Kinematic Analysis for planetary gearbox applied in Hybrid Electric Vehicles

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ABSTRACT

In the context of global warming and searching for alternative sustainable mobility, hybrid and electric vehicles have been seeing as a promising solution. The majority of these vehicles are power-split hybrid vehicles using planetary gear as speed coupling. This paper studies the kinematic behavior of this type of coupling by extensive literature research about hybrid and electric vehicles, transmission gearbox, power-split control systems, and planetary gear. A method to calculate the mechanism kinematics is presented in order to determine torque, angular speed, power factors, global gear ratios, and efficiency distribution of a planetary mechanism. Also, analysis and comparison of the transmission used in the Chevrolet Volt and in the Toyota Prius are carried out. As a result, the paper shows the Volt gives preference to a series operating mode at low speeds, using in most situations only the Electric Motors to provide torque, while Prius, even in urban velocities, has to use an Internal Combustion Engines to avoid drops in EMs efficiency. Thereby, it provides a guide for engineers in several fields to easily understand the function of mechanical design and open possibilities to discuss new operation modes and control settings aiming to increase efficiency and performance in the hybrid and electric vehicle transmission.

Keywords: Vehicle dynamics. Kinematic Analysis. Hybrid Electric Vehicles. Planetary gearbox.

INTRODUCTION

Aiming to mitigate the global warming effects and strengthen the countries' response to climate changing, the Paris Agreement [1], [2], established actions to reduce gas emissions and control the temperature rising of the planet. Worldwide, several countries (including Brazil) have proposed action strategies to reduce their high carbon emissions and increase their energy efficiency. Following these guidelines, it has happened an increase in the use of electricity as an alternative power source by the automakers, such as Tesla, Jaguar, Mercedes-Benz, BMW, Porsche, and Audi in ABB Formula-E.

Several countries in the European Union have already imposed many regulations on Internal Combustion Engines (ICE) aiming to reduce carbon emissions through the next decades. Because of that, automakers are thoroughly improving technology in Hybrid Electric Vehicles (HEVs) and Electric Vehicles (EVs), which appear as a good alternative to substitute the conventional vehicles [3]. Besides, the HEVs deserve to be highlighted, since they combine the ICE with Electric Motors (EMs), matching the large autonomy of a combustion powertrain with high efficiency of an electric powertrain and being a solution to attend both economic and environmental reasons [4].

The challenges for this evolution to take place involve infrastructure and local maintenance for these new cars, reduction of the production cost of these new technologies, the transition between fleets, etc. [5]. Although countries in Asia and Europe have preferred the EVs, it is very likely that some countries that still very dependent on fuels, like Brazil, will opt for HEV and Plug-in Hybrid Electric Vehicles (PHEV) [6]. The latest release and probably one of the most dominant HEV in the Brazilian market for the next few years is the Toyota Corolla Altis Hybrid.

HEVs usually have their gearbox or power coupling system equipped with one or more planetary gear trains, due to its advantages, such as the high transmission power, the compact structure, the high transmission efficiency [7], the high power density and ability to operate as a two-speed transmission [8]. Besides, this HEV and planetary gear combination imply the engine speed and torque of power-split hybrid powertrains are decoupled from the wheels, allowing the engine to operate at more efficient points [9].

Knowing the relevance of the subject presented above, this paper aims to compare the main features of Planetary Gearbox (PG) and shows a kinematic analysis for this type of coupling applied in HEVs, allowing obtaining characteristics such as torque, angular speed, power factors, global gear ratios, and efficiency distribution of a planetary mechanism. To achieve this goal, firstly, this paper presents the most relevant works in this research area at “State of the Art” and the principal concepts used at "Planetary Gears". Then, different configurations are explored at "Transaxle HV 4th Gen" and at "Chevrolet Volt". After, a "Modeling Algorithm" section explains the method developed, and the model achievements are shown in the "Results" section. Lastly, the "Conclusions" provides an overview of the work main contributions.

