A REVIEW OF MATHEMATICAL TRANSFORMS FOR PHASE-LOCKED LOOPS CONTROL

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Abstract: The mathematical transforms used in Phase-Locked Loops (PLLs) play a crucial role in their performance and functioning. Depending on the format of the control loop's input signal, noise level requirements and other requirements such as available computational power, one must choose which topology best suits a given application. This article will address the most used transforms and filters in the most common PLL topologies, such as the Clarke Transform, the Park Transform, the Second-Order Generalized Integrator (SOGI) filter and the Hilbert Transform. Finally, a comparison of the advantages and disadvantages of each of these methods is discussed.

Keywords: Phase-Locked Loop; Park Transform; Clarke Transform; Second-Order Generalized Integrator; Hilbert Transform.

UMA REVISÃO DAS TRANSFORMADAS MATEMÁTICAS PARA CONTROLE DE PHASE-LOCKED LOOPS

Resumo: As transformadas matemáticas utilizadas em Phase-Locked Loops (PLLs) possuem um papel crucial em seu desempenho e funcionamento. A depender do formato do sinal de entrada da malha de controle, dos requisitos de nível de ruído e outros requisitos como poder computacional disponível, deve-se se escolher qual topologia se adequa melhor a determinada aplicação. Esse artigo irá abordar as transformadas e filtros mais utilizados nas topologias mais comuns de PLLs, como a Transformada Clarke, a Transformada Park, o filtro Second-Order Generalized Integrator (SOGI) e a Transformada Hilbert. Ao final, um comparativo das vantagens e desvantagens de cada um desses métodos é abordado.

Palavras-chave: Phase-Locked Loop; Transformada Park, Transformada Clarke, Second-Order Generalized Integrator, Transformada Hilbert.

1. INTRODUCTION

The Phase-Locked Loop (PLL) is a control loop system, where the output phase is compared with the phase of an input signal [1]. The purpose of this closed loop is to reduce the phase error between a reference signal and a generated signal, changing its frequency. The application of this component can vary from demodulators, frequency synthesizers, synchronization circuits and frequency followers.

There are many types of PLL topologies and each one of them has its own advantages and disadvantages [2-5]. This article will present a technical comparison between the most used PLL topologies focusing on the mathematical transformations and filters used on them.

2. METHODOLOGY

The main objective of this article is to present a review of the scientific literature found mainly in the IEEE Xplorer database and a comparison between the mathematical transformations used on the most common PLL topologies.The research was made with a focus on articles that show the details of these techniques as well as the mathematical approach.

The following specific objectives will be discussed in this article:

- Review of the most used PLL topologies and which transformations they use;
- Review of the theoretical aspect of each transformation and their usage;
- Comparison of these transformations with regard to their advantages and disadvantages.

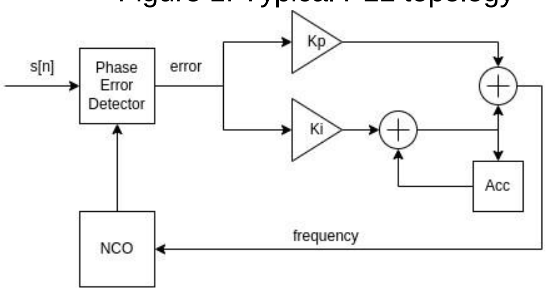
3. RESULTS AND DISCUSSION

The classical architecture of the PLL is made up of three major components: the phase detector, the loop filter and a voltage or numeric controlled oscillator (VCO/NCO) depending if this module is analog or digital respectively.

The phase detector, as the name says, is responsible for estimating the phase shift between the reference and the generated signals. The loop filter is a digital controller, usually a proportional-integral (PI) to adjust the output signal frequency to reduce the phase error. And finally the NCO is responsible for generating the output signal based on the given phase and a lookup-table of the desired signal.

An architecture of a PLL can be seen in the figure 1.

Figure 1: Typical PLL topology



There are many topologies of PLLs and usually the difference is related to the phase detector module, which is the most important block in this component.

Some of the most important topologies are the following:

- Synchronous Reference Frame Phase-Locked Loop (SFR-PLL) [2];

- Second Order Generalized Integrator Phase-Locked Loop (SOGI-PLL) [3];
- Double Second-Order Generalized Integrator-Quadrature Signal Generator PLL (DSOGI-QSG-PLL) [4];
- Frequency Fixed Second-Order Generalized Integrator Phase-Locked Loop (FFSOGI-PLL) [5];
- Cascaded Second-Order Generalized Integrator Phase-Locked Loop (CSOGI-PLL) [3];
- Second Order SOGI-PLL [3];
- Modified SOGI PLL [3];
- Mixed Second-and Third-order Generalized Integrator (MSTOGI-PLL) [3];
- Hilbert Transform Based PLL [1].

