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A STUDY OF FATIGUE DAMAGE WITH APPLICATION TO VIBRATION TESTING

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SUMMARY

This paper begins with a study of fatigue damage and how fatigue damage is quantified. Then, this paper describes the application of such a study to vibration testing, especially with respect to testing a product for its expected lifetime according to the product's end-use environment.

INTRODUCTION

Fatigue damage deals with the accumulation of damage to a product over time due to the application of repeated loads, repeated stress-inducing vibration patterns which weaken the product and, if the stress levels are high enough, can initiate a crack in the product, propagate that crack, and eventually induce product failure. Product failure due to fatigue arises not from a one-time application of a crippling shock (e.g., dropping a crystal glass on a hard floor) but from the accumulation of damage as the product experiences more and more stress-inducing vibrations, or loads. For example, a crack might initiate in a fatigued axle worn over and over by use, that crack might propagate by continued use, and eventually that axle might snap while in use causing a terrible accident—the Versailles rail accident (Reference [1]).

MIL-STD-810G (Reference [2]) states:

> The major cause of items failing to perform their intended function is material fatigue and wear accumulated over a time period as a result of vibration-induced stress (MIL-STD-810G 514.6A-3).

Thus is demonstrated the importance of a study of and interest in fatigue damage.

1. A Study of Fatigue Damage

Again, fatigue damage involves the accumulation of damage to a product over time due to the application of repeated loads. Over time, a crack might initiate, continue to propagate, and eventually cause product failure, as mentioned above. A product's fatigue behavior is quantified by an S-N curve (also called a Wohler curve). This curve plots stress versus number of cycles required to cause product or material failure at each respective stress level. Such a curve is generated empirically. For instance, consider the example S-N curve in Figure 1, taken from Reference [3].[1]