STATE OF THE ART

In 1997, Toyota launched the first generation of Toyota Prius that was considered one of the best cars that year and made other automakers interested in design Hybrid Vehicles [10]. Besides Prius, other hybrid cars became popular such as the Volt from the Chevrolet (also known as Chevy), Ford Escape [11], Ford Fusion, and many others. Those vehicles have different types of systems, although the majority uses a power-split device to couple the motors and engines. The popularity of the power-split hybrids can be attributed to their capability to take advantages of both series and parallel configurations when their power-management algorithms are properly designed [12].

Toyota Prius is a series-parallel hybrid that uses the Toyota Hybrid System (THS) to couple two Motor Generators (MGs), called MG1 and MG2, and an ICE into the output from the vehicle [10]. The first THS was a single PG with an MG1 coupled in sun gear, an ICE in the carrier, and the output coupled with an MG2 in the ring. Because the MG2 is coupled in the output, this type of power split is considered an input series-parallel HEV.

Chevy Volt [14] also uses a power-split device as a series-parallel hybrid. However, it has not the same type of couplings and operates differently. In this case, the sun gear continues attached to MG1, the ring gear is coupled in MG2 and ICE while the output shaft is coupled into the output from the system towards the wheels. Unlike Prius, which does not use any clutch, Chevy Volt has three clutches (CL1, CL2, and CL3) in its system that makes possible four operating modes in this output series-parallel hybrid [12] whereas Prius using no clutches has only one operating mode.

Besides these notorious transmission models from General Motors (GM) and Toyota, Wang *et al.* [15] introduce different models. The hybrid market vehicles, presented by Wang *et al.* [15], and their main configurations are summarized in Tab. 1. The Chery Arrizo 7e uses only one PG with an ICE applied in the sun, an MG applied on the ring, and the output followed by a Continuously Variable Transmission (CVT) in the carrier. Ford Escape uses a single PG followed by a conventional reduction in a fixed ratio, very similar to Prius 4th Gen, as will be shown. Vehicles as Lexus RX400, Toyota Camry, and Highlander, used as a power split device the THS-II, which consists of a double PG. GM also applied double PG power split GM Alisson on Malibu Hybrid and some Sport Utility Vehicles (SUVs). Renault produced the IVT (Infinitely Variable Transmission), also a double PG to some cars. Those double PG Power Split devices are very similar to each other in configurations, differing mainly in position and number of clutches. There are projects in GM to use three or more PGs, to be applied in trucks and SUVs.

Table 1 - Planetary Hybrid Powertrain System.

Vehicle Model	Planetary Hybrid Architectures			
	Nº of MG	PGs	Name	*OT
Chevy Arrizo 7e	1 MG	Single	-	CVT
Chevy Buick Velite5	2 MGs	Dual	-	-
Chevy Malibu	2 MGs	Dual	-	-
Chevy Tahoe	2 MGs	Dual	-	-
Chevy Tahoe (*NV)	2 MGs	Triple	AHS2	-
Chevy Volt	2 MGs	Single	-	-
Ford Escape	2 MGs	Single	FHS	*FG + OTM
Ford Fusion	2 MGs	Single	-	-
GMC Yukon	2 MGs	Dual	-	-
GMC Yukon (*NV)	2 MGs	Triple	AHS2	-
Lexus GS450h	2 MGs	Dual	THS-II	-
Lexus LS600hL	2 MGs	Dual	THS-II	-
Lexus RX400	2 MGs	Dual	-	-
Toyota Camry	2 MGs	Dual	-	-
Toyota Harrier	2 MGs	Dual	THS-II	-
Toyota Highlander	2 MGs	Dual	-	-
Toyota Kluger	2 MGs	Dual	THS-II	-
Toyota Prius	2 MGs	Single	THS	-

*OT = Other transmissions associated

*NV = New Version

*FG + OTM = Fixed Gears and Output Torque Multiplier

Concerning the types of planetary, Wang *et al.* [15] provides a review work on architectures of planetary transmissions currently used in HEV. Furthermore, Zhang *et al.* [16] raised all the possible operation modes of simple PG with the removal and addition of clutches in Chevy Volt and Toyota Prius models, evaluating their performance. Moreover, many analyses are possible, as the kinematic and torque formulation proposed for different types of planetary gear sets by Amaral [17], the dynamic programming and the Genetic Algorithm to obtain better efficiency in series-parallel HEVs shown by Kabalan *et al.* [18] and the method to evaluate the efficiency of gear transmission in a PG provided by Petrescu *et al.* [19]. Besides, Morais [20] and Kouroussis, Dehombreux and Verlinden [21] shows some application in automatic gearboxes.

