LONGITUDINAL DYNAMIC MODELING OF A LIGHTWEIGHT OFF-ROAD VEHICLE USING JULIEN'S THEORY

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ABSTRACT

With the computational advance, the virtual simulation of vehicle systems has become of extreme importance, due to the fact of providing reduction of the cost and operational times. Thus, models that represent more reliable physical phenomenon are required. In this way, this paper presents a modeling, developed in Blocks Diagram, of the longitudinal dynamics of a lightweight off-road vehicle with Rear-Wheel Drive (RWD) and CVT transmission, using a tire model. The model consists of the engine, CVT transmission and a fixed gearbox, linked to semi-axles, which are connected to the wheels. For a better understanding of the physical phenomenon, a model of the tire-soil interaction is required. Therefore, Julien's Theory is used, a semi-empirical model that relates the traction force to the longitudinal slip at the contact region between the tire and the ground, allowing us to verify with greater proximity to the reality, the behavior of the transmission system as well as the behavior of the longitudinal dynamics of the vehicle.

INTRODUCTION

The Vehicle Dynamics knowledge is necessary for the understanding of the vehicle-terrain interaction. The literature divides the study of vehicle dynamics in three parts: vertical, lateral and longitudinal. However, in this paper, only longitudinal dynamics is considered.

Longitudinal dynamics is function of driveline forces (including engine and powertrain) and motion resistances (rolling resistance, aerodynamic drag and climbing resistance) [1]. Thus, vehicle longitudinal dynamics studies the understanding of the behavior of the vehicle when subjected to acceleration and braking forces.

These forces influence the transfer of loads to the ground, as well as the adhesion of the tire to the ground, causing the forces acting on the region of contact between the tire and the ground influence the car motion [1, 2, 3].

Only acceleration efforts are considered in this paper. So, longitudinal slip may occurs due to the stretching and compression of the tire at the contact patch. Thus, for predicting forces acting on the tire, a semi-empirical model developed by Julien is used in this paper.

Furthermore, the transmission system also influences the vehicle dynamics. Its use is greatly criticized for the environmental damage that causes [4]. With this, projects have been developed with the intention of changing this panorama in order to promote the reduction of the emission and consumption of fuels. Thus, Continuously Variable Transmission (CVT) was developed, altering the transmission ratio continuously, making possible the operation of the engine close to its ideal condition, reducing the emission of fuel [5, 6, 7].

Tests on physical prototypes are widely used in vehicle development, in spite of the fact that are highly time-consuming and financially expensive, requiring considerable time to prepare and create. In this way, technological advancement has allowed the development of computational models, making tests on physical prototypes be largely used as final tests that allow the validation of previously performed computational tests.

A methodology using block diagrams is adopted for the proposed model, which allows the subdivision of a system into small parts, where from the knowledge of each one of them, one can know the behavior of the complete system from cause and effect relations between the small parts [8]. Therefore, a computational model built in Simulink/MATLAB™ which allows the use of block diagrams is used in this paper.

1. DRIVELINE SYSTEM

According to [3], the acceleration achieved by a vehicle is dependent on two factors: traction force at tire prints and torque at driving wheels. The first one is function of the tire-road friction and the other one is function of driveline performance [3]. The driveline system consists of an engine and powertrain system [5]. Both of them have the function to transfer torque to the driven wheels, overcoming the motion resistances [1].

A second-order polynomial (Equation 1) can represent the torque curve in [N.m] of an engine by taking into account the engine angular velocity input [3].

$$T_m = P_1 + P_2\omega_m + P_3\omega_m^2 \tag{1}$$

For spark ignition engines, P_1, P_2 and P_3 can be measured by:

$$P_1 = P_{max}/\omega_{max} \tag{2}$$

$$P_2 = P_{max}/\omega_{max}^2 \tag{3}$$

$$P_3 = -P_{max}/\omega_{max}^3 \tag{4}$$

Where, P_{max} and ω_{max} are the maximum engine power [kW] and the angular velocity [rad/s] at which the engine reaches this maximum engine power, respectively.

In this paper is used a spark ignition engine with maximum torque $P_{max} = 7{,}46$ kW and $\omega_{max} = 3800$ rpm ≈ 398 rad/s. The generated torque curve can be seen in Figure 1.