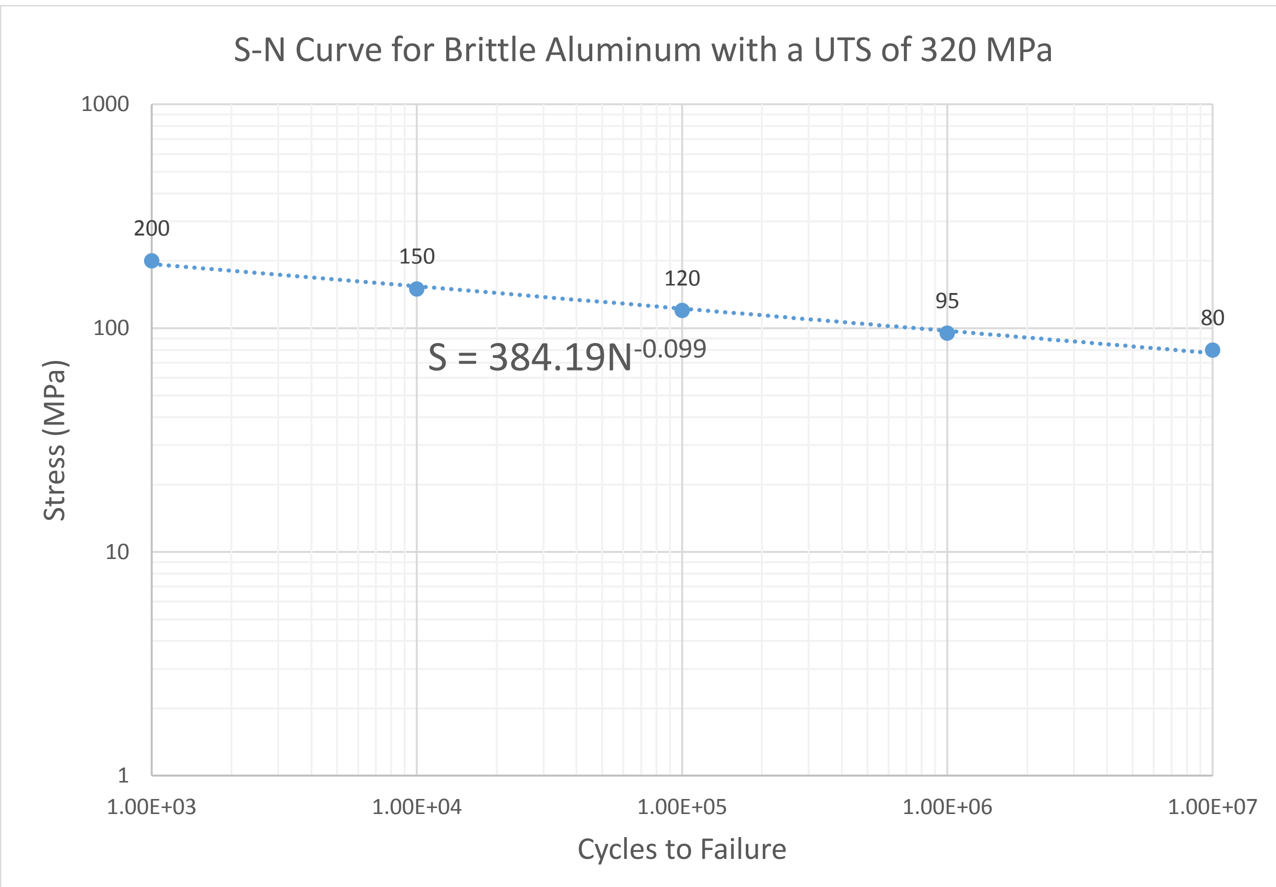

Figure 1 S-N Curve for Brittle Aluminum with a UTS of 320 MPa (Stress levels noted above data points)

Now, Miner's rule. Rychlik says (Reference [4]),

[1] The exact specification of the material under test as well as the setup and variables involved in the generation of this S-N curve are outside the scope of this paper.

> Most often, the linear Palmgren-Miner (P-M) rule is employed to compute a fraction of total life “consumed” by the load. The rule postulates that the order of cycles is irrelevant and that the total damage is a sum of the damages due to individual cycles. One predicts the fatigue failure if the accumulated damage exceeds some critical threshold (Rychlik 9/14).

Thus, it doesn’t matter in what order cyclic loads are applied. A cyclic load applies the same amount of fatigue damage no matter when it occurs. Second, total damage is a sum of the damages due to individual cycles. Miner’s rule makes sense—it’s intuitive. The mathematical representation makes sense too, as given in the paper written by Henderson and Piersol (Reference [5]):

$$D = \sum_i \frac{n_i}{N_i}$$

Equation 1 Miner's Rule

where:
n_i = number of cycles applied with peak stress S_i
N_i = number of cycles with peak stress S_i needed to cause failure
D = total damage (failure occurs when $D \approx 1$) (Henderson and Piersol 21).

Equation 1 states that total damage equals the sum of the fractional, or partial damages contributed at each stress level that the product is subject to. Fatigue damage accumulates as more and more cyclic stresses are applied, as each partial damage (for as many stress levels i as are applied) increases. Failure occurs near when this sum equals one. In the simplest case, when cycles at only one stress level are applied, failure occurs when $n \approx N$, that is, when the same number of cycles are applied as are needed to cause failure at the one stress level. The information necessary to apply Miner’s rule is obtained through the production of an S-N curve. Recall Figure 1. Suppose we applied 2500 cyclic loads to the same brittle aluminum at the 150 MPa level and 500,000 cyclic loads at the 95 MPa level. According to Miner’s rule,

$$D = \frac{2500}{10{,}000} + \frac{500{,}000}{1{,}000{,}000} = 0.75.$$

That is, we’ve consumed 75% of the brittle aluminum’s fatigue life (we’re dealing with approximations here). Again, failure occurs when $D \approx 1$, when we’ve consumed 100% of a product or material’s fatigue life.