PLANETARY GEARS

In addition to conventional gears, there is a class of gear train known as an epicyclic or planetary gear train. Norton [22] says that this type of device has wide applications being defined as a device with two degrees of freedom. Therefore, two inputs are required to obtain a predictable output. Planetary gears have several advantages compared to conventional ones, including obtaining higher transmission ratios in smaller volumes, bidirectional power flow and simultaneous and concentric outputs from a single unidirectional input. A planetary train is characterized by a gear that can rotate around another axis in addition to rotating around itself [17].

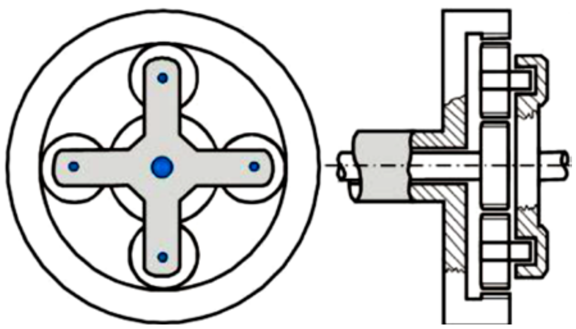

Figure 1 - Example of a Planetary Gearbox [22].

By analogy to the solar system, on the planetary train or PG, the central gear is called solar and the gears that revolve around it are the planets or satellites, shown on Fig. 1. There are other components that make up a PG, such as the ring gear and the carrier of the planets [17]. A single PG has differential or power addition capabilities, it can have a system with an input and two output (Fig. 2a), allowing the blocking of one of the outputs, braking or immobilizing its element, or even using them in the desired proportion. Another alternative is to design a simple PG with two inputs and one output with the same control possibilities (Fig. 2b).

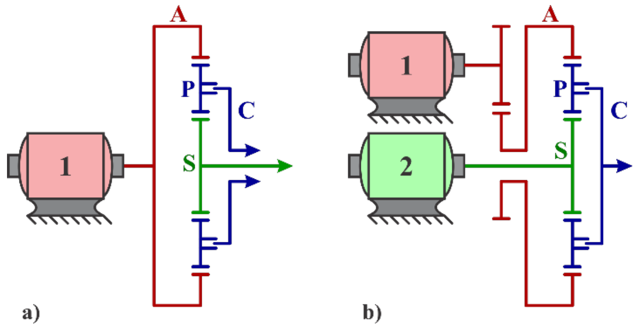

Figure 2 - Possibilities PGs configurations (a) one input and two outputs and (b) two input and one output.

Using an analytic method, it is possible to obtain the main equation and ratio of a simple PG, adopting the nomenclature shown in Fig. 3. The same figure shows the indications of the primitive radii and rotations of the sun (R_s, ω_s), planet (R_p, ω_p), ring (R_r, ω_r), and carrier (R_c, ω_c). Fig. 3 also indicates the velocities of the sun V_s, the ring V_r and de carrier V_c.

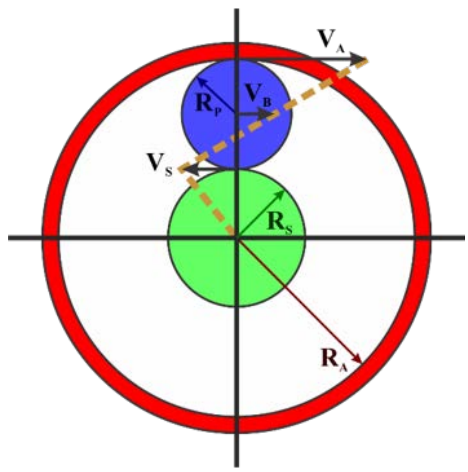

Figure 3 - Relative velocities and radii indicated on a single Planetary Gear scheme.

