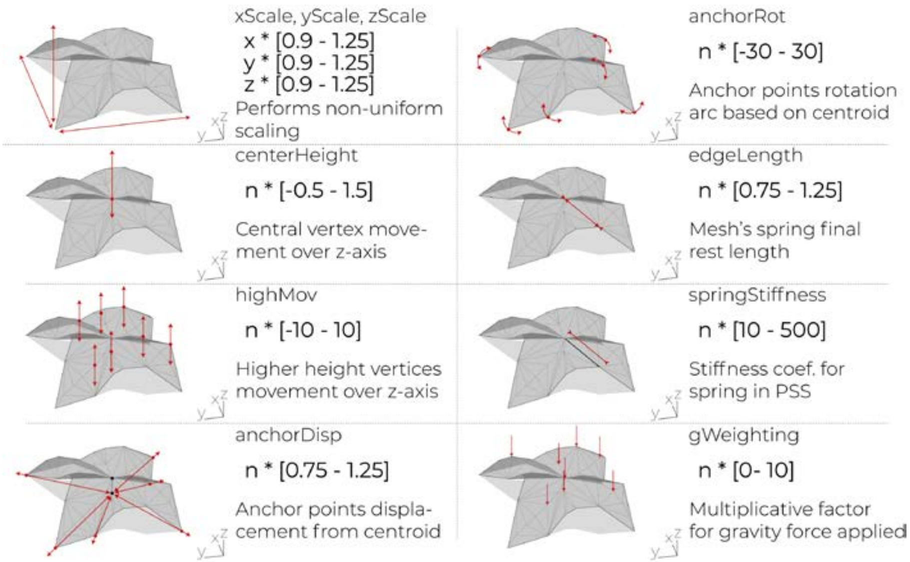

Figure 5. Control parameters for shape manipulation.

3 Results

The output of this optimization process includes two shapes and their structural analysis values. This method preserves both initial and optimized shapes. Table 1 illustrates the starting and optimal state for a sample of four selected candidates, along with geometrical data. Figure 6 shows a graphical representation of results for candidates' indexes 0, 6, and 9.

Table 1. Optimization process results. x.0 is for initial, x.1 for optimal. Unit = meters.

Cand.	Adm. Def.	Max. def.	Max. disp.	Span [m]	Area [m2]	Height[m]
0.0	0.078934	0.080066	0.056025	23.68	363.98	6.02
0.1	0.038362	0.004628	0.003359	11.51	91.75	8.42
1.0	0.104915	0.127241	0.085691	31.47	572.88	7.78
1.1	0.079660	0.025192	0.021269	23.89	369.24	12.09
2.0	0.082351	0.134658	0.086158	24.71	441.17	6.58
2.1	0.063621	0.007427	0.006199	19.08	233.92	8.97
3.0	0.119807	0.159968	0.103694	35.94	875.59	9.16
3.1	0.070184	0.020363	0.016352	21.05	332.13	9.74
4.0	0.150231	0.197391	0.112374	45.07	1454.29	7.98
4.1	0.112370	0.009578	0.007794	33.79	777.51	13.68
5.0	0.160869	7.494493	5.286791	48.26	1082.44	11.48
5.1	0.092992	0.044065	0.032396	27.90	514.61	17.84
6.0	0.174753	5.630786	2.821389	52.42	2054.74	9.60
6.1	0.093922	0.024891	0.013518	28.18	521.31	13.79
7.0	0.081190	5.594725	3.563433	24.36	556.19	14.61
7.1	0.042346	0.094241	0.050281	12.70	69.09	23.96
8.0	0.078934	5.383503	3.395665	23.68	524.11	11.22
8.1	0.039073	0.040855	0.024325	11.72	59.51	24.97
9.0	0.137246	4.450724	3.224402	41.17	750.51	9.51
9.1	0.097414	0.016789	0.010268	29.22	604.67	13.72