2. MINER’S RULE, THE S-N CURVE, AND HENDERSON AND PIERSOL

The sketch above describes the fundamental concepts at work in fatigue damage spectrum. MIL-STD-810G states that

> The major cause of items failing to perform their intended function is material fatigue and wear accumulated over a time period as a result of vibration-induced stress

The S-N curve usually bears (or, is idealized to bear) a common shape—a line when displayed with logarithmic axes, as portrayed in the S-N curve for the brittle aluminum (Figure 1)—and this can be and is greatly utilized. A line has a slope, and it is the slope of the S-N curve upon which everything hangs. The S-N curve (or, the line that best fits the data points, or, the idealized S-N curve) can be defined by an equation. As stated by Henderson and Piersol (20),

$$N = cS^{-b}.$$

Equation 2 Equation of S-N Curve

In Figure 1 we are given the equation $S = 384.19 * \mathrm{N}^{-0.099}$ for the brittle aluminum, which can be rewritten as $N = \frac{1}{384.19} * S^{-10.1}$. Here, the brittle aluminum's material parameter $b = 10.1$. Recall Equation 1. Henderson and Piersol go on to show that

$$D = \sum_i \frac{n_i}{N_i} = \sum_i \frac{n_i}{cS_i^{-b}} = \sum_i \frac{1}{c} n_i S_i^b$$

Equation 3 Miner's Rule + S-N Curve

by substituting Equation 2 for N_i (21).

Having Equation 3, we're able to calculate (another way to say it—"accumulate") the damage applied to a product given the product's S-N curve and the number of cycles applied to the product at each stress level for all stress levels.

3. TEST ACCELERATION

Accelerating a test (reducing test time) while maintaining the same amount of fatigue damage is essentially a matter of moving up and down the S-N curve. Equation 2 says

$$N = cS^{-b}.$$

Suppose we have $N_1 = cS_1^{-b}$ and $N_2 = cS_2^{-b}$ (for a particular frequency bin). Then,

$$\frac{N_2}{N_1} = \frac{cS_2^{-b}}{cS_1^{-b}} = \left[\frac{S_1}{S_2}\right]^b.$$

Equation 4 Cycle-Ampltiude Relationship

Notice that the constant c is cancelled out in Equation 4. Thus, test acceleration is not dependent on c, but only on the material parameter b which is derived from the slope of the S-N curve.

Since number of cycles is proportional to time, this can be written

$$\frac{t_2}{t_1} = \left[\frac{S_1}{S_2}\right]^b.$$

This agrees with the formula expressed in MIL-STD-810G,

$$\frac{t_2}{t_1} = \left[\frac{S_1}{S_2}\right]^m,$$

where m is used in place of b. MIL-STD-810G states:

> m = a value based on (but not equal to) the slope of the S-N curve for the appropriate material where S represents the stress amplitude and N represents the mean number of constant amplitude load applications expected to cause failure…The value of "m" is strongly influenced by the material S-N curve, but fatigue life is also influenced by the surface finish, the treatment, the affect of mean stress correction, the contributions of elastic and plastic strain, the waveshape of the strain time history, etc. Therefore, the value of "m" is generally some proportion of the slope of the S-N curve, known as the fatigue strength exponent and designated as "b" (MIL-STD-810GA-4).

In terms of power spectrums, MIL-STD-810G states (MIL-STD-810GA-4):

$$\frac{t_2}{t_1} = \left[\frac{W(f)_1}{W(f)_2}\right]^{\frac{m}{2}}.$$

Equation 5 Time-Power Relationship

In this equation, the exponent is halved because the power spectrum W (power in general) is proportional to the square of amplitude (squaring the fraction inside the brackets while keeping the right-hand side of the equation the same means we have to divide the exponent by two).

Equation 5 is the formula defining time reduction (accelerated testing) while maintaining the same amount of fatigue damage. The fundamental premise employed throughout is the Miner-Palmgren rule of damage in tandem with the idealized S-N curve (i.e., a line in a log-log plot).

Notice especially the exponent in Equation 5. Again, and this can't be overstated, everything hangs on an accurate material parameter, an accurate m (based on b). This m defines how time and amplitude vary.