The relations between the velocities V_c and V_s, and between V_r and V_c are used to obtain Eq. 1 and 2, which results in Eq. 3. The basic ratio (b) is given by Eq. 4.

$$R_s\omega_s = R_c\omega_c - R_p\,\omega_p \tag{1}$$

$$R_c\omega_c + R_p\,\omega_p = R_r\omega_r \tag{2}$$

$$R_s\omega_s + R_r\omega_r = 2R_c\omega_c \tag{3}$$

$$b = \frac{\omega_r - \omega_c}{\omega_s - \omega_c} = -\frac{R_s}{R_r} = -\frac{N_s}{N_r} \tag{4}$$

The basic ratio (b) of a PG is the main term of this system. It refers to the cases that relate the ring and sun when the carrier is locked. In this case, being the input in the sun, the simple PG works like a common reducer and its relationship is given by the negative ratio between the teeth of the solar and ring gears (or by its radius). The kinematic equation for a simple PG is given by Eq. 5.

$$\omega_r + (b-1)\omega_c - b\omega_s = 0 \quad (5)$$

In order to observe the torque balance inside a PG, it is used a torque diagram. Thus, using Fig. 2a as an example the elements ring (R), carrier (C), planet (P), sun (S), input torque (T_{in}), output torque (T_{out}), resistance torque (T_{res}) and the gearing points (E_1 and E_2) are indicated in the diagram in Fig. 4. The numbered torque are the variables of the system.

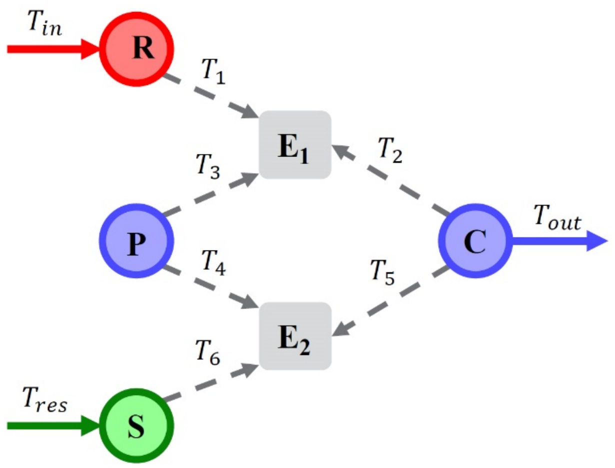

Figure 4 - Torque diagram for a single PG with ring input and fixed sun.

Also, it is important to define the ratios r_1 and r_2, as seen in Eq. 6 and 7, where N_S, N_P and N_R are the teeth number of sun, planet and ring, respectively.

$$r_1 = \frac{N_R}{N_P} \quad (6)$$

$$r_2 = -\frac{N_P}{N_S} \quad (7)$$

The diagram in Fig. 4 provides Eq. 8-15 that can be rearranged as a matrix equation.

$$T_1 + r_1 T_3 = 0 \quad (8)$$

$$T_2 + (1 - r_1)T_3 = 0 \quad (9)$$

$$T_4 + r_2 T_6 = 0 \quad (10)$$

$$T_5 + (1 - r_2)T_6 = 0 \quad (11)$$

$$T_6 - T_{res} = 0 \quad (12)$$

$$T_3 + T_4 = 0 \quad (13)$$

$$T_1 - T_{in} = 0 \quad (14)$$

$$T_2 + T_5 + T_{out} = 0 \quad (15)$$

TRANSAXLE HV 4th GEN

The transaxle HV 610 is applied in the fourth generation of the Toyota Prius and in the Corolla Altis Hybrid [23], a model recently launched by Toyota in Brazil, being the first hybrid that uses a flex engine. The system is a simple planetary with two EMs and an ICE.