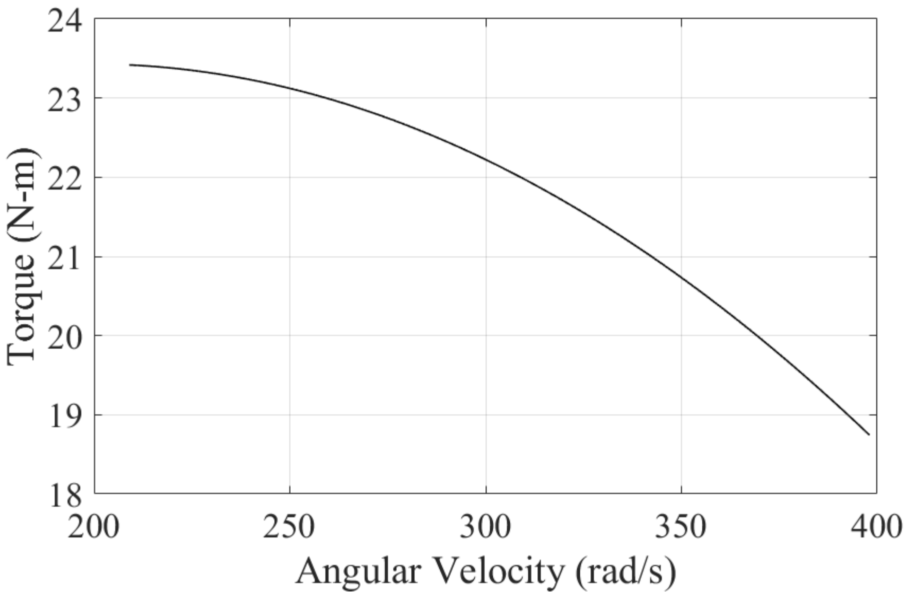

Figure 1: Torque performance of the engine

The transmission of the proposed vehicle consists of a CVT transmission, where two pulleys with variable diameters (primary and secondary) are connected by a timing belt. A CVT transmission is characterized by provides continuously varying combinations of transmission ratios, allowing the opening of one of the pulleys and the closing of the other, changing simultaneously the multiplication of torque and ensuring better fuel economy.
It is adopted the hypothesis of continuous operation at maximum power, making the engine run for most of the time in this maximum rotation. Therefore, considering the hypothesis defined, which is ideal, the transmission ratio provided by CVT (i_{CVT}) can be defined by Equation 5.

$$i_{CVT} = \frac{\omega_{max}}{\omega_{cx}} \tag{5}$$

Where ω_{cx} is the angular velocity output of the secondary transmission, as will be seen later. Thus, following the flux of power method, the CVT transmission, can be modelled by:

$$T_{CVT} = i_{CVT} T_m \tag{6}$$

$$\omega_{CVT} = i_{CVT} \omega_{cx} \tag{7}$$

Where, T_{CVT} and ω_{CVT} are the torque and angular velocity of the CVT output, respectively and T_m and ω_{cx} are the engine torque output and the angular velocity output of the secondary transmission, respectively. The transmission ratios defined for the CVT are 3.71:1 (maximum) and 0.69:1 (minimum).

The torque from the CVT is transmitted by the first drive shaft through the first pair of gears to a second drive shaft. Upon reaching the second drive shaft, the torque is reduced by a second pair of gears coupled to the third drive shaft, where the semi-axles are fixed. In this paper, equal gear ratios for the gear pairs are defined (3.18:1). Equations 8 and 9 show the relations between torque and angular velocity of input and output for a gearbox.

$$T_{cx} = i_{red1} i_{red2} T_{CVT} \tag{8}$$

$$\omega_{cx} = i_{red1} i_{red2} \omega_{Wheel} = i_{red} \omega_{Wheel} \tag{9}$$

Where, T_{cx} and ω_{cx} are the torque output and angular velocity output of the gearbox, respectively and T_{CVT} and ω_{Wheel} are the CVT torque output and the angular velocity output of the driven wheels, respectively. Finally, i_{red1} and i_{red2} are the transmission ratios of the first and second pair of gears, respectively.

