Pixel-Based Geometric Decoding of Mondrian Compositions

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Abstract. This study explores the use of a computational method for decoding and encoding an art composition in the digital environment. Geometric decoding includes subdivision through color and shape. Five artworks of Mondrian, which contain simple but intense geometric data and have a unique composition style, are discussed in this context. Matrix-based expression of composition in terms of color and form with a code-based analysis allowed to definition of a control mechanism for existing geometric data. The aim of this decoding process, which follows segmentation and fragmentation methods, is to capture the common composition approach and particular comparison approach. Visual encoding, as the reverse of the decoding process, is the interpretation of the data obtained as a result of the analysis by assigning a color map. By this meanings, the matrixes and visual outputs enable the artworks to be compared according to the main composition style, or the similarities between them.

Keywords: Geometric decoding, Mondrian, Pixel-based, Segmentation, Fragmentation

1 Mondrian in Computable Art

Evaluating artworks with computational approaches allows analyzing existing by transferring them to digital media as well as the creation of new artworks. The question of whether there is a sub-text or a mathematical structure in the way the artists handle the artwork is the backbone of the analysis made in the digital environment. The aesthetics and originality in the artwork are difficult to be reproduced by artificial intelligence (Hertzmann, 2018), and one of the ways to make these qualities understandable is to develop the algorithms for existing artifacts (De Silva Garza & Lores, 2005; Wang & Xie, 2020). However, people who create art in new media, including the production of artificial intelligence as well as human-made algorithms, mostly focus on contemporary art approaches rather than the roles of computer programmings or digital design tools (Greenfield, 2006). Understanding the uniqueness of the artwork and the compositional idea behind it, are one of the big issues to evaluate the endeavor of software developers or directly a computer to produce works like an artist. By this meaning, Mondrian compositions have been a subject that researchers from many different fields from architects to computer engineers work on. Mondrian's composition style with a white background and horizontal and vertical black lines have three decisive colors: Red, blue, and yellow. These colors fill one or more than one space between black lines that intersecting each other at right angles. The balance between the size and position of the elements in the composition supports the dynamism of the style (Park, 2020). The clarity and balance in the relationship between the color, line, and shape of the elements that define the style, complete the order and organization of these elements (Cleveland, 2008).

With generative and transformative approaches, it is possible to create reproductions by attributing the rules of the artist's style as an algorithm to encode with computational rules (Zhang et al., 2012). In previous studies, researchers have conducted different analysis studies on Mondrian compositions using computer-aided design tools and computational methods (Feijs, 2019). Monica is a computer program that produces images in Mondrian's style, and it has an algorithm that works randomly without being bound by any predetermined rules (de Silva Garza & Lores, 2004; de Silva Garza & Lores, 2005). This program is based on using the evolutionary algorithm for Mondrian-style artwork generation. It works based on the principle of random decisions within the rules that will consider the style, color, size, and position of the rectangles to be used in a table to be produced in a digital environment. Feijs (2004), on the other hand, made a classification as planes, lines, and their combinations while analyzing Mondrian compositions and developed a computer model that he could create reproductions in the digital environment (Feijs, 2004). In his work published in 2020, he divided the compositions into sub-parts using green lines that were never used in compositions through the black lines in the compositions. As a result, splittingness and complexity values were analyzed (Feijs, 2020). Computational geometric visualizations using the kd-tree data structure are involved in researches on the representation of Mondrian tables (LeFevre et al., 2006; Roy & Teh, 2008, Wang et al., 2015) on a three-dimensional cube (Skrodzki & Polthier, 2018). Andrzejewski et al. (2010) focused on creating an educated productive model by using machine learning method based on the principles behind the compositions such as the intensity of use of colors to be able to recognize the original Mondrian artworks.

Mondrian style is one of the interdisciplinary subjects studied continuously due to developments in programming languages, and digital design tools. A wide range of research methods specific to the Mondrian style was used both in the analysis of existing works and in the creation of reproductions. Paintings, which were reduced to sub-sections from line thickness to the position and size of colored components, enabled the digital production of compositions with similar styles. However, the purpose of this study is to create a comparison algorithm for the common language of existing works, rather than suggesting a new analysis method or computer program for reproductions. The process of separating a whole into geometric parts horizontally and vertically offers a quantitative decoding alternative (Klee & Moholy-Nagy, 1953). For this comparison process, a novel and simple pixel-based decode method was suggested. Instead of separating each component in the compositions as geometric elements such as rectangular, square, or line, the algorithm built on the smallest building blocks of these geometries, pixels, which are already represented with different colors.