The system, called the Hybrid Synergy Drive (HSD), has undergone some changes since the first Prius model launched in 1997. The first Toyota Hybrid System (THS) model is a series-parallel system coupled to a planetary system with a MG coupled to the sun (MG1), an ICE to the planetary carrier, and the ring section coupled to the second EM (MG2) and the outlets for the wheels. The 2nd and 3rd generations used systems with two PGs, with the second PG working just as a reduction between the second engine and the output to the wheels. The 4th gen uses a counter gear between the ring section from the PGs and the shaft coupled to the second EM. Different from previous ones that kept MG1 below MG2 on assembly, this counter gear adds the torques from MG2, ICE and MG1 and drives the output to the differential and wheels. This power split device is shown in Fig. 5.

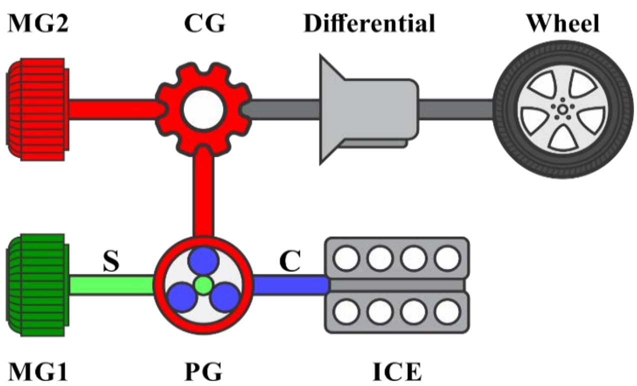

Figure 5 - Scheme from Prius 4th Gen Transaxle HV.

The Toyota Corolla Altis, launched in 2019, is a version that uses the hybrid Prius transaxle of the fourth generation, being the first flex hybrid focused in the

Brazilian market. A kinematic analysis of each situation on the operation mode from HSD is carried out in this paper, through the formulation for a single PG in Eq.1-5, analyzing the operation method for each of the system's operating states.

For a stationary state, it starts moving using only MG2. Cinematically observing, the same counter-gear is applied to MG2 and the planetary ring section. As there is no decoupling system, the solar gear connected to the MG1 must be running even if MG1 is not providing any power.

In this analysis it is used a negative basic ratio i, instead of b, for each reduction. It is the direct ratio between the ring and the solar radii, as seen in Eq. 16 and 17. In the 4th gen Prius, it refers to the basic reduction of PG and to the shaft coupled in the MG2 ratio to planetary ring rotation, respectively.

$$i_1 = \frac{N_R}{N_S} \tag{16}$$

$$i_2 = \frac{N_{Rext}}{N_{MG2}} \tag{17}$$

Appling i_1 and i_2, instead of b, in Eq. 5 and starting with the MG2 alone or using both EMs, the Eq. 18-19 are obtained. The solar gear must rotate if the ring gear turns, thus, it is possible to use both MGs.

$$\omega_r = \frac{1}{i_2}\omega_{MG2} \tag{18}$$

$$\omega_r = -\frac{1}{i_1}\omega_{MG1} \tag{19}$$

In case the Battery State of Charge (SoC) is very low, the system may turn on the ICE even in a stationary state in order to recharge. In this case, MG1 is responsible for this task and their rotation in a stopped car ($\omega_r = 0$) is shown by Eq. 20-21.

$$\omega_{ICE} = \frac{1}{i_1 + 1}\omega_{MG1} \tag{20}$$

$$\omega_{MG1} = (i_1 + 1)\ \omega_{ICE} \tag{21}$$

When there is a greater demand for power, the ICE is powered and the carrier is no longer in a stationary condition. In this case, the MG2 engine output, which has a proportional speed to the car's, is shown in Eq. 22 as an adaptation of the Eq.5.

$$\omega_{MG2} = i_2\left[\frac{i_1 + 1}{i_1}\omega_{ICE} - \frac{1}{i_1}\omega_{MG1}\right] \tag{22}$$

When the vehicle slows down or when the speed is constant and there is a decrease in power demand, there is a recovery by the MGs. During the slowdown or regenerative brake, the combustion engine goes into a cut-off regime and the polarity is reversed in the generators to recover part of the energy. The speed ratios are similar to Eq.15 in this case. In the case of non-regenerative brakes, the ICE keeps on and follows Eq. 21.