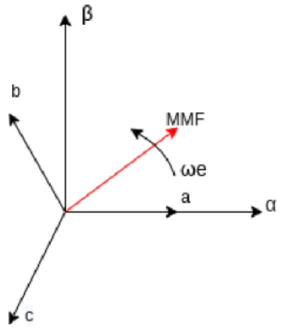
The main idea of a PLL system, as said before, is to reduce the phase error between a generated signal and a reference signal. One of the simplest topologies is the SFR-PLL, and many others are just a variation of it.

There are four main transformations used in PLLs: the Clarke Transform (abc-αβ)[2], the Park Transform (αβ-dq)[2], the Hilbert Transform [1] and the SOGI Filter [3], used to generate orthogonal signals. These transformations will be discussed further on.

3.1.CLARKE TRANSFORM

The Clarke Transform is a transformation of a stationary three phase system (ABC) to a stationary biphasic system (αβ) [6]. These new coordinates are also in quadrature, i.e., they have a phase difference of 90° between them, where α is the abscissa (real part) and β the ordinate (imaginary part). The difference between both frames is presented in figure 2, as well as the magnetomotive force (MMF)[6].

Figure 2: Representation of the αβ axes



A simple way of understanding this transformation is relating it with an electrical motor. The force produced by the coil of a balanced three-phase motor is the vector sum of the force of each phase. In the equations 1 and 2, it is shown the force of each axis in the Clarke frame. They are generated by the vector decomposition of a three-phase system.

$$F_\alpha = F_a\cos(0) + F_b\cos\left(\frac{2\pi}{3}\right) + F_c\cos\left(\frac{4\pi}{3}\right) = F_a - \frac{1}{2}F_b - \frac{1}{2}F_c \quad (1)$$

$$F_\beta = F_a\sin(0) + F_b\sin\left(\frac{2\pi}{3}\right) + F_c\sin\left(\frac{4\pi}{3}\right) = 0 + \frac{\sqrt{3}}{2}F_b - \frac{\sqrt{3}}{2}F_c \quad (2)$$

Simplifying the equations above, we arrive at the matrix representation of the Clarke Transform [6]:

$$\begin{bmatrix} f_\alpha \\ f_\beta \\ f_o \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \quad (3)$$

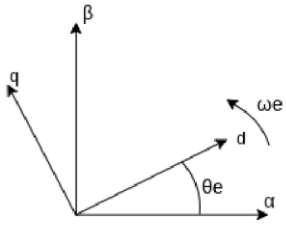
Note the inclusion of the zero component (f_0), which is just a mathematical

device for the transform to have an inverse form, i.e., the matrix must be square.

3.2.PARK TRANSFORM

The Park Transform [6] is similar to Clarke's, but it transforms the stationary Clarke αβ axes into a rotational two-phase system with fictitious DQ coordinates, as can be seen in Figure 3.

Figure 3: Representation of the DQ axes



The difference of this transform is that the DQ axes rotate at the same frequency as the system, e.g., the frequency of the electric network or rotor from which the DQ axes originated. Therefore, the DQ components become continuous, with D being the component of the direct axis and Q being the quadrature axis [6].

The transform can be done in two ways: ABC-DQ or αβ-DQ. The equations for both can be seen below [6].

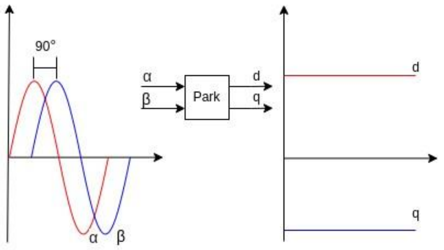
$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{N_{abc}}{N_{\alpha\beta}}\begin{bmatrix} \cos(\theta_e) & \cos\left(\theta_e - \frac{2\pi}{3}\right) & \cos\left(\theta_e - \frac{4\pi}{3}\right) \\ \sin(\theta_e) & \sin\left(\theta_e - \frac{2\pi}{3}\right) & \sin\left(\theta_e - \frac{4\pi}{3}\right) \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(\theta_e) & \sin(\theta_e) \\ -\sin(\theta_e) & \cos(\theta_e) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (5)$$

where the $N_{abc}/N_{\alpha\beta}$ ratio is the ratio between the number of turns of the coil on the ABC axis and αβ. This ratio can assume two values depending on the type of transformation. For an invariant amplitude transformation, $\frac{2}{3}$ is used and for an invariable power transform, $\sqrt{\frac{3}{2}}$ is used instead.

