MODELING, CONTROL AND SIMULATION OF A SYSTEM INVOLVING A CONICAL TANK

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Abstract: The present paper aims to model, through a theoretical approach followed by virtual simulation, two different control systems involving a conical tank – a level control done by a pump and a flow rate control done by a valve. Mathematical approximations were used to represent the elements of the plant. The tank's model was linearized through deviation variables and Taylor Series. Proportional and proportional integral (PI) compensations were designed with the aid of the root locus diagram and controller synthesis. Results confirmed the controllers' proper action around the linearization point. The paper concludes with the proposal of some improvement ideas and a caution regarding the limited usability of the root locus.

Keywords: modeling; control; nonlinear system; root locus; conical tank.

MODELAGEM, CONTROLE E SIMULAÇÃO DE UM SISTEMA ENVOLVENDO UM TANQUE CÔNICO

Resumo: O presente artigo visa modelar, através de uma abordagem teórica seguida de simulação virtual, dois sistemas de controle envolvendo um tanque cônico – um controle de nível efetuado por uma bomba e um controle de fluxo efetuado por uma válvula. Aproximações matemáticas foram usadas para representar os elementos da planta e o modelo do tanque foi linearizado por meio de variáveis de desvio e Série de Taylor. Compensações proporcional e proporcional integral (PI) foram projetadas usando o diagrama do lugar das raízes ou a síntese de controlador. Os resultados confirmaram a atuação adequada dos controladores em torno do ponto de linearização. O trabalho conclui com a proposta de algumas ideias de melhoria e um alerta sobre a limitada usabilidade do lugar das raízes.

Palavras-chave: modelagem; controle; sistema não-linear; lugar das raízes; tanque cônico.

1. INTRODUCTION

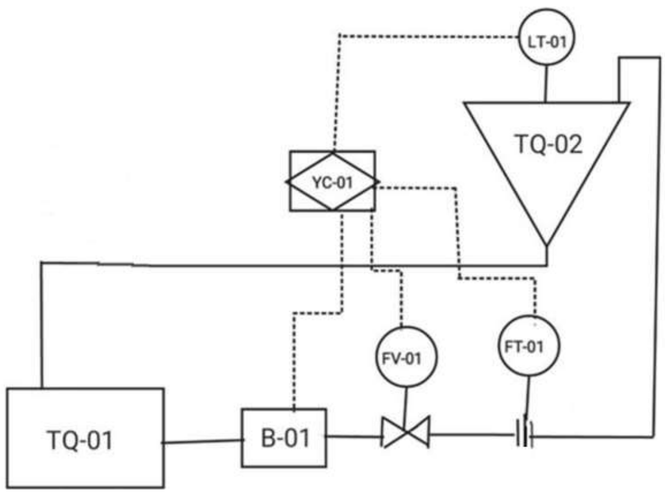
Controlling the flow rate and fluid level in a tank is a recurring practice in the industry – for example, when dealing with reboilers or evaporators – and therefore these critical process variables need to be carefully monitored and adjusted [1,2]. When further considering environment disturbances and nonlinearities associated with real-life systems, it becomes evident the necessity of a control that accounts for these unwanted conditions.

For that purpose, mathematical models may be used to approximately and abstractly represent a plant through equations, should these be available or obtainable [3,4]. The implementation here proposed thus seeks to illustrate how an automated process may be designed to correct errors and deal with a nonlinear plant through its model.

Despite considering a not so critical scenario, involving the control of flow and water level in a conical tank, this paper chose to focus on a simplified scenario to first solidify a basic understanding as to how these controls work and how one may approach when designing them.

Two systems will be considered: a level control by varying the pump B-01 speed and a flow rate control by adjusting the valve FV-01. A generic tank TQ-01 is coupled with a conical one TQ-02 like shown in Figure 1, which additionally specifies a level sensor LT-01 and a flow sensor FT-01, both associated with their respective actuators through a microcontroller YC-01.

Figure 1. Schematic of the theoretical system



2. METHODOLOGY

Mathematical models will first be derived through mass balance and first-order approximations of Ordinary Differential Equations (ODE), in an effort to describe the dynamic behavior of the system's components. Due to the nonlinear nature of the conical tank in this system, a technique of linearization will also be applied, consisting on the use of the Taylor Series alongside deviation variables [3,4].