4. FROM THE ABSTRACT TO THE CONCRETE (1)

One example of the ways in which these concepts can be employed is Vibration Research Corporation's Fatigue Damage Spectrum, the process of which is displayed in Figure 4.

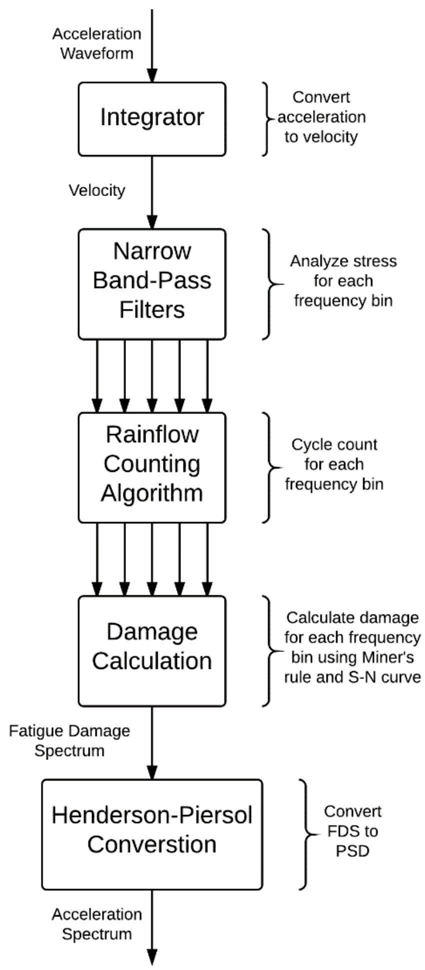

Figure 2 Flowchart of VRC's FDS

First, the imported acceleration waveform is integrated in order to obtain a velocity waveform. This alligns with Henderson and Piersol's claim that "it is velocity rather than acceleration that has a direct relationship to stress," (Henderson and Piersol 21) although it should be noted that the PSD test profile produced from the fatigue damage spectrum calculation will be the same whether the fatigue calculation employs acceleration, velocity, or displacement.

Second, the velocity waveform is narrowband-filtered. FDS is just that—a spectrum—and it displays a waveform's damage value for each frequency bin. A random waveform (or any waveform for that matter) can be considered the sum of many waveforms, each with a different frequency, which, when added together, produce the original waveform (this is Fourier analysis, and is fundamental to vibration testing). So, the narrowband filters "pull" these constituent waveforms out of the original waveform, and the damage analysis is performed for each waveform (that is, for each frequency bin).

This next step is a crucial one. Recall that in order to accumulate damage we must know the material parameter and the number of cycles applied to the product at each stress level for all stress levels. Rainflow counting takes care of the latter requirement. Although it might be easy to count the number of cycles in a sine wave by eye (since it's so regular), counting the number of cycles in a waveform with random amplitude is more difficult. The cycle-counting algorithm is designed to break down a nearly-sinusoidal waveform with randomly varying amplitude into its constituent cycles and these cycles' corresponding magnitudes, providing the set of cycles and stress levels that we need in order to accumulate damage. Again, this cycle-counting is performed for each frequency bin (i.e., for each filtered waveform).

Having the cycles and stress levels for each frequency and having the material parameter of the product, these are substituted into Equation 3 to produce a fatigue damage amount for each frequency bin, giving us the fatigue damage spectrum.

Finally, using the work of Henderson and Piersol, a corresponding PSD test profile is calculated from the fatigue damage spectrum which applies the same amount of damage in the same amount of time. Although the original, imported waveform might be a random wave with high kurtosis, the calculated PSD test profile converted from the fatigue damage spectrum will be a test profile with kurtosis 3 because of the assumptions and derivation of Henderson and Piersol.