Geometric decoding has been used as an analysis tool for the part-whole relationship in the composition and directly to the proportion of the whole composition. This process is handled in two different ways: Segmentation and fragmentation. The comparison matrixes created from the selected artworks represented the pixel-based values of the works according to each of the decoding. Matrixes have the numerical expressions of the composition, but they are also visualizable data about the style. The visualized data created according to the similarities and differences in the common composition approach are the results of an empirical method for the encoding process.

2 Research Design

Within the scope of the study, we selected five artworks completed between 1938 and 1943 which contains red, blue, and yellow colors on the white canvas with black stripped (Figure 1). Paintings produced by the artist in different periods may also have different small nuances within themselves. In order to minimize these differences, his artworks in a certain period were preferred. These compositions also have visible similarities.

As a subjective analysis model, it is possible to read the main composition style in the sub-divisions of the artwork (Thompson, 1977). The sub-division methods in the pictures examined in this study also correspond to the segmentation and fragmentation methods described as geometric decoding. This process indicates in which geometries the selected images will be divided into parts. To narrow the scope of the geometric decoding concept, which has a very wide framework, a pixel-based method was followed.

First, in the segmentation method, all geometric components with the same color in the composition were considered as a whole. In the composition consisting of red, blue, yellow, black, and white colors, the percentages in color usage derive from the pixels, regardless of the position and form of the components. Secondly, the fragmentation method was used. Regarding the fragmentation method, the composition was considered as a whole and in each painting, the user was allowed to create as many horizontal or vertical fragments as desired. The user was able to decode the picture from the band gaps of equal width, fragments, by dividing the picture into two parts or by dividing it into a hundred parts.

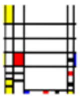
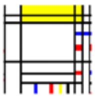
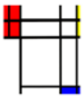
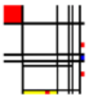
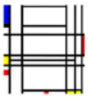
	Selected Compositions		Year	Dimension [cm]
No. 1		Trafalgar Square	1939 - 1943	145,5 x 120
No. 2		Place de la Concorde	1938 - 1943	102,5 x 103
No. 3		Composition of Red, Blue, Yellow and White: Nom III	1939	69,5 x 63
No. 4		Composition No 8, with Red, Blue, and Yellow	1939 - 1942	75 x 68
No. 5		Composition No 10, with Blue, Yellow and Red	1939 - 1942	79,5 x 73

Figure 1. Selected Mondrian compositions in the context of this study.

During the research, P5.js, an open-source JavaScript library, was used for the decoding. The intensity of the colors was increased in the digital environment to regulate minimal differences in R, G, and B values. The height of each picture was fixed to 500 pixels. Since the aspect ratio of the pictures is different from each other, the width of each composition is in different pixels in accordance with the proportions of the original painting. After the compositions were decoded as segmentation and fragmentation with p5.js, the numerical data obtained were expressed as matrix-based on the same platform. For both methods, the number of pixels belonging to the specified intervals and its ratio to the overall composition was printed.

The base of the comparison concept is the data of matrix-based calculations. These calculations have proceeded with the same calculation logic in both segmentation and fragmentation methods. The data of geometric decode processes were used for the matrix-based calculations. Pixel matrixes of each painting are analyzed with algorithms prepared in p5.js. The concept of comparison has been examined with both the particular comparison and the common composition approach. In both approaches, the differences between matrixes are calculated for comparison of the paintings. Since the five colors are calculated in separate columns in the matrixes, it does not make a difference for comparison whether the value obtained is positive or negative. For this reason, absolute values are taken as a basis in matrix operations.