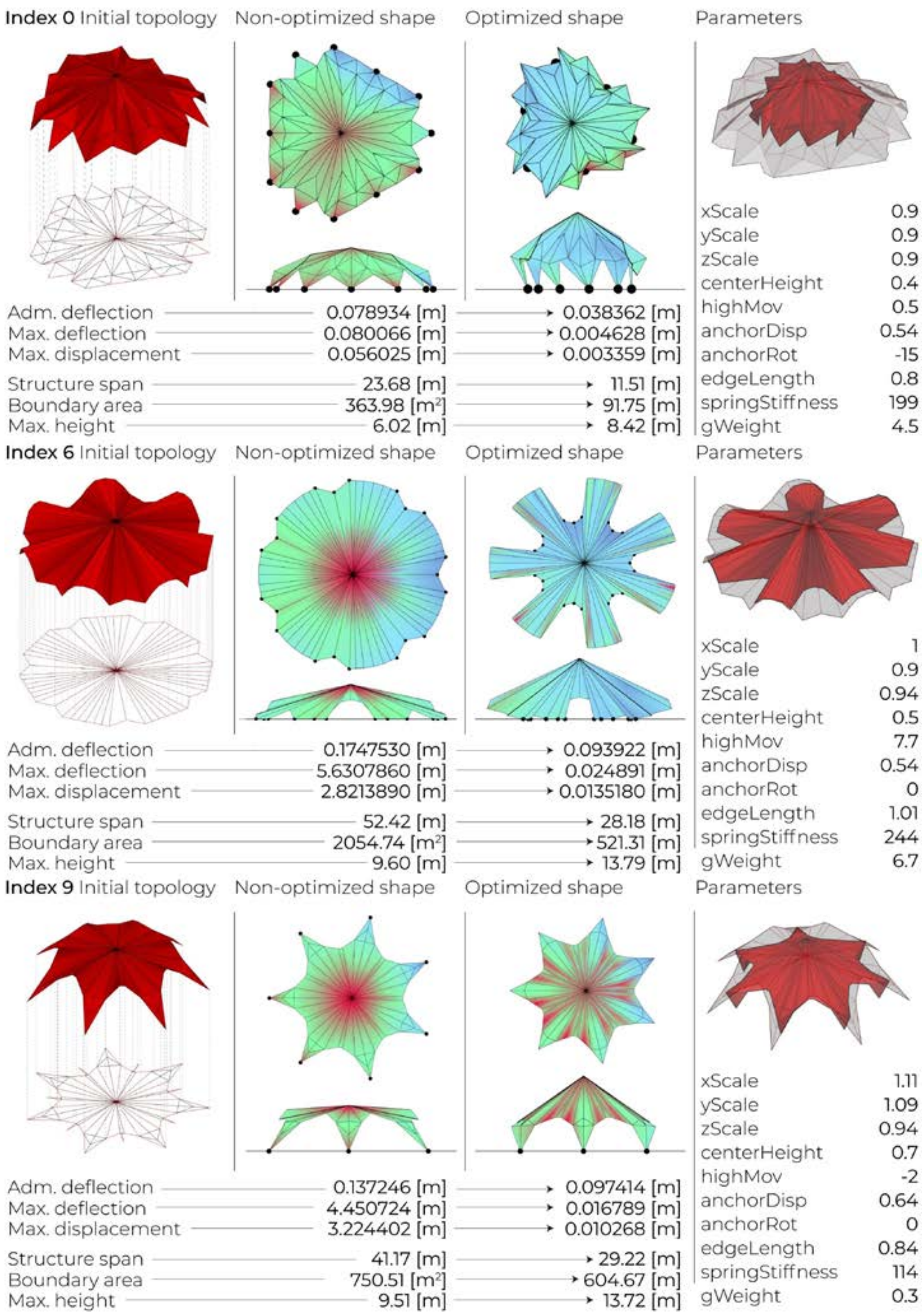

Figure 6. Optimization results for candidates index 0, 6 and 9 from table. Additional final parameter configuration is shown.

3.1 Initial Structural Performance by Polygonal Family

The base geometrical configuration for the studied solutions in the design space is a regular n-gon polygon. The current domain goes from three to eight sides. We group all solutions by their initial polygonal configuration to study the overall structural behavior before pre-evaluation and pre-optimization. Grouping by family leads to six groups of different sizes since the number of sides is a variable for the custom design space. Table 2 details the total length for each family based on the initial number of sides.

Table 2. Polygonal family size data. From the total samples lists, partial samples represent the suitable solutions to optimize due to their non-optimal deformation values (def. > l/300). Excluded solutions satisfy this requirement already.