2. JULIEN'S THEORY APPLIED TO THE DRIVEN WHEELS

The driven wheels receive the generated torque from the driveline system, causing a compression effect in front of and within the contact region of the tire [1, 2, 7]. Thus, the tire travels less when subject to a driving torque that when it is in free rolling [2]. A phenomenon known as longitudinal slip [7]. The longitudinal slip (i) can be predicted by:

$$i = \left(1 - \frac{v_v}{r_e \omega_{roda}}\right) * 100\% \tag{10}$$

Where, v_v is the longitudinal speed of the vehicle and r_e is the static radius of the tire.

Thus, considering the longitudinal slip, Julien's tire model was developed. In this theory, the tire tread is considered elastic and the normal pressure on the tire-ground interface is uniformly distributed. In addition, the tire-ground interface is divided into two regions that define the tire-soil interaction: adhesion region and sliding region [2, 7]. The first one is dependent on the elastic properties of the tire, and the second one is further dependent on the properties of the tire-soil interface.

The total tractive force due to the adhesion and sliding regions is given by Equation 11 and it is dependent on the coefficient of adhesion of the ground (μ_p), the vertical load on the tire (W) the total length of the tire-soil interface region (l), the slip (i), the tangential stiffness coefficient of the tire (k), and the coefficient (λ), which can be determined by Equation 12:

$$F_x = \mu_p W - \frac{\lambda(\mu_p W - lk\lambda i)^2}{2l^2 k\lambda i} \tag{11}$$

$$\lambda = \frac{\mu_p W + \sqrt{2(1 - \mu_s/\mu_p)\mu_p W l^2 k}}{lk} \tag{12}$$

Where, μ_s is the sliding coefficient of the tire.

In this paper, the coefficients used in Julien's Theory are values taken from [2] (Table 1).

Table 1: Julien's Theory coefficients

Coefficients
$\mu_p = 0,68$
$\mu_s = 0,65$
$k = 3930000$ kN/m²

3. **MOTION RESISTANCES AND MOTION EQUATIONS**

In this paper is considered two motion resistances: aerodynamic drag and rolling resistance. The first one is dependent on the drag caused by the air flow over the vehicle, which generates vortices at the rear of the vehicle [1, 7]. In addition, small vortices are also produced in regions such as wheels and rear-view mirrors, adding value to aerodynamic drag. Due to aerodynamic drag be a complex movement, a semi-empirical model (Equation 13) is commonly used to its prediction.

$$R_A = \frac{1}{2} A\rho c_d v_v^2 \tag{13}$$

Where, A is the front area of the vehicle (0,98 m²), ρ is the density of the air and for this paper is assumed to be equal and constant to 1,255 kg/m³, c_d is the aerodynamic drag coefficient and it is defined as the value of 1.1 for the vehicle used in this paper. And, finally v_v is the longitudinal speed of the vehicle in m/s.

The other motion resistance considered in this paper is the rolling resistance. When the tire begins rolling, the carcass is deflected in the contact patch area [2]. Thus, the result of carcass deflection is that the center of normal pressure shifts in the direction of rolling, producing a moment, known as rolling resistance moment, about the axis of rotation of the tire [1, 2, 3, 7]. To maintain equilibrium during a rolling, a horizontal force must exist, known as rolling resistance force that can be predicted by:

$$R_R = f_R W_t \tag{14}$$

Where, f_R is the rolling resistance coefficient and W_t is the total weight of the vehicle.

The rolling resistance factor (f_R) is influenced by a number of factors such as temperature, structure of the tire, internal tire pressure as well as vehicle speed [1, 2 7]. Thus, considering a tire rolling on a medium hard soil (off-road) with an internal pressure close to 10 PSI ≈ 69,5 kPa, the value of 0.1 for this factor is obtained [2].

Considering a flexible tire rolling, its motion equation can be represented by Equation 15. Where J_{wheel} is the tire inertia$\dot{\omega}_{wheel}$ is the angular acceleration of the tire, T_{wheel} is the is the torque coming out of the gearbox multiplied by the efficiency of the transmission system, F_x is the traction force, r_d is the dynamic radius of the tire and M_R is the rolling resistance moment, given by multiplying the rolling resistance force by the dynamic radius of the tire.

$$J_{wheel}\dot{\omega}_{wheel} = T_{wheel} - F_x r_d - M_R \tag{15}$$

In the vehicle motion equation, we have the difference between the tractive force and motion resistances (Equation 16):

$$m_t\ddot{x} = F_x - R_a - R_R = F_x - F_R \tag{16}$$

Where, m_t and $\ddot{x}$ are the total mass and the longitudinal acceleration of the vehicle, respectively. F_R is the total resistance motion over the vehicle.