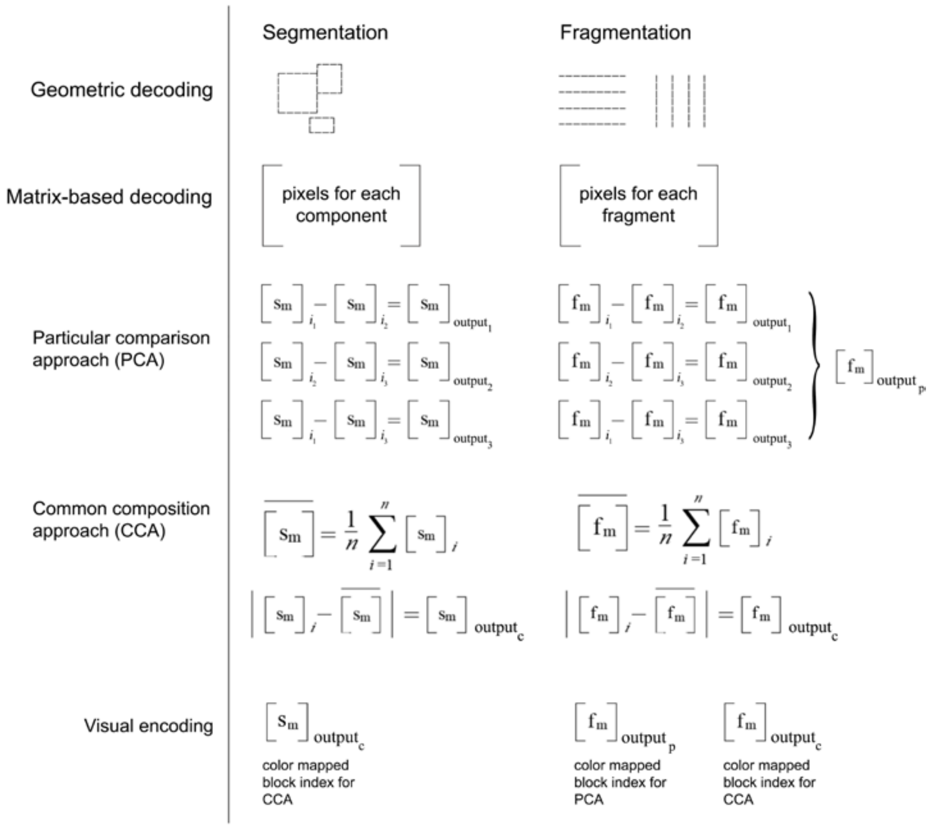


Figure 2. The process from geometric decoding to visual encoding. *n* is the number of artworks selected, *i* is the index processed, PCA refers to particular comparison and CCA refers to common composition approaches. For each selected artwork, s_m represents the segmentation matrixes, and f_m represents the fragmentation matrixes.

Particular Comparison Approach (PCA): PCA is based on calculation of numerical difference between pixel values by creating binary combinations of any number of paintings. The difference values obtained in PCA calculations allow comparison of two paintings in terms of pixel-based similarity.

Common Composition Approach (CCA): CCA employes the arithmetic average of each calculated matrix. The difference of the matrix of a selected painting from this common composition matrix shows the similarity and difference degrees of the painting with the common language on a pixel basis.

Gray-scale color mapped has been applied in order to present a more clear comparison of matrix-based calculations.

3 Geometrical Decoding of Mondrian Compositions

In the segmentation process, the paintings are classified according to the pixel values in a single piece without any sub-part. Since the pixel locations in the paintings are eliminated, only pixels in the x and y coordinates are controlled. The distribution of the five colors used in the Mondrian style in the artworks selected for the work corresponds to a 5x5 matrix.