The controller design will be done on MATLAB either via controller synthesis or the root locus method applied to the plant’s open-loop transfer function – the latter’s use facilitates the manipulation of closed-loop poles, and consequently the system's behavior, by adjusting the compensator gain [4,5].

Despite having suggested that a microcontroller would be the one responsible to apply the designed forms of compensation, it is outside the scope of this paper to consider the root locus dealing with sampled data, which would be required if a physical model were to be constructed. Nonetheless, the analysis that will be made in the continuous domain still serves not only as a good initial approach to understand the functioning of the system, but also as a great approximation for digital signals with low sampling periods [6].

Lastly, a closed-loop simulation will also be made with the help of Simulink, where the main components of each control scenario will be represented by their time-domain mathematical models – as to better portray their associated nonlinearities –, and a step response will be obtained for values both around and distant from the linearization point, in order to validate the theoretical design.

2.1 Conical Tank

The tank may be modeled by its mass balance represented in Equation (1), which is considering the water density as a constant, and by some constitutive relations associated with a cone [3], shown in equations (2), (3) and (4).

$$\frac{dV(t)}{dt} = Q_i(t) - Q_o(t) \tag{1}$$

$$Q_o(t) = K\sqrt{h(t)} \tag{2}$$

$$V(t) = \frac{h(t)\pi r^2(t)}{3} \tag{3}$$

$$r(t) = \frac{R}{H}h(t) \tag{4}$$

, where

$V(t)$: fluid volume $[m^3]$;
$Q_{i/o}(t)$: input/output flow rate $[m^3/s]$;
K: flow rate and height proportionality constant $[m^{2.5}/s]$;
$h(t)$: fluid height $[m]$;
$r(t)$: fluid base radius $[m]$;
R: tank base radius $[m]$;
H: tank height $[m]$.

Thus, the mathematical model of a conical tank may be represented by the Equation (5).

$$\frac{\pi R^2}{3H^2}\frac{dh^3(t)}{dt} = Q_i(t) - K\sqrt{h(t)} \tag{5}$$

Due to the cubic and squared heights, this model is strongly nonlinear and should be linearized in order to allow the design of a controller via root locus. The method applied considered the height as $h(t) = \bar{h} + \hat{h}(t)$, where $\bar{h}$ represents a stationary height – also serving as the point of interest for the linearization, around where the controller will work as expected, and considered as half the tank's total height in this paper – and $\hat{h}$ represents the deviation variable. Similarly, the input flow should be treated as $Q_i(t) = \bar{Q}_i + \hat{Q}_i(t)$, after which the Taylor Series may be applied for both the cubic and squared terms, resulting in Equation (6).

$$\frac{\pi R^2 \bar{h}^2}{H^2}\frac{d\hat{h}(t)}{dt} = \hat{Q}_i(t) - \frac{K}{2\sqrt{\bar{h}}}\hat{h} \tag{6}$$

2.2 Pump and Valve

Both actuators may be approximately described as an output flow exclusively dependent on the controller signal, having their transient characteristics represented by a first order system [3-5], shown in (7).

$$\tau_{p/v}\frac{dQ_o(t)}{dt} + Q_o(t) = m(t) \tag{7}$$

, where

$Q_o(t)$: output flow rate $[m^3/s]$;
$m(t)$: control output signal $[m^3/s]$;
$\tau_{p/v}$: pump/valve time constant $[s]$.

One point to note is that this application considers that there always will be water present in the pipes, guaranteeing a minimum pressure for the pump to operate normally. This justifies a model that associates its output flow exclusively to the signal coming from the controller [5], implying that the contribution of the generic tank TQ-01 is insignificant in the mass balance and may be disregarded in the system modeling.

2.3 Level and Flow Sensors

While the pump and valve were represented by a first-order system, intending to approximate them to a more realistic behavior (i.e., not instant), both sensors may indeed be modeled as simple static gains, since their transient response are much faster if compared to the other system elements [5]. Considering a unity gain, their models may also be completely neglected.

2.4 System Parameters

All mentioned parameters are summarized in the following Table 1. Constants were chosen based on approximations of real possible values, such as of an electromechanically-actuated control valve or a standard centrifugal pump [2-5].

Table 1. Constants used and their values

$H\ (m)$	$R\ (m)$	$K\ (m^{2.5}/s)$	$\tau_p\ (s)$	$\tau_v\ (s)$
1	0.25	$1\text{x}10^{-5}$	0.5	2.5

The final system response should take into account not only desired characteristics, but also the components' capability to react properly to any changes and disturbances.