4. **RESULTS**

Figure 2 shows the driveline system layout used in this paper. As previously stated, the model consists of a spark ignition engine, CVT transmission and a fixed gearbox, linked to semi-axles, which are connected to the wheels.

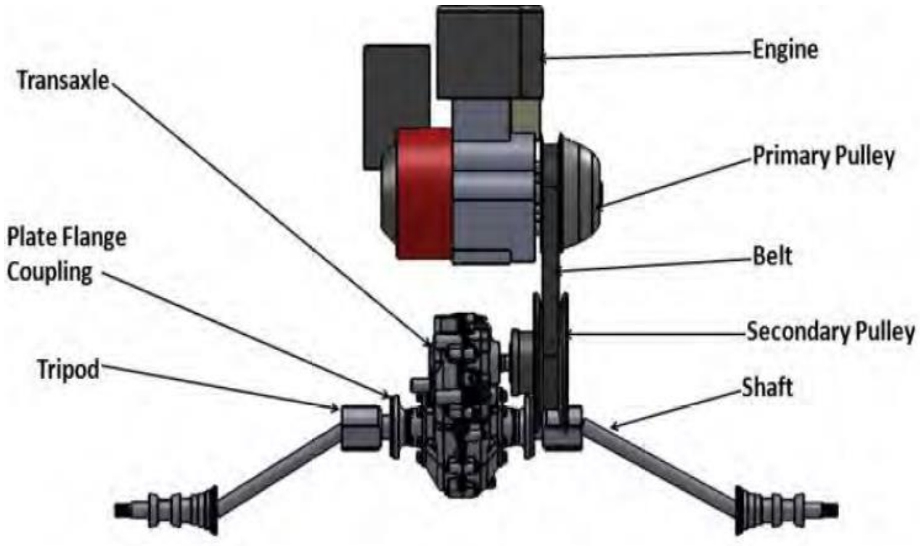

Figure 2: Driveline system used in this paper [9]

To verify the modeling, parameters of Table 2 are used, which presents the values adopted for the simulation of the studied vehicle.

Table 2: Vehicle parameters

Parameters	Values
Vehicle total mass	$m_t = 270$ kg
Weight distribution Rear-Front	$60 - 40$
Tire radius	$r = 0.30$ m
Tire inertia	$J_{wheel} = 0.8$ kg.m²
Driveline efficiency	85%

Figure 3 shows the longitudinal dynamics model developed using block diagrams in the Simulink/MATLAB program. The engine delivers torque to the CVT transmission and consequently to the gearbox, finally reaching the wheels. In response, the tractive force is generated as an input to the motion equation of the vehicle.