The ratios for Transaxle HV are afterward applied into differential before reaches the wheels. The fixed reduction from the ring gear to wheels i_W such as i_1 and i_2 for 4th Gen Prius are i_1 = 2.6, i_2 = 3.12 and i_W = 3.47.

In order to obtain the torques, it is necessary to define some considerations. Firstly, a problem with constant speeds for the power calculation is adopted and the system is considered with no power losses. The power balance is shown in Eq. 23-24, where the power on the wheels P_W is the sum of the powers from the MG1, ICE and MG2.

$$P_W = P_{MG1} + P_{ICE} + P_{MG2} \tag{23}$$

$$T_W\omega_W = T_{MG1}\omega_{MG1} + T_{ICE}\omega_{ICE} + T_{MG2}\omega_{MG2} \tag{24}$$

The Eq. 22 applied within Eq. 24 and then the car's control strategy provides the relations necessary to describe all kinematics in the Transaxle HV. As also known by previous relations, $\omega_W = \omega_{MG2}/i_2 i_W$. In case there is a sprint without ICE, the relations between torques are shown in Eq. 25-26.

$$T_W = T_{MG1} i_1 i_W + T_{MG2} i_2 i_W \tag{25}$$

$$\omega_{MG2} = -\frac{i_2}{i_1}\omega_{MG1} \tag{26}$$

CHEVROLET VOLT

Recently, GM announced that is going to migrate their new releases directly towards pure EVs, deciding to extinguish new HEV projects. Thus, the Chevy Volt had its production line disabled for GM to creates an electric SUV. However, it is still interesting to study this model and its differences to THS.

The Volt model also uses a simple planetary in the power split, however, the system output is given in the planetary carrier, with the MG1 located in the sun and the ICE coupled in line with the MG2 in the ring gear [14]. The Volt also has clutches that can ground some components or decouple it. CL 1 and CL 2 are always reversed, while one is open, the other is closed. These clutches allow four operating modes for power split device, discussed in Tab.2. For this case, i_1 is a planetary positive basic ratio. The Volt's power split device is shown in Fig. 6.

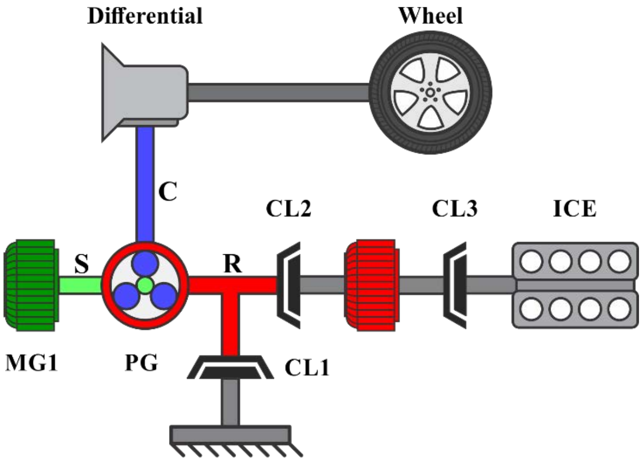

Figure 6 - Scheme from Chevy Volt power split device.

In this case, there may be an initial start-up with only one EM rotating due to the clutches, unlike the Prius system, just ground the CL1 ($\omega_r = 0$) and release the CL2 with CL3 blocked. In Eq. 27, ω_O is the planetary output rotation towards the wheels.

$$\omega_O = \frac{1}{i_1 + 1}\omega_{MG1} \tag{27}$$

When there is a need for MG2 power, CL1 and CL2 invert the states, disengaging the ring and coupling MG2 to the ring, and the ω_O is obtained as shown by Eq. 28.

$$\omega_O = \frac{i_1}{i_1 + 1}\omega_{MG2} + \frac{1}{i_1 + 1}\omega_{MG1} \tag{28}$$

If there is still a need to recharge batteries or more power start, the CL3 is closed and the ICE is coupled to the MG2 in the same rotation as the ring. Regeneration occurs in the same way, the output rotation ends up supplying more torque and turning the EMs into generators while the ICE is decoupled. In Tab. 2 are indicated the four operating modes in this vehicle.