Figure 4 shows the DQ components generated by the Park Transform. One of the advantages of being continuous components is that it facilitates the operations and complexity of control loops.

Figure 4: Representation of the Park Transformation



It is worth mentioning that when the quadrature component (Q) is zero, it means that the rotational axis DQ is in phase with the reference input signal.
Therefore, this transform is widely used in network synchronization systems.

3.3.HILBERT TRANSFORM

Digital Hilbert Transforms are a special class of all-pass digital filter whose characteristic is to introduce a π/2 radians phase shift of the input signal [7]. This transformation can be represented as a digital filter or a convolutional operator as seen in the equation 9 and 10 [7], which represents the Hilbert kernel and the frequency response respectively.

$$x_o = x(n) * h(n) \quad (6)$$

$$where\ h(n) = \begin{cases} \frac{2}{\pi n} & n\ is\ odd \\ 0 & n\ is\ even \end{cases}$$

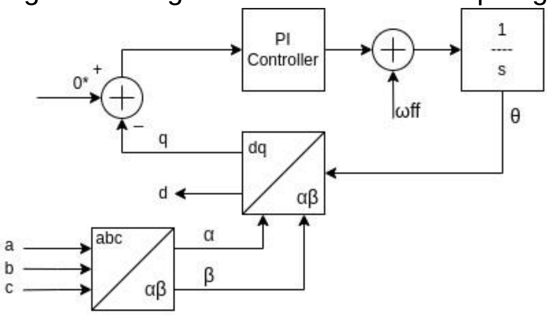
$$H(f) = -jsgn(f) \quad (7)$$

As it is a convolutional operator, it can be represented as a Finite Impulse Response (FIR) Filter. The Hilbert Transform can be used to generate a quadrature signal, an alternative to the Clark Transform.

3.4.SFR-PLL

The SRF-PLL is one of the simplest topologies based on the Clarke and Park transforms [2]. The schematic can be seen in figure 5.

Figure 5: Diagram of the SFR-PLL topology



The working principle of this PLL assumes that when the signals are in phase and the quadrature component, i.e., Vq, will be zero. The PI controller changes the angular frequency in order to set Vq to zero, thereby setting the phase difference between the two signals to zero. To perform the feedback of the control loop, the angular frequency integral is performed, which results in the phase that is used for the Park transform and the generation of the output signal.

This structure, although simple, has disadvantages for having a high error for unbalanced three-phase systems and containing harmonics.

3.5.SOGI-PLL

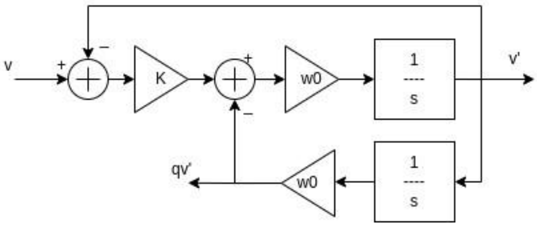
The SOGI-PLL architecture [3] is an enhancement of the SRF-PLL. As there aren’t three phases to perform the Clarke Transform and generate the quadrature signal, the SOGI filter itself is used for this purpose.

The SOGI filter is a bandpass filter centered on a frequency ω0 and a damping factor k = 1/Q. This filter has two particularities:

- Its two outputs have unity gain at the center frequency;
- It is possible to adapt its center frequency without having to recalculate its coefficients.

The filter architecture and its transfer functions can be seen in figure 6, where v' and qv' are the quadrature signals.