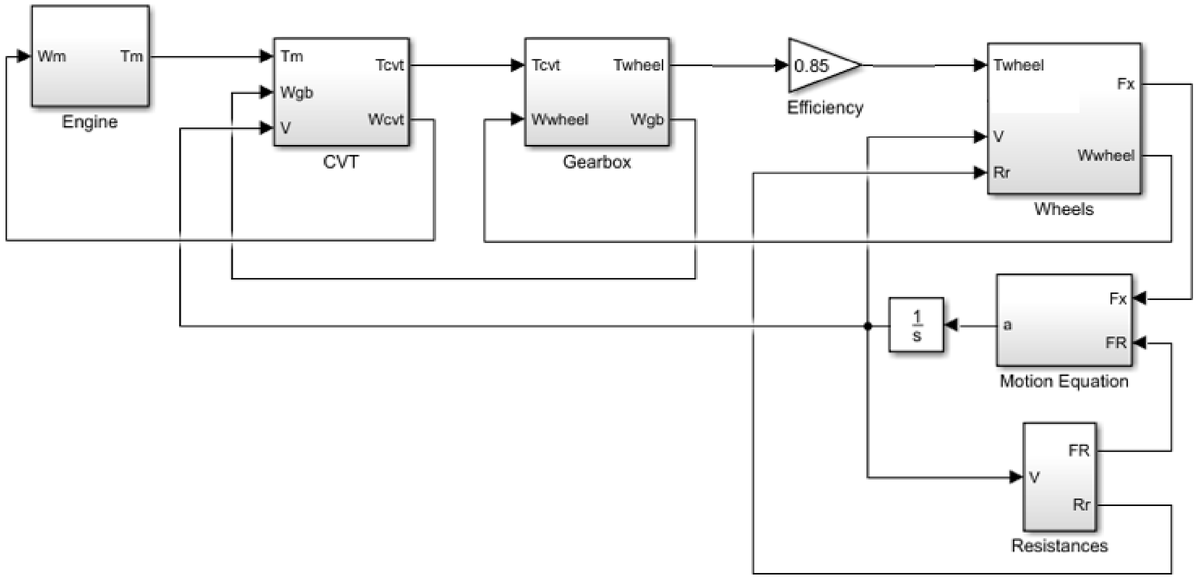

Figure 3: Longitudinal dynamics model using block diagrams

Figures 4 to 8 present the results for the model developed in the Simulink/MATLAB program, version R2017b, executed on a computer with an Intel Core I5 processor and 8 Gb of RAM. In order to solve the differential equations of the model, the Runge-Kutta method of 4th order with variable step was used.

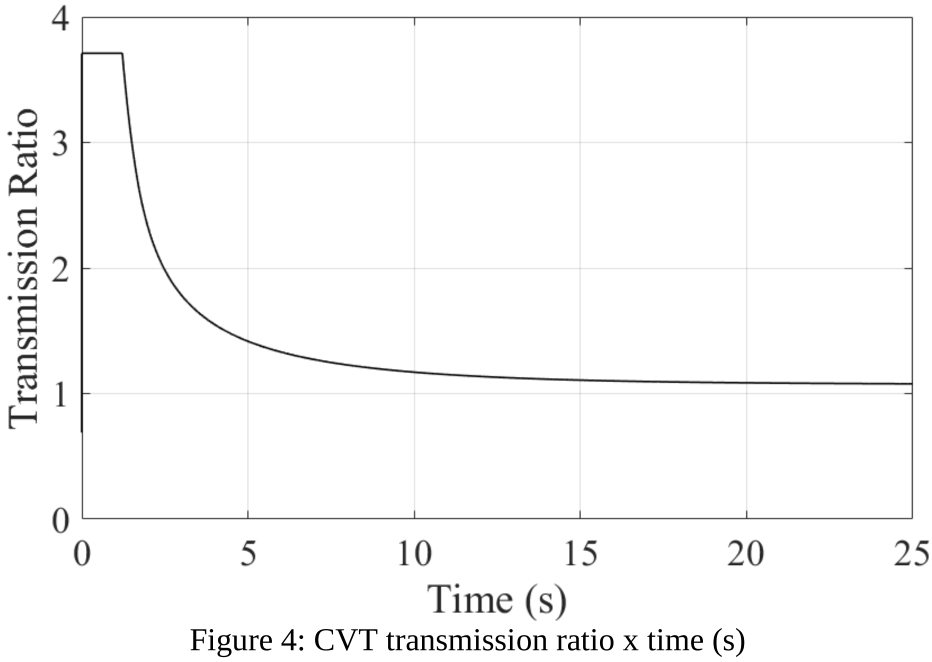

Figure 4: CVT transmission ratio x time (s)

It may be noted that the minimum CVT transmission ratio (0,69:1) is not reached, due to the fact that the terrain is uneven, causing the required minimum speed not be reached. In addition, the behavior in hyperbole of this curve, due to Equation 5, is verified. This behavior can be observed in numerous works, among them [4, 7], where the idea of hypothesis of continuous operation at maximum power is used.

In order to corroborate that presented in Figure 4, the variation of the vehicle speed as a function of time is shown in Figure 5. The maximum speed reached by the vehicle is approximately 44 km/h and tends to be lower compared to vehicles on paved roads.

