MATHEMATICAL MODELING IN A NON-LINEAR MULTIVARIABLE COUPLED TWO TANKS SYSTEM

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Abstract: The practice of system simulation based on mathematical modeling is growing every day, whether for simple processes with few variables to complex ones with multiple variables, helping to understand the results when applied to authentic systems. Nonlinear multivariable systems are regular on an industrial scale, especially in the chemical, petrochemical, and food sectors, and are often taught in higher education courses in theoretical without practical application, often not achieving adequate understanding. This project aims to present a system simulation and its validation applied in multivariable nonlinear models for a coupled tank system, experimentally demonstrating its results improving learning.

Keywords: Coupled Tanks; Mathematical Modeling; Multivariable Systems; Simulation.

MODELAGEM MATEMÁTICA EM UM SISTEMA DE DOIS TANQUES ACOPLADOS MULTIVARIÁVEL NÃO LINEAR

Resumo: A prática de simulação de sistemas a partir de modelagem matemática vem crescendo a cada dia mais, seja para processos simples com poucas variáveis, até os complexos com múltiplas variáveis auxiliando na compreensão dos resultados quando aplicados em sistemas reais. Os sistemas multivariáveis não lineares são bastante comuns em escala industrial, sobretudo nos setores químico, petroquímico e alimentício, e muitas vezes são ensinados no curso superior de forma teórica sem aplicação prática, muitas vezes não alcançando o entendimento adequado. Este projeto visa, apresentar uma simulação de sistemas e sua validação aplicada em modelos não lineares multivariável para um sistema de tanques acoplados, demonstrando de forma experimental seus resultados melhorando a aprendizagem.

Palavras-chave: Modelagem Matemática; Sistemas Multivariável; Simulação; Tanques Acoplados.

1. INTRODUCTION

Men study and try to understand phenomena and systems that guide them to solve problems in their routines. Therefore, a mathematical model is needed, that is, a mathematical representation of these systems. Mathematical modeling can be performed from equations that describe the physics of the process, however, in some circumstances the system presentation is complex or does not allow us to obtain an interpretation based on physical phenomena. A probable solution to this problem is to obtain the mathematical model from the treatment of data collected from the system, providing a statement of its dynamics. This procedure is called system identification.

The use of this type of study is important and applicable, mainly on an industrial scale, in petrochemicals and food companies, for example, but it is also widely used because it is easy to build and presents a relatively simple modeling, which can be assembled and tested in different ways.

The practice of simulation grows every day, being applied in simple to the most complex processes, helping the understanding of basic states and occurrences in real processes. During the covid 19 pandemic, it became even more important to carry out process simulations, especially when analyzed from an academic perspective, which was hampered by the lack of practical classes in laboratories.

This work presents a mathematical modeling process of two coupled tanks, on an experimental laboratory scale, built for the evaluation of a nonlinear multivariable process, simulated in the discipline of Transport Phenomena taught by Professor Michell Thompson F. Santiago at the Federal University of Bahia in the Chemical Engineering department during the process of synchronous remote classes.

2. METHODOLOGY

The coupled tank system adopted in this study is represented in Figure 1 and can be described by the following equations:

$$\frac{dh_1}{dt}A_1 = Q_{in1} - K_1\sqrt{h_1} \pm K_{ac}\sqrt{|h_1 - h_2|} \qquad (1)$$