Polygon	Partial Samples	Excluded	Total Samples
3	29	1	30
4	22	2	24
5	9	1	10
6	11	0	11
7	22	1	23
8	29	1	30

The structural optimization goal for vault-like shapes is to minimize the values of maximum deflection and displacement. These values determine the total deformation for a given structure since this study focuses on improving the performance of the admissible value for maximum deflection, Table 3 shows the extracted statistical data for performance-related metrics of the three-side polygonal shape.

Table 3. Statistical performance data for each polygonal family.

Polygon	Best	Worst	Median	Average	Std Dev
3	0.016638	5.594725	0.946254	1.489829	1.691695
4	0.050561	7.494493	0.773972	1.421536	1.758439
5	0.127241	2.919028	0.816094	1.039575	0.902222
6	0.012225	2.224369	0.516139	0.629128	0.617678
7	0.01184	5.630786	0.791589	1.498345	1.789054
8	0.009809	3.233603	0.930108	1.020305	0.86184

To further test the flexibility of this technique, this study defines an architectural context suitable for vault-like shapes as shell structures. This test defines two additional requirement values: to minimize the distance between the top height initial vertex and a 10 [m] fixed height vertex and to minimize the distance between anchor points and an inner circumference diameter of 40 [m] to cover. Table 4. shows the initial and optimized maximum deflection values for a three-sided polygon candidate.

Table 4. Initial and optimized deflection values for the top three best candidates from the 3-sided polygon family.

Polygon Family	Index from DS	Initial Max Deflection	Initial Adm Deflection	Final Max Deflection	Final Adm Deflection
3	117	0.20155	0.153333	0.016247	0.125018
3	105	0.272733	0.153333	0.048419	0.130540
3	54	0.353605	0.153333	0.046816	0.118067

3.2 Results analysis and observations

The first set of results for selected candidates show that the top five performing candidates can increase their initial performance, decreasing maximum deflection and displacement to 9.74% of their initial values on average. The bottom five performing candidates greatly enhance their initial values to 0.77% of final maximum displacement. For the in-context specific approach, topologies of hexagon and octagon have considerably better initial performance, but triangles also have very compelling solutions. Optimization results for triangle candidates show a similar enhancement level of 13.02% final value from the initial one. Figure 7 shows the initial and optimized shape for a three-sided polygon.

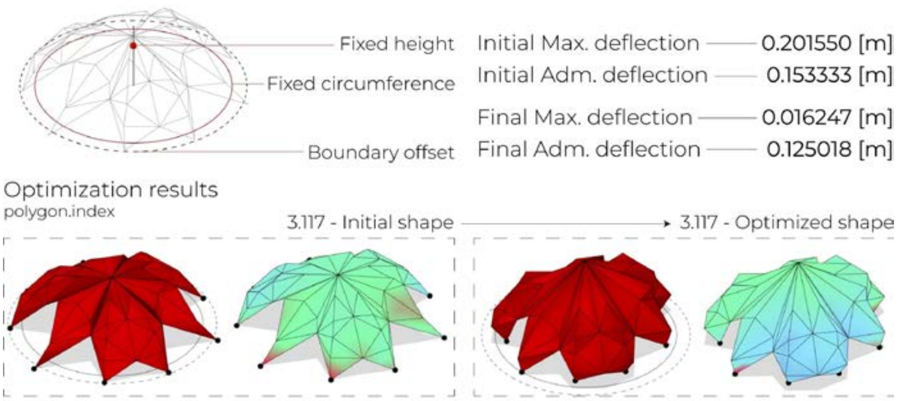

Figure 7. Initial and optimized shape for candidate 3.117 under the architectural context.

Regarding the first section of results, final values notoriously show structural enhancement for total deformation values. However, the lack of external spatial constraints enables high shape manipulation freedom to morph the input vault-like shapes into solutions with the best possible deformation values for the process until stopped. This condition exposes a pattern of shape optimization: reducing the total span and boundary area while increasing the final maximum height value. This behavior is consistent with the expected structural properties for vaults.