<u>A Qualification</u>

Equation 3 involves a constant c. VRC's FDS doesn't ask for c. Further, the fatigue damage values present in the spectrum are much greater than 1, whereas Equation 1 (Miner's rule) is designed such that $D \approx 1$ indicates imminent failure. What's going on here? Simply put, VRC's FDS does not correspond to the actual, fractional damage D that is present in Miner's rule. Although Miner's rule is employed in the generation of the FDS, there is no correspondence between actual D and the values of the FDS. However, the c constant is cancelled out during the conversion from the FDS to its corresponding PSD test profile. So, although not having c precludes the strict application of Miner's rule (so that $D \approx 1$ indicates likely failure), the PSD test profile converted from the FDS will still, if an accurate b is used, accurately reflect and apply the amount of damage present in the original waveform. Essentially, VRC's FDS calculates a unit-less damage amount present in the original waveform(s)—call it A, and calculates the PSD test profile that would apply that same amount of damage A.

VRC's FDS is used to equate two waveforms according to fatigue damage—the field data with the random test. This is why c doesn't matter here. In a manner similar to the way that $FDS(wave_1) = FDS(wave_2) \Rightarrow \frac{FDS(wave_1)}{1000} = \frac{FDS(wave_2)}{1000}$, the constant c is cancelled out in the equivalency.

5. FROM THE ABSTRACT TO THE CONCRETE (2)

Having an understanding of fatigue damage theory, and having a means whereby we are able to accumulate a waveform's fatigue damage amount (again, an amount not referring to the fraction of life a waveform consumed by a waveform as is the case with Miner's rule proper), we can make application to testing a product according to its end-use environment. That is, we can develop a test profile that applies to the product the damage equal to the damage the product would experience during its expected lifetime of use. First, we collect a waveform or waveforms representative of the product's expected operating environment (i.e., containing the vibration patterns we expect our product to experience in its lifetime). Then, we send the waveform or waveforms through a process like that depicted above in Figure 4 and explained in the figure's accompanying paragraphs. Having the fatigue damage spectrum, the damage values can be linearly extended according to the product's lifetime of use (e.g., the representative waveform might only have a length of 10 minutes, whereas the product's expected lifetime in the environment represented by that waveform might be 500 hours—the damage amounts in the spectrum must be increased accordingly). Then, via Henderson and Piersol, the damage spectrum is converted into the corresponding PSD test profile which applies the same amount of damage present in the original waveform or waveforms.

In the case where a product's end-use environment is represented by more than one waveform, each waveform's damage spectrum can be linearly extended accordingly (as described above), and then the damages contributed by each waveform can be combined into one, net damage spectrum, the corresponding PSD test profile of which can then be calculated.

Further, with the PSD test profile having been calculated, the lifetime test can be accelerated via Equation 5. The PSD test profile consists of as many points as there are points in the damage spectrum, and each point in the profile is accelerated based on Equation 5. With test acceleration, the damage spectrum does not change. With test acceleration the same amount of damage is applied in less time, meaning the power spectrum profile increases.

The most important parameter in all of this is the product's material parameter. The process depicted in Figure 4 depends on this parameter, b. Test acceleration depends on b. The accuracy of this application—the accuracy of the resultant test—is only as accurate as that of b.

6. FROM THE ABSTRACT TO THE CONCRETE (3)

What follows is an application of the theory and practice described above using actual data and Vibration Research Corporation's FDS.

A composition of 13 waveforms was assembled that summarized the vibrations the product would experience in its expected lifetime. Each representative waveform had a corresponding number of repetitions—the amount of that waveform to be present in the fatigue damage spectrum (the amount of that waveform the product was expected to experience)—this is the linear damage extension described above. VRC's FDS reflects this linear damage extension in its Target Life setting. As an example, suppose we import three waveforms A, B, C. Suppose we consider the first point in each waveform's damage spectrum, a_1, b_1, c_1. Suppose we increase A's target life by a factor of j (i.e., j repetitions), B's target life by a factor k, and C's target life by a factor of l. The first point in the combined damage spectrum would equal $j * a_1 + k * b_1 + l * c_1$, and such a linear combination would apply to every point in the combined damage spectrum (in other words, the combined spectrum would be a linear combination of the other spectrums).

Importing the 13 waveforms using VRC's FDS import method with a provided material parameter b, and adjusting each waveform's target life accordingly, we have the screen pictured in Figure 6.