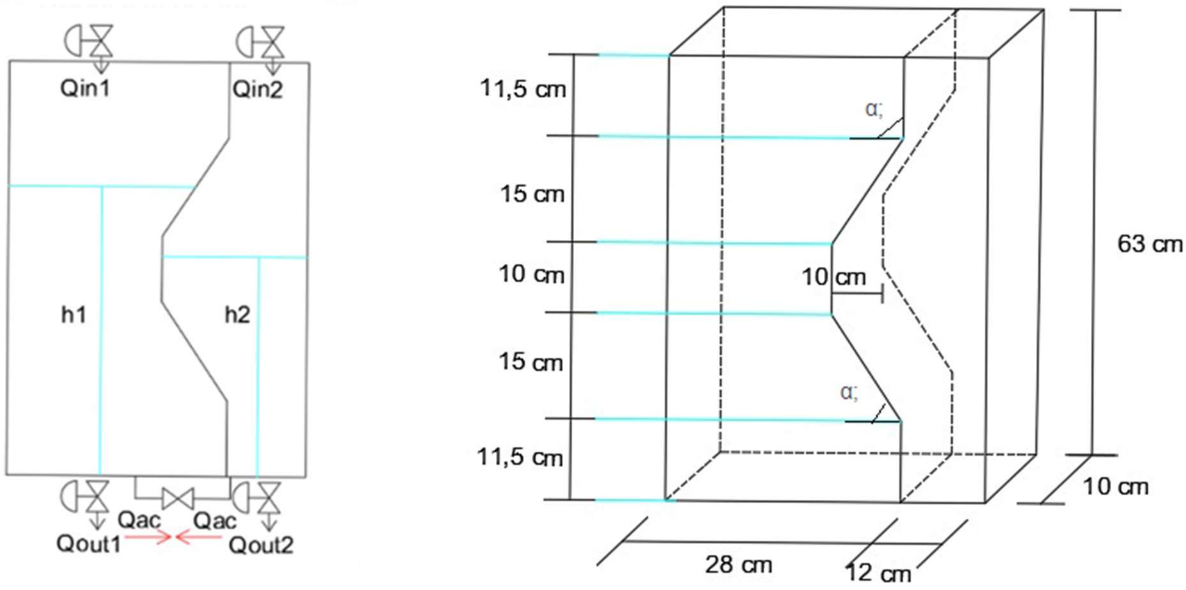
$$\frac{dh_2}{dt}A_2 = Q_{in2} - K_2\sqrt{h_2} \pm K_{ac}\sqrt{|h_1 - h_2|} \qquad (2)$$

With h1(t) and h2(t) being the levels, Qin1(t) and Qin2(t) the inflows and A1(t) and A2(t) the section areas of tanks 1 and 2 at heights h1(t) and h2(t) respectively. The parameters K1, K2 and Kac are the constants associated with the flow rates of tank 1 of tank 2 and coupling between the tanks, respectively, and will be used in the simulations to obtain the curves of the tank levels over time.

ISSN: 2357-7592

Figure 1. Illustrative diagram of the coupled tank system. Source: The authors.

For tank areas, the following applies:



$$A_1(t) = g(h_1(t)), \qquad A_2(t) = 40\,.\,10\ - g(h_2(t)) \qquad (3)$$

With g(h) in cm², with h in cm, given by:

$$g(h) = \begin{cases} 120 & 0 \le h \le 11{,}5 \\ 120 + 10\,.\,\dfrac{(h - 11{,}5)}{\tan(\alpha)} & 11{,}5 \le h \le 26{,}5 \\ 220 & 26{,}5 \le h \le 36{,}5 \\ 220 - 10\,.\,\dfrac{(h - 36{,}5)}{\tan(\alpha)} & 36{,}5 \le h \le 51{,}5 \\ 120 & 51{,}5 \le h \le 63 \end{cases} \qquad (4)$$

In addition, the tangent of the angle α:

$$\alpha = atan(2) \Leftrightarrow tan(\alpha) = 2 \qquad (5)$$

The set of equations (1) to (5) was implemented in a model in Simulink, to evaluate the nonlinear model presented, as outlined in Figure 2:

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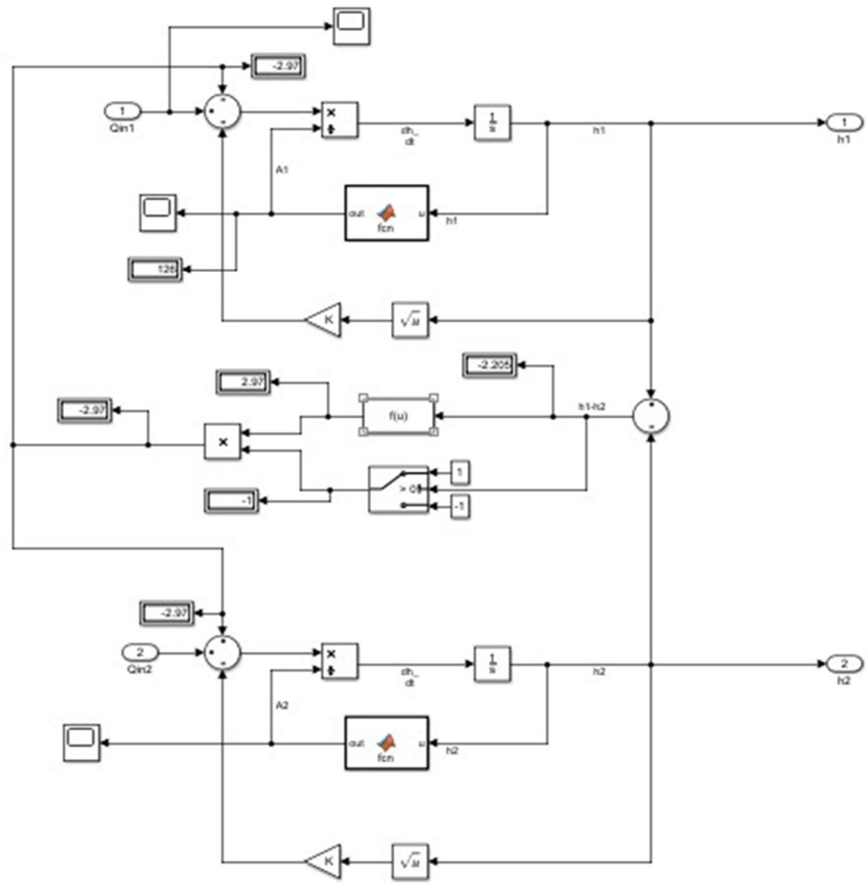
Figure 2: Mathematical modeling of the coupled tank system in Simulink



The input Qin1, Qin2 output Qout1 Qout2 and Qac coupling flow rates are described by the following equations:

$$\begin{cases} Q_{in1} = K_1.\sqrt{h_1} + K_{ac}\sqrt{h_1 - h_2}\,, & h_1 \geq h_2 \\ Q_{in2} = K_2.\sqrt{h_2} - K_{ac}\sqrt{h_1 - h_2}\,, & h_1 \geq h_2 \\ Q_{in1} = K_1.\sqrt{h_1} - K_{ac}\sqrt{h_1 - h_2}\,, & h_1 < h_2 \\ Q_{in2} = K_2.\sqrt{h_2} + K_{ac}\sqrt{h_1 - h_2}\,, & h_1 < h_2 \end{cases} \tag{6}$$

$$Q_{out1} = K_1.\sqrt{h_1} \tag{7}$$

$$Q_{out2} = K_2.\sqrt{h_2} \tag{8}$$

$$Q_{ac} = K_{ac}.\sqrt{h_1 - h_2} \tag{9}$$

In turn, different operating conditions of the tank valves were considered, varying the inlet flow through constants K1 and K2 which, for simplification purposes, were considered equal, in addition, the coupling flow between them was also varied in the same order of magnitude. 12 simulations were performed, which are summarized in table 1:

Table 1: Simulations

Simulação	Kac [cm²/s]	K [cm²/s]	h1o [cm]	h2o [cm]	h1f [cm]	h2f [cm]
1	2,0	2,5	0,0	0,0	23,0	35,0
2	2,0	2,5	5,0	16,0	25,0	45,0
3	2,0	2,5	25,0	17,0	41,0	52,0
4	20,0	25,0	0,0	0,0	15,0	19,0
5	20,0	25,0	12,0	4,0	48,0	22,0
6	20,0	25,0	7,5	30,0	15,0	45,0
7	200,0	250,0	0,0	0,0	27,0	31,0
8	200,0	250,0	41,0	37,0	54,0	52,0
9	200,0	250,0	2,0	5,0	22,0	18,0
10	2000,0	2500,0	0,0	0,0	34,0	46,0
11	2000,0	2500,0	6,0	2,0	18,0	31,0
12	2000,0	2500,0	27,0	30,0	35,0	38,0