In the fragmentation process, the paintings uploaded to p5.js are examined as separate fragments horizontally or vertically. Fragment width is determined by user input. In the scope of this study fragment division input was taken as 20. Therefore, the height value of the paintings, 500 pixels were divided into 25 fragments in total. The same algorithm also changes the coordinate values for horizontal and vertical fragments. While controlling the pixels according to the five colors in the compositions, a difference of 50 pixels was determined in the R, G, B values in order to reduce the error margin of the algorithm (Figure 3).

```
Inputs: 5 artworks of Mondrian to upload
a for user input for the width of fragments
k for the number of fragments
for  from initializing point to the k value, increase the array number of the matrixes
  for from initializing point of the array number of the matrix to the end of the
      fragments, increase the matrix elements values according to the pixels on y axis
      for from initializing point of the array number of the matrix to the end of
          the uploaded image width, increase the matrix elements values according to the
          pixels on x axis
              getting the pixels from x,y
              declare variables for Red, Green, Blue values in the image
      if B and G values are in between 0-50, and R value is in between 205-255
         increase red value
      else if R and G values are in between 0-50, and B value in between
              205-255
                  increase blue value
      else if R value is in between 0-50, and B and G values are in between
              205-255
                  increase yellow value
      else if Red, G and B values are in between 0-50
                  increase black value
      else if Red, G and B values are in between 205-255
                  increase white value
      else
                  increase other values

    for from initializing point to the k value, increase the array number of ratio matrix

Outputs: Pixel and ratio matrixes according to the value of user input as the number of
fragment, for each of the uploaded artworks
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Figure 3. The algorithm for fragmentation of the pictures according to the user input
Source: Authors.

4 Visual Encoding of the Outputs

The visual encoding process is considered as the visualization of matrix-based pixel evaluation results with gray-scale color mapping according to the numerical values of the matrix elements. Regarding the visualization, the normalization process was carried out such that the value of the largest of the matrix elements is 0 and the value of the smallest one is 255, in line with the RGB values. As the similarity increases, that is, as the relevant matrix element value decreases, a lighter color is used. In the matrix outputs of the encoding operation, R is for red, B is blue, Y is yellow, L is black, and W is used for white.

As shown in Figure 4, S_{output} p, which is the visualized version of the matrix generated with the segmentation method and common comparison approach, has revealed a result that can be seen with the naked eye when the selected artworks are viewed separately.