FILENAME	REPETITIONS
A	400
B	90
C	100
D	2000
E	4000
F	16
G	200
H	200
I	8
J	1800
K	1200
L	1800
M	1600

Figure 3 Table of Target Lives

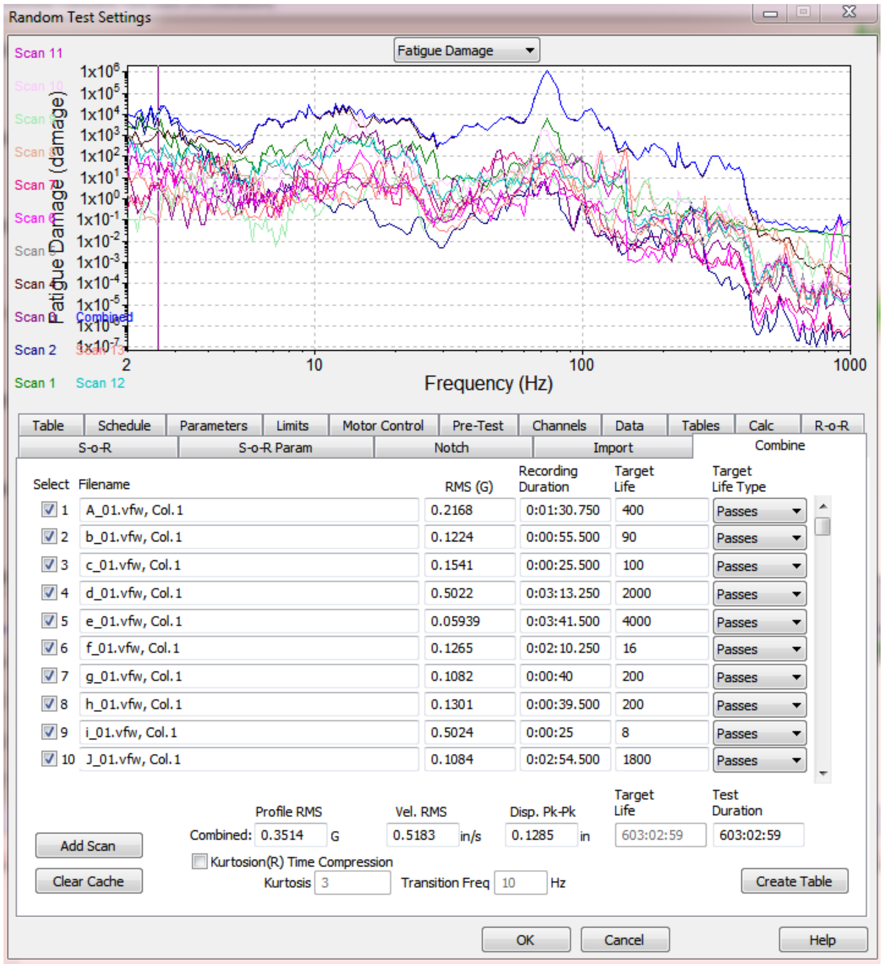

Figure 4 VibrationVIEW FDS Screenshot with Waveforms and Target Lives

In Figure 6 we see each waveform's fatigue damage spectrum (after its target life was set) and the combined, net fatigue damage spectrum. Notice, however, that the resultant test duration after target lives were set is some 603 hours. A lengthy test, to be sure. However, the test duration can be reduced according to the relationship of Equation 5. Setting the test duration to, say, 120 hours and creating the table yields the PSD test profile displayed in Figure 7. Notice that with the test acceleration (test duration reduction), the combined profile RMS increases from 0.35 to 0.53 G (while damage applied remains the same)