3. RESULTS AND DISCUSSION

The images in figures 3 to 14 illustrate the results that were obtained in the simulations, for the different operating conditions. From the behavior of the curves, it can be seen that the model under analysis is a first-order system, whose response will depend on the dynamic variables involved in the process, for a small flow, modeled in simulations 1, 2 and 3 has it is assumed that the response of the system will tend to be greater, and it will take a longer time to stabilize, in steady state, (around 1000 seconds, approximately), in view of this, new tests were carried out, gradually increasing the flow (simulations 4 to 12) observing a faster system response which was approximately 100 seconds in simulations 4 to 6 10 seconds in simulations 7 to 8 and 2 seconds in simulations 9 to 12 , it should also be noted that different conditions were considered initial levels, such as an empty or half-full tank. It should also be remembered that the main purpose was to simulate different response times for the valves, which is normally done in the industry, adjusting their opening, or closing, which can be done manually or automatically, with the help of control techniques (which will not be covered in this article).

Figure 3: Simulations 1

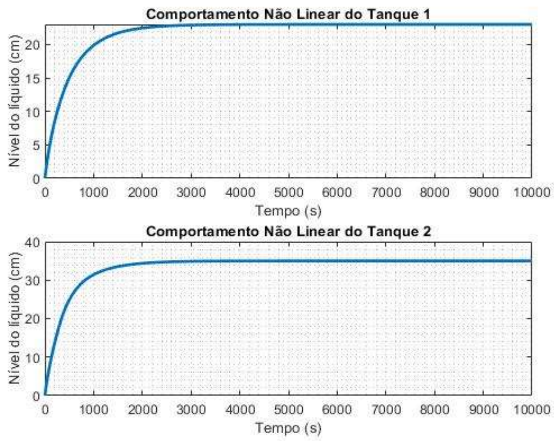


Figure 4: Simulations 2

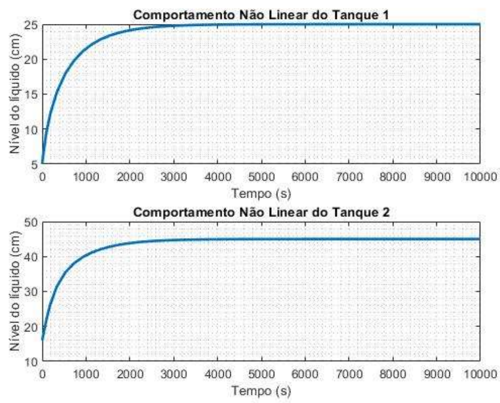


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Figure 5: Simulations 3

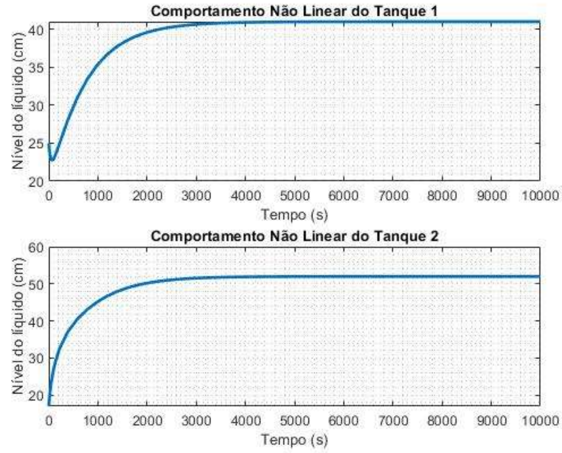

Figure 6: Simulations 4

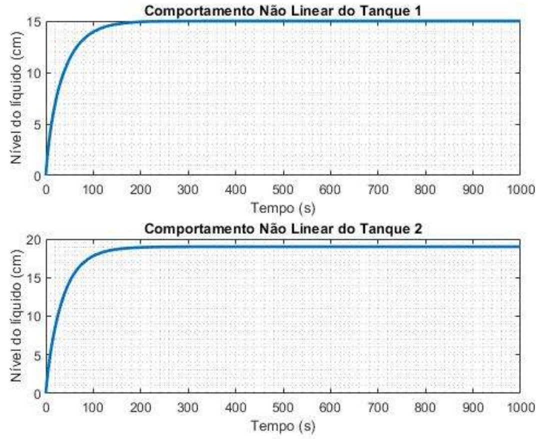

Figure 7: Simulations 5

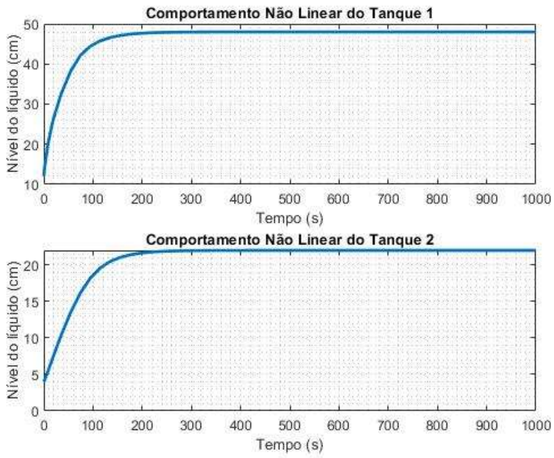

Figure 8: Simulations 6

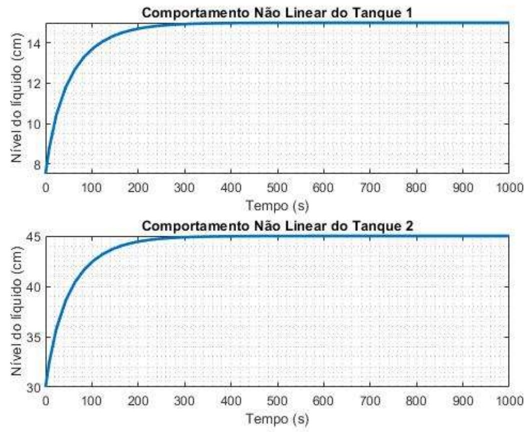

Figure 9: Simulations 7

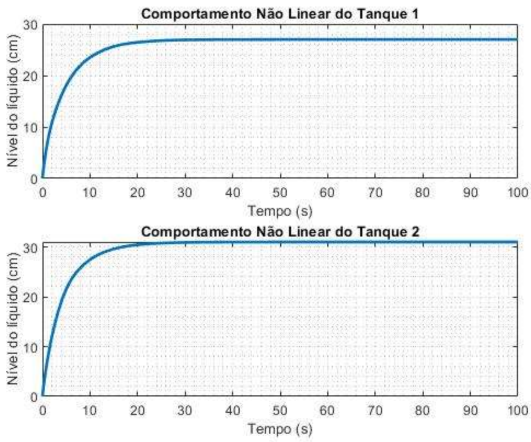

Figure 10: Simulations 8

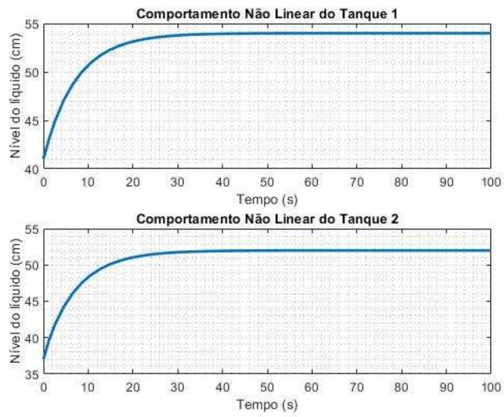

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Figure 11: Simulations 9

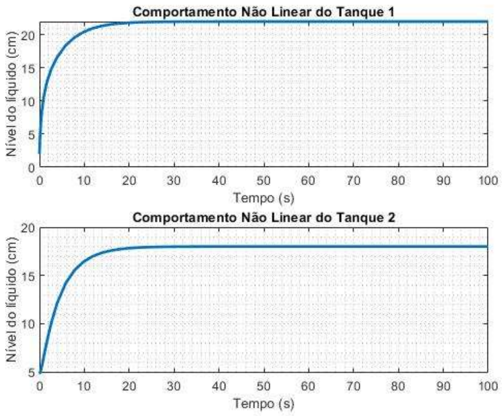


Figure 12: Simulations 10

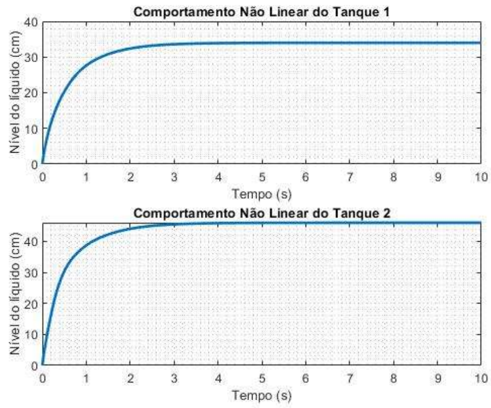