Figure 6: Diagram of the SOGI filter and its transfer functions



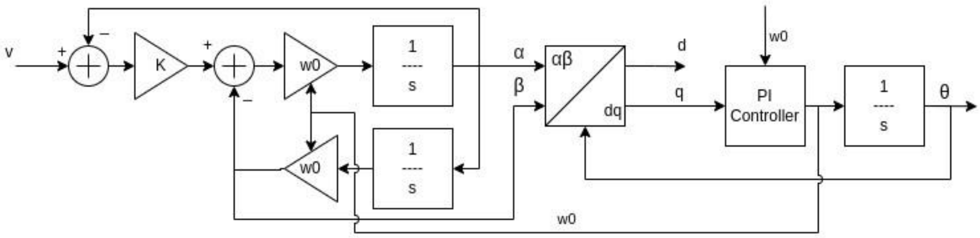
$$D(s) = \frac{v'}{v}(s) = \frac{k\omega' s}{s^2 + k\omega' s + \omega'^2}$$

$$Q(s) = \frac{qv'}{v}(s) = \frac{k\omega'^2}{s^2 + k\omega' s + \omega'^2}$$

It is important to note that the output qv' will always be 90° out of phase with the output Vd, regardless of the input frequency. However, their amplitudes will be different only if the input signal frequency is different from the center frequency.

In the SOGI-PLL architecture, the output of the SOGI filter is used as input to the SRF-PLL. Furthermore, the frequency estimated by the PLL is used as feedback to adjust the central frequency of the SOGI filter in order to keep the input signal always with unity gain regardless of its frequency. This is shown in figure 7.

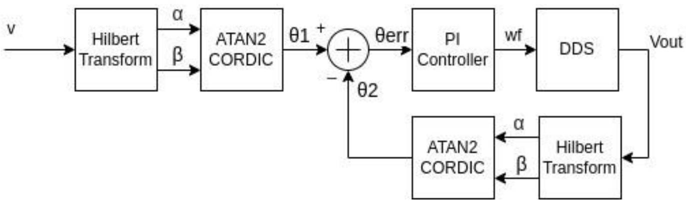
Figure 7: Diagram of the SOGI-PLL



3.6.HILBERT BASED PLL

The Hilbert Based PLL uses the Hilbert Transform as a quadrature signal generator. The architecture is shown in figure 8.

Figure 8: Diagram of the Hilbert Based PLL



The phase shift between the reference and generated signal is estimated using the Coordinate Rotation Digital Computer algorithm (CORDIC). This algorithm has been a trusted technique, which has been used to compute elementary trigonometric functions by means of planar rotation or vectoring process [1]. In this case, the function implemented is the *arctangent2*, which receives an input signal (sine) and its quadrature (cosine).

The Direct Digital Synthesizer (DDS) shown on the last diagram is an external hardware to synthesize the output signal. And the PI controller is used to control the output signal frequency based on the phase shift between the signals.

3.7.DISCUSSION

The Clark Transform is usually used with the Park Transform to generate the DQ axes. It does not require a very high computational power to perform them, as they are based only on the solution of sines and cosines. However, the Clarke transform has the disadvantage of needing a three-phase input to generate the quadrature signals. In addition, it's not robust to noise, harmonics and offset in the input signal. On the other hand, the Park transform generates DC output signals, which makes it easier to design the PLL proportional-integral (PI) controller.

The Hilbert Transform requires a high computational power to calculate it, since it's based on a convolutional operation. It means that the higher the order of the FIR filter used to implement it, the higher the computational cost will be as well, but in an exchange of a better filter response. The same concept applies for the CORDIC algorithm, since it's an iterative process. The more iterations, the more reliable the approximation of the desired trigonometric function will be.

The SOGI filter is a very restricted bandpass filter, high frequency noise and low frequency oscillations are attenuated. The version presented in this article is still susceptible to offset in the Q component, but there are variations as shown in [3], which make it immune to offset, that add a zero to its transfer function by adding another integral feedback. The disadvantage is that if there is a subtle high frequency jump in the input signal, the filter will not be able to adjust its resonant frequency, as the input signal will be completely attenuated by being too far out of resonance. This can be mitigated by reducing the quality factor of the filter, but resulting in a lower attenuation of frequencies outside the desired actuation range.

4. CONCLUSION

The proposed work reviewed and compared some PLL architectures based on different mathematical transformations, which involves the Clarke Transform, the Park Transform, the Hilbert Transform and the SOGI filter. They can be used independently or together depending on the architecture chosen, but with the same purpose of generating a quadrature signal. The PLLs that use the Clarke and Park Transforms are less computationally demanding but less robust to noise, while the Hilbert Transform is more computationally demanding but has a better filter response and its resolution scales with the number of iterations of the CORDIC algorithm and the number of coefficients. The SOGI filter is really robust against low and high frequency noises. In addition it does not demand a high computing effort, but this topology is not good for subtle frequency variations.

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