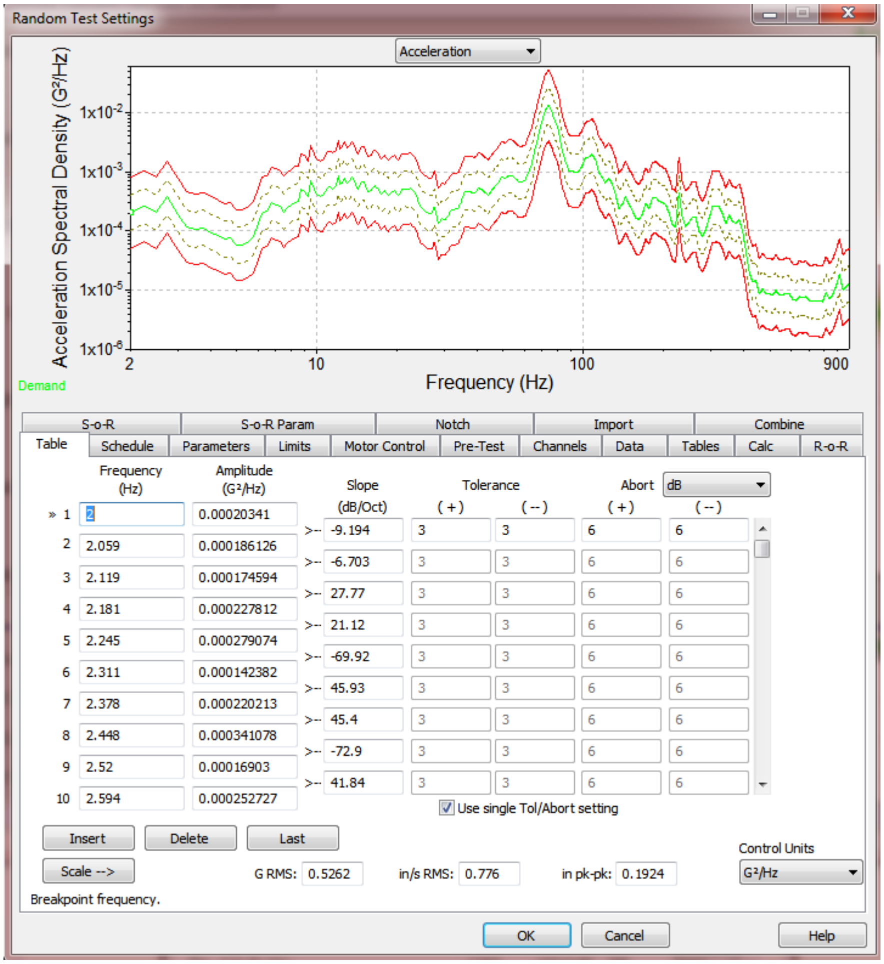

Figure 5 Resultant PSD Test Profile and Breakpoint Table

Thus, not only do we have a random test profile that would accurately simulate the end-use environment of the product for that product's expected lifetime of use (by applying the same amount of damage that would be applied to the product by its environment throughout the product's expected lifetime), we have also an accelerated test—a test that accurately simulates the product's end-use environment (applying the same amount of damage) but in significantly less time.

By selecting VRC's Kurtosion® Time Compression (see bottom of Figure 6), the G RMS of the accelerated test can be reduced while maintaining the same amount of fatigue damage. This option brings back into the test more of the lifelike, large amplitude peaks that are minimized with a standard, Gaussian, kurtosis-3 test, and bringing these peaks back increases the test's damage while preserving the test's G RMS level. This means the same amount of damage can be applied with less G RMS, hence the G RMS reduction accompanying Kurtosion® Time Compression.

CONCLUSION

This paper began with a study of fatigue damage and how fatigue damage is quantified. From this study, this paper demonstrated a process whereby the study of fatigue damage can be applied to vibration testing, especially with respect to testing a product for its lifetime of use according to its end-use environment and in the light of fatigue damage. With such a process, the damage that a product would experience throughout its life is quantified and a corresponding PSD test profile is generated that applies to the product that same amount of damage (a “life-dose” of damage). In addition, the test can be accelerated such that the test duration decreases while the amount of damage applied remains the same. All of this requires an accurate material parameter, ultimately based on the slope of the product’s S-N curve.

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