Figure 13: Simulations 11

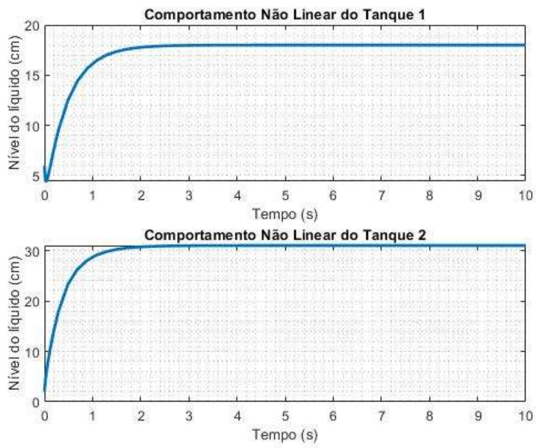
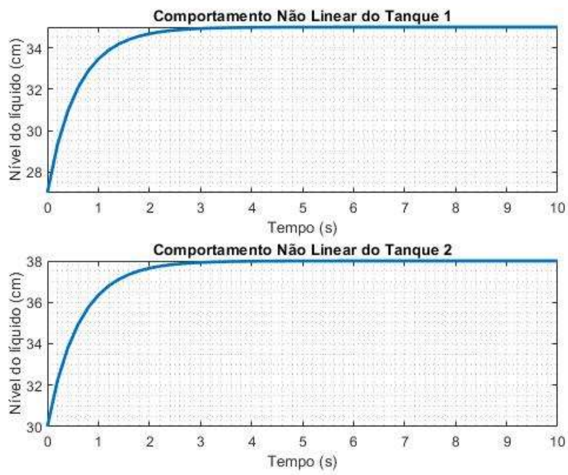


Figure 14: Simulations 12



4. CONCLUSION

After the study and results obtained, it is clear that through the modeling of the system in question it is possible to have a better understanding of the physical phenomenon, such as the relationship of constants linked to flow and level stabilization time, which facilitates the use of such structures in the industrial scope and in the learning and better understanding of such phenomena, from which it is possible to obtain satisfactory results and conclusions and which add experimental knowledge without the need for physical design.

However, it can be concluded that it was possible to better understand and study the phenomenon of coupled tanks through mathematical modeling and the aid of the simulink software, considering the results obtained in this article as satisfactory.

5. REFERENCES (ARIAL 12)

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