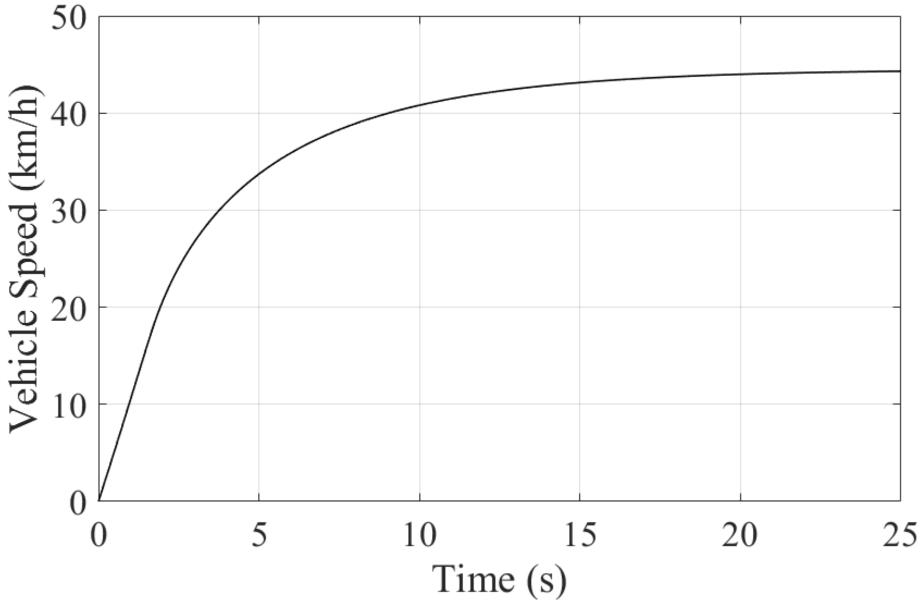

Figure 5: Vehicle speed (km/h) x time (s)

With the advancement of the vehicle, the wheel slips into the ground, as previously stated. Thus, Figure 6 shows the percentage of slip as a function of time and Figure 7 shows the variation of the tractive force developed at the tire-soil interface as a function of longitudinal slip.

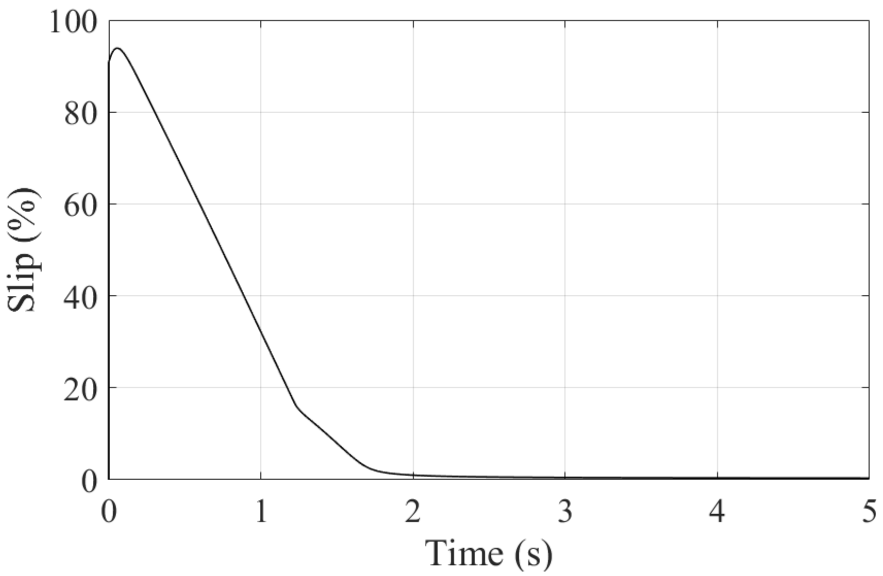

Figure 6: Slip (%) x time (s)

Due to the equation presented by Equation 20, when increasing the value of the normal load on the wheel, consequently, the area of contact between tire and soil is increased, thus increasing the developed tractive force.

Despite this, a higher resistance to movement is generated, thus decreasing the maximum longitudinal speed achieved by the vehicle.