The level control presented, for instance, may demand unreasonably high-capacity pumps in order to handle any exceedingly low settling times, which should not even be a requirement in the first place, due to the non-time-critical nature of its controlled variable. A response, then, in the order of dozens of seconds should be enough when designing the controller for it. On the other hand, the flow rate control mainly needs to account for its slow-acting valve response. The criterion of a two-times-slower process was then arbitrarily defined for this compensation design.

3. RESULTS AND DISCUSSION

3.1 Level Control

The water level being measured must be sent in a closed-loop to the controller, which will compare it with a reference value and send a signal to control the flow out of the pump, trying to minimize the error. Since the valve is kept completely open, its influence on the flow rate as a small friction loss was considered negligible and its model disregarded in this system.

3.1.1 Controller Design

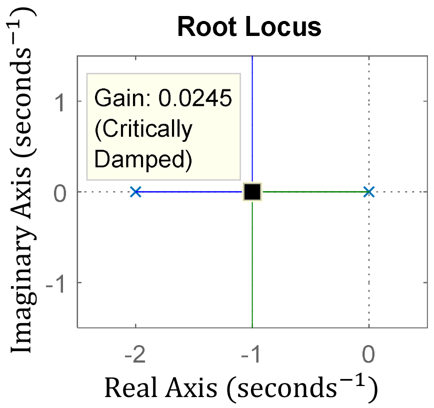
Based on Equations (6) and (7), it is possible to represent in the frequency domain the tank $T(s)$ and the pump $P(s)$, shown in (8) and (9), respectively.

$$T(s) = \frac{1}{\pi\left(\frac{R\bar{h}}{H}\right)^2 s + \frac{K}{2\sqrt{\bar{h}}}} \tag{8}$$

$$P(s) = \frac{1}{\tau_p s + 1} \tag{9}$$

The product of (8) and (9) results in the open-loop equation $G(s)$, from which the root locus diagram shown in Figure 2 may be obtained.

Figure 2. Root locus for level control

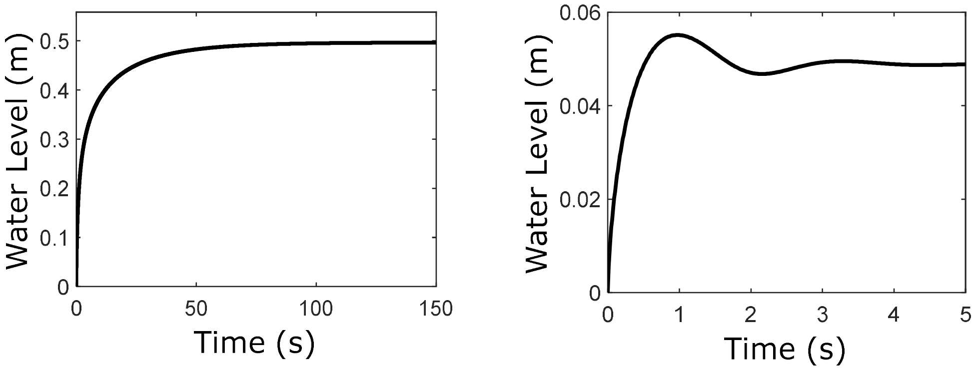


Since the pole is already so close to the origin, an integrator characteristic may be drawn on and a simple proportional controller ends up being enough, thus the gain may be adjusted until a desired response is obtained. Following the previous defined requirement for a settling time of dozens of seconds, the gain was empirically adjusted to $2\text{x}10^{-3}$, having as a basis that at $2.45\text{x}10^{-2}$ the response would become critically damped.

3.1.2 Step Response

Figure 3(a) illustrates how the controller was able to work as designed with a settling time of approximately $60\ \text{s}$, but only around the linearization point of $0.5\ \text{m}$, and Figure 3(b) depicts a much faster underdamped response at a reference of $0.05\ \text{m}$. Even with just this proportional gain, the steady state error was practically non-existent.