However, embedding an architectural context as an external requirement proves that additional constraints can drive the optimal solutions into very compelling results while preserving the desired functionality. This flexibility is possible due to the number of available parameters that manipulate the shapes. Constraining the higher vertex and anchor points to a more fixed position in space can reach a "limit" in terms of a fitness function. With enough given parameters, the implementation can perform well enough to minimize the desired structural behavior values.

4 Discussion

This study aims to develop a design workflow for early stages as a three-step modeling technique: generation, selection, and optimization. This implementation focuses on the pre-evaluation and pre-optimization of vault-like shapes. The process runs with a defined set of geometric input parameters to manipulate the shape to satisfy a custom structural fitness function.

The solution sets from this optimization technique allow an understanding of the structural enhancement behavior from the starting conditions. Implementing this technique, either with no external requirements or with functional program constraints, shows that shapes can greatly increase the initial structural performance under low challenging scenarios. All solutions tend to retract their anchor points to the centroid and raise their maximum height. This condition aligns with an optimal state for funicular shapes as passive structures but may vary if by designer's choice constraints are less permissive.

Implementing a searching process that operates from topology to typological level would enlarge the initial design space for a given optimization problem. This structural optimization technique aims to set a fast and optimal search for ideal solutions from the beginning. At a higher level, the pre-evaluation method will execute regardless of the generative process. The design's early stage can enable the process integration through modularity. For this approach, in particular, a module that generates and another module that analyzes allows shape manipulation. Given its functionality, one can consider other structural typologies to generate under specific design parameter choices and submit them to pre-evaluation based on the user's requirements to optimize.

References

Adriaenssens, S., Block, P., Veenendaal, D., & Williams, C. (2014). *Shell Structures for Architecture: Form Finding and Optimization*. Routledge. https://doi.org/10.4324/9781315849270

Akbarzadeh, M., Van Mele, T., & Block, P. (2014). Compression-only Form Finding through Finite Subdivision of the Force Polygon. In Proceedings of the IASS-SLTE 2014 Symposium: *"Shells, Membranes and Spatial Structures: Footprints."* The International Association for Shell and Spatial Structures (IASS).

Álvarez, M., Bernal, M., & Castro, C. (2020). Modeling technique for vault-like structure generation through topological manipulation. In *Congreso SIGraDi 2020: Transformative Design* (pp. 238–245). Blucher.

Bernal, M., Okhoya, V., Marshall, T., Chen, C., & Haymaker, J. (2020). Integrating expertise and parametric analysis for a data-driven decision-making practice. *International Journal of Architectural Computing*, 1478077120940975.

Brown, N., & Mueller, C. (2018). Design variable analysis and generation for performance-based parametric modelling in architecture. *International Journal of Architectural Computing*, 17(1), 36–52. https://doi.org/10.1177/1478077118799491

Chaszar, A., & Joyce, S. C. (2016). Generating freedom: Questions of flexibility in digital design and architectural computation. *International Journal of Architectural Computing*, 14(2), 167–181. https://doi.org/10.1177/1478077116638945

Cohn, M. Z. (1994). Theory and practice of structural optimization. In *Structural Optimization* (pp. 20–31). Springer-Verlag.

Firl, M. (2010). Optimal shape design of shell structures. Doctoral dissertation, Technische Universität München.

Fuchkina, E., Schneider, S., Bertel, S., & Osintseva, I. (2018). Design Space Exploration Framework—A modular approach to flexibly explore large sets of design variants of parametric models within a single environment. *Proceedings of the 36th ECAADe Conference*, 2, 367–376.

Kilian, A., & Ochsendorf, J. (2005). Particle-spring Systems for Structural Form Finding. *Journal of the International Association for Shell and Spatial Structures,* 46(148), 77-84.

Ramm, E., Bletzinger, K.-U., & Reitinger, R. (1993). Shape optimization of shell structures. *Revue Européenne des Éléments Finis*, 2(3), 377–398. https://doi.org/10.1080/12506559.1993.10511083

Rippmann, M., & Block, P. (2013). Funicular Shell Design Exploration. In *ACADIA 13: Adaptive Architecture* (pp. 337–346).

Vuèiæ, M., Tepavèeviæ, B., Stojakoviæ, V., Jovanoviæ, M., Mitov, D., & Bajšanski, I. (2018). Topology design of form-active gridshell structures. *Structure Optimisation* (pp. 103–110).