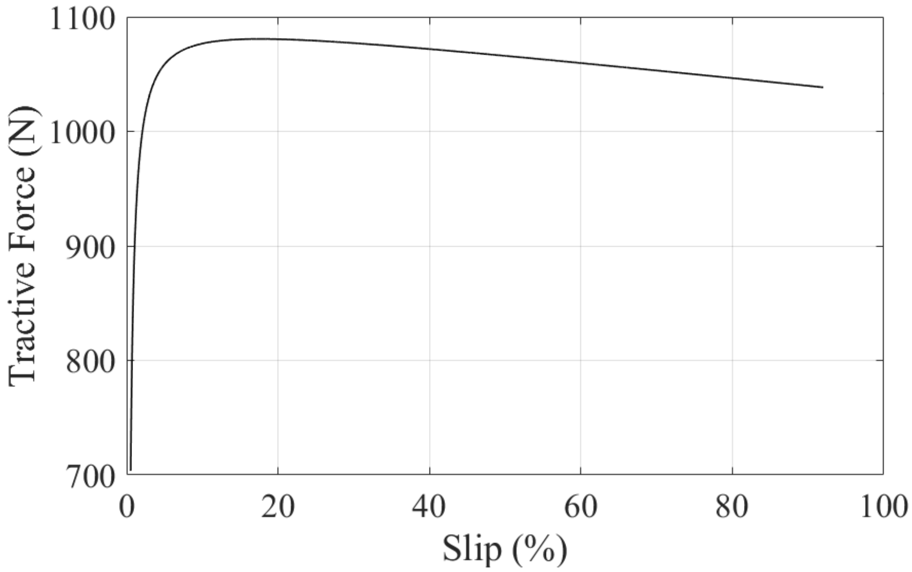

Figure 7: Tractive force (N) x slip (%)

It is also noted that due to the slip decreases rapidly in time between 0 and 5 seconds, we have at that moment the largest value of the tractive force as seen in Figure 8. Thus, at the beginning of the simulation, the tractive force is higher and decreases with the simulation time due to the reduced value of the longitudinal slip.

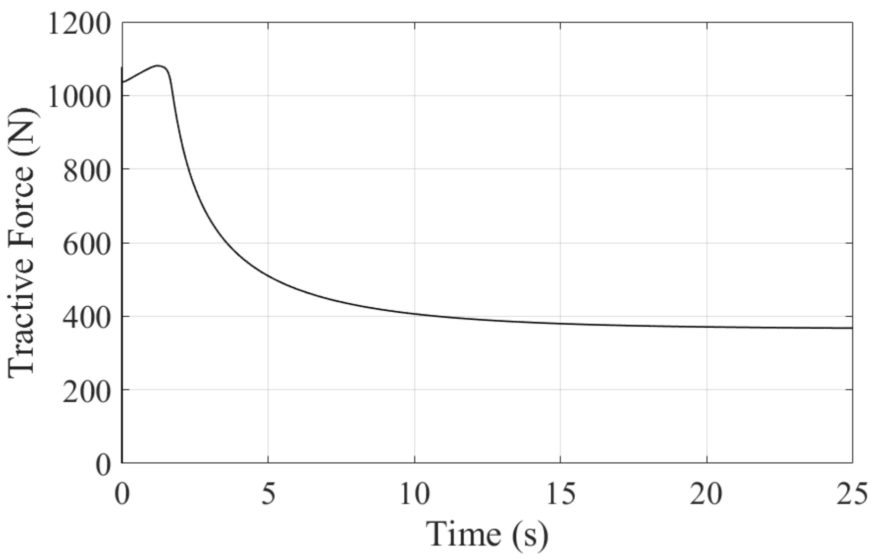

Figure 8: Tractive force (N) x slip (%)

CONCLUSION

This paper presents a longitudinal dynamics model of an off-road vehicle, developed through the knowledge of the flow of power methodology, in order to verify the behavior of the vehicle dynamics during the passage over an off-road terrain.

A vehicle based on Baja competitions promoted by SAE was used for this modeling. The model consists of a spark ignition engine, CVT transmission and a fixed gearbox, linked to semi-axles, which are connected to the wheels.

The results presented by the vehicle in study were satisfactory based on the last competition Baja SAE Brazil 2018 [10], where the final longitudinal speed reached by those prototypes vehicles were in the range between $34{,}4 \leq v\ (km/h) \leq 52{,}6$. The behavior in hyperbole of the transmission ratio of the CVT is shown in different papers such as [4, 7], serving as an excellent comparative to the result presented by this paper.

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