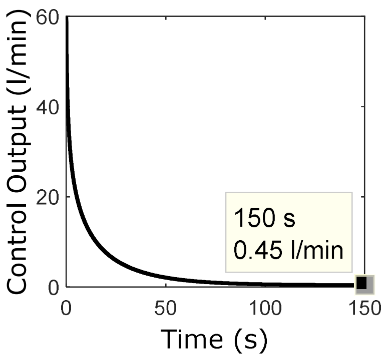
Figure 3. Step response of the first control to level references of (a) $0.5\ m$ and (b) $0.05\ m$



(a) (b)

Figure 4 also shows the controller output converted to l/min, considering the $0.5\ \text{m}$ water level scenario, which assures that the design would be possible in a real-life situation – the controller would only need to provide a flow rate up to $60\ \text{l/min}$ for a brief amount of time.

Figure 4. Step response of the controller output for a $0.5\ \text{m}$ reference level



3.2 Flow Rate Control

The measured flow rate will also be compared to a reference value in order to control the valve. Since the water level is now irrelevant, so will be the contribution of the conical tank in this system. Without its nonlinearity, this control may be considered weakly nonlinear, unlike the previous one.

3.2.1 Controller Design

Based on Equation (7), the valve may be represented as in (10).

$$V(s) = \frac{1}{\tau_v s + 1} \tag{10}$$

The open-loop equation $G(s)$ would be approximately equal to $V(s)$, as the valve now represents the only dominant pole in the system. However, it is important to note that this approximation will only hold true if the added controller gain is sufficiently small, since the plant's real poles will initially approach each other with increasing gains [4,5] – this could cause the previously neglected flow sensor to have an impact on the system response, as its pole would not be dominated by the valve anymore.

Without the tank's near-origin pole, it would be a good idea to implement some kind of integral compensation to reduce the steady state error. Due to the loop containing a single pole, a root locus analysis seemed redundant and thus a proportional integral (PI) controller $G_c(s)$ was designed through controller synthesis, where the integral time is set to τ_v, and τ represents the closed-loop time constant [5], as shown in (11).

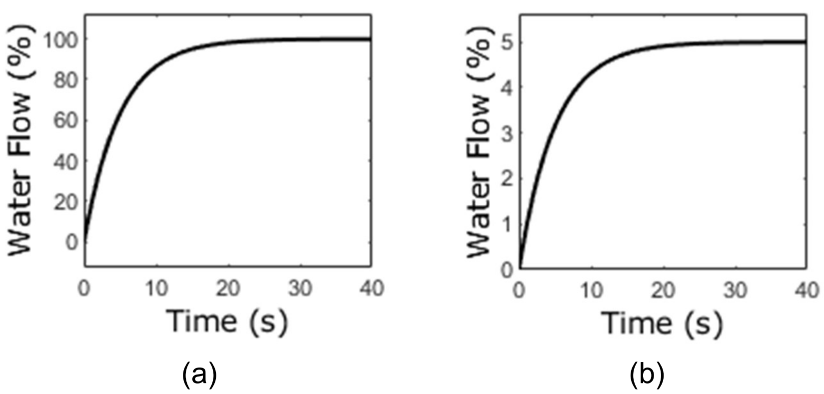
$$G_c(s) = \frac{\tau_v}{\tau}\left(1 + \frac{1}{\tau_v s}\right) \tag{11}$$

As the requirement is to have a total response time two times slower than the valve (i.e., $\tau = 2\,\tau_v$), the controller's gain may be directly inferred as $1/2$, considering that the plant is now acting as an integrating system.

3.2.2 Step Response

Since this system is theoretically linear, its step response will be the same whichever reference is chosen, seen in Figure 5. The controller now directly outputting a flow rate signal would also look like these.

Figure 5. Step response of the second control to flow rate references of (a) 100% and (b) 5%



4. CONCLUSION

It was possible to display control actions being used in two different scenarios, and how each one of them was designed depending on their intrinsic characteristics. Likewise, it has been observed how strongly nonlinear systems deviate from this design when operating far from their equilibrium – perhaps a multi-point linearization method could be considered.

It is important to note that a non-critical scenario such as the one presented here allowed for more freedom of choice on the controller design and thus the criterion for a slow enough system could be achieved. However, real life systems may require more tradeoffs involved with time-critical underdamped responses that were not considered in this paper.

Further empirical tests could also be made to obtain more accurate constants, and since a lot of approximations were considered in favor of simplicity – such as not considering the impacts of friction loss or the contribution of TQ-01 –, a more complete analysis may be necessary if physical tests were to be made.

At last, it is import to note that despite the root locus being used throughout this paper, with great utility in visualizing the process of tuning a controller, in practice it is known that its applicability may be very limited due to the requirement of the plant's mathematical model, something that in most cases is not readily available in the industry.

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