Although this method does not produce original information, it offers an important output in terms of ensuring the algorithmic control of existing visual information. According to Figure 4, painting No.3 has the closest value to the white calculated on average as a common composition approach. It is noticeable that the white canvas is visibly less dense in compositions 1 and 5.

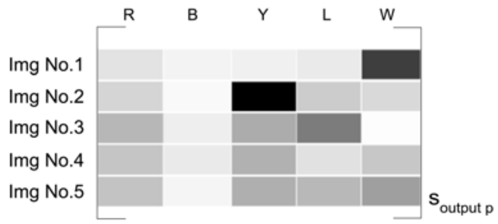

Figure 4. The visual representation of the output of segmentation method, PCA approach for each composition. Source: Authors.

Paintings decoded with fragmentation method enabled a more comprehensive analysis. The matrix including the comparative results of the first three images is shown in Figure 5 (f_{output} p). This output matrix has been prepared using Particular Comparison Approach. Numbers between 0 and 24 on the left refer to 25 fragments in which the processed paintings are divided horizontally by the algorithm. The matrix comparison values in the vertical direction for each fragment number show the difference for each horizontal fragment of the painting No. 1 and 2, 2 and 3, and 1 and 3, respectively. According to this visual encoding, it is seen that the difference between painting No. 1 - 2 and the painting numbers 2 - 3 is quite high in the first 5 fragments. Besides, the light colors in fragments 10, 11, and 12 indicate that these fragments contain values close to each other, which means these areas in the paintings are much more similar. When all the color pixels in the comparison matrix are added together, the highest difference is seen in paintings No.1 and No.2, and the least difference is in between No. 1 and No.3.

The f_{output} c shown on the right in Figure 5 is a visual prepared with the fragmentation method and common composition approach. This comparison is only the comparison of No.1 painting, with the common composition average obtained from five paintings, in horizontal fragments. One of the striking points is the abundance of places that are very close to white, that is, completely suitable for the common composition. Particularly in the use of blue and yellow, painting No.1 is quite similar to the other four paintings between fragments 9 and 21. This result confirms the total white difference in Figure 5 and also represents its distribution over fragments.

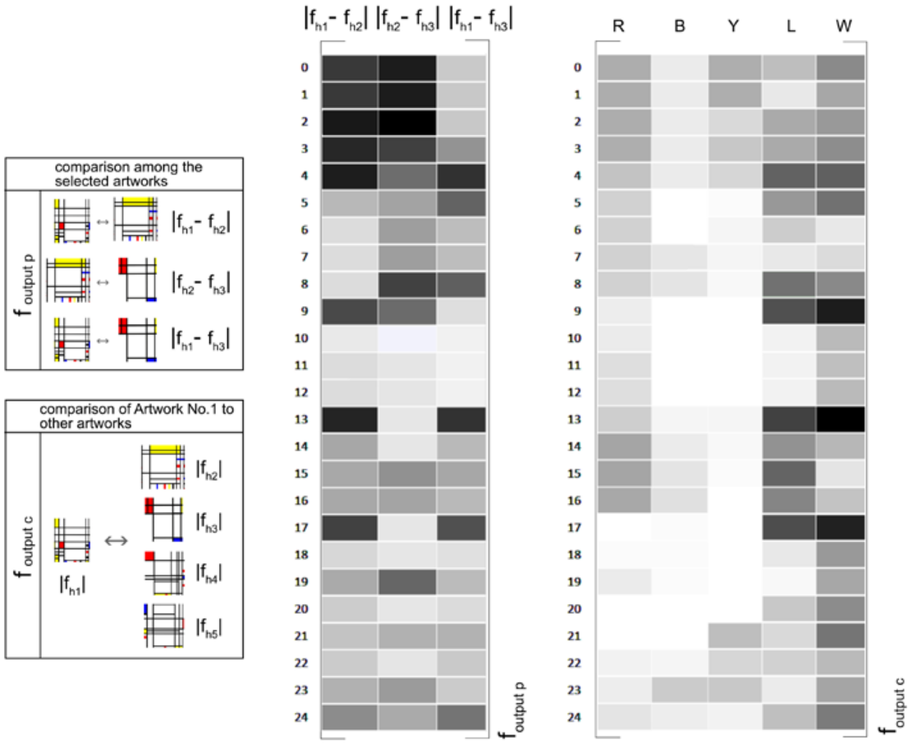

Figure 5. The horizontal fragments of the outputs according to the user input=20. f_{hi} refers to the matrix of the horizontal fragments of the related picture with number *i*. f $_{output}$ p shows the result of particular comparison among the selected picture 1, 2 and 3 (on the left). f $_{output}$ c shows the result of the comparison of No.1 and the matrix value derived from CCA within the horizontal fragments (on the right). Source: Authors.

The axes of the fragmentation were changed to decode the compositions in the vertical direction. However, in this case, the fragment values differ than each other due to varying width of the original paintings. The number of vertical fragments of the paintings are 21, 25, 22, 23 and 23 respectively.

In Figure 6, according to the comparison of the vertical and horizontal visual encodings of the fragments, that the first 6 fragments contain dark colors. This means that both vertical and horizontal paintings contain different color pixels in areas close to the starting coordinate of the composition. The middle parts of the vertical fragments of the three paintings have a more balanced distribution than the horizontal fragments and the different ratios are closer to each other. The resulting image of vertical fragmentation is f_{output} c, on the right in Figure 6. The use of black and white color is similar to the common matrix in the 6 - 18 while the fragments in the beginning and end parts of the composition differ greatly in 3rd, 21st, and 22nd fragments.

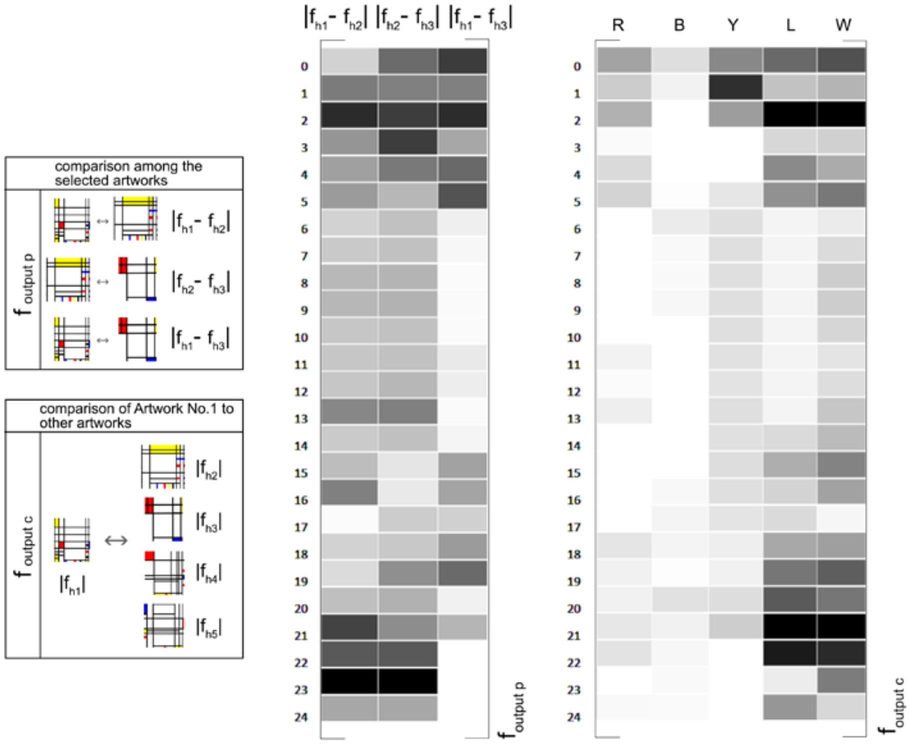

Figure 6. The vertical fragments of the outputs according to the user input=20. Source: Authors.

The f_{output} c matrixes present the horizontal and vertical similarities separately. As a result of the superposition of these two data, Figure 7 was generated. A red color overlay is applied to the grayscale output that represents the horizontal fragment differences. The blue color is assigned to the visual of the vertical fragment. In these two different colors the superposition table, the purple color scale was automatically created according to the intensity of the colors.

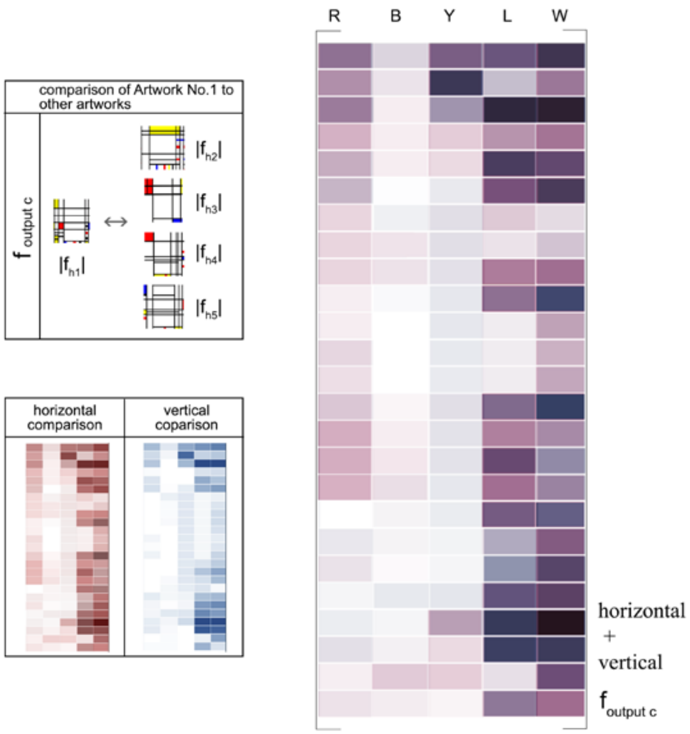

Figure 7. The superposition of the horizontal fragment output with red color overlay, and vertical fragment output with blue color overlay. Source: Authors.

We can also compare the relative similarity of a selected artwork to different artworks, based on the percentages of fragment-based color usage differences. In the comparison table of No.1, No.2 and No.3 color differences in vertical fragments, the percentage of total usage differences of R, B, Y, L, W colors in 24 vertical fragments is shown. In this table, we can say that No.2 artwork is more similar to No.1 in terms of using red in vertical fragments, and more similar to No.3 in using yellow. The use of blue, black and white is quite close to each other in percentage.

Table 1. The comparison of color usage in vertical fragments of Artwork No. 2 to No.1 and No.3. Source: Authors.

Compared Artworks	R	B	Y	L	W
No.1 - No. 2	%5,3	%2,1	%23,4	%25,2	%44
No.2 - No. 3	%10	%3	%17,2	%26	%43,3

5 Discussion and Conclusion

This study presents a computational comparison method for how Mondrian's composition style can be divided into parts by deduction and how the separated parts can be approached to the original style by induction. The segmentation and fragmentation methods reveal the visual information contained in the compositions belonging to the same style, which cannot be distinguished directly by the eye but can be produced by making assumptions. Therefore, apart from an analysis method for new reproductions, in this research, a decoding method was proposed for the comparison of existing artworks.

The method proposed in the study has been tested with 5 artworks of Mondrian. The geometrical shapes of Mondrian compositions and the color distribution used in the artwork contributed to obtaining relatively meaningful results. However, the validity and usability of the method needs to be tested with a larger and diversified sample set. Another limitation of the study is that the debateful concepts such as originality, reproducibility, relationship with context or authorship of works of art are neglected.

In the future studies, integration of the proposed analysis method and advanced computational methods and techniques (cellular automata, shape computing, image processing, pattern recognition, machine learning) might provide a basis for investigation of novel generative and transformative design models.

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