For the Chevrolet Volt, the difference is that both MG2 and MC are connected to the same shaft, so the power ratio is given by Eq. 29-30.

$$P_W = P_{MG1} + P_{ICE} + P_{MG2} \tag{29}$$

$$T_W\omega_W = T_{MG1}\omega_{MG1} + \omega_{ICE}(T_{MG2} + T_{ICE}) \tag{30}$$

It is important to highlight the ratio between output and the wheels because of tires and differential (i_W). For the Eq. 30, $\omega_W = i_W\omega_O$, with ω_O as shown by Eq. 28.

Table 2 –Operating modes in Chevrolet Volt [16].

Operating Mode	Clutches state	Description
1.Series Mode ICE Off	CL 1 - Closed CL 2 - Open CL 3 – Open	MG1 drives the car, ring gear static
2. Combined (output split) ICE off	CL 1 - Open CL 2 - Closed CL 3 – Open	Both EMs drives the vehicle.
3. Series Mode ICE on	CL 1 - Closed CL 2 - Open CL 3 – Closed	Works as a series hybrid, ICE charges MG2, MG1 drives.
4. Series-Parallel Hybrid	CL 1 - Open CL 2 - Closed CL 3 – Closed	All motors provides power to vehicle

MODELING ALGORITHM

In order to obtain a method to calculate the torque and rotation from each component and to study ways to observe and possibly control the system to better efficiency and performance, it is necessary to apply computational tools to the problem. A possible flow to this problem is modeling a Hybrid Transmission in blocks, as shown in Fig. 7. This general model is applied in both vehicles analyzed in this paper using MATLAB/SIMULINK® software.

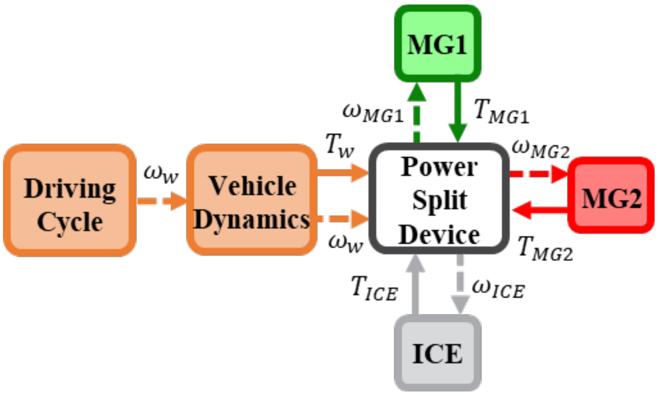

Figure 7 - Flow in an ordinary algorithm in a HEV.

For modeling the Toyota Prius and the Chevrolet Volt, it is proper to define which of variables shown in Fig. 7 will be evaluated through Power Split Device relations and the ones to be arbitrated. The torques to be evaluated by equations are related to the components that are used in the whole vehicle operation range, MG2 in Toyota Prius and MG1 in Chevrolet Volt. Concerning the rotations, the ones to be arbitrated are related to ICE operation as it is the most important component in the overall energy efficiency.

Simulating a simple driving cycle, it is used a ramp entrance of velocity in both cars. Prius vary from 0 to 80 km/h and Volt from 0 to 120 km/h. It is necessary to reach higher velocity in Volt to observe the ICE operation that only works as a series-parallel vehicle at higher speeds or torque demands. The torques in the wheels for each car are settled as shown in Eq. 31 and 32 for Prius and Volt, respectively.

$$T_W = 1050 + 0.017\, V^2 \tag{31}$$

$$T_W = 1200 + 0.025\, V^2 \tag{32}$$

Here, V represents velocity of the car and the ICE, in both cases, has the same torque curve. The torque is considered to be 0 until reaches 900 rpm in its rotation. As it turns on, its torque curve has a linear profile starting with 60 N.m at 900 rpm and reaching 140 N.m at 3600 rpm. The torque curve of ICE versus car's velocity is indicated in all torque graphs. The remaining EMs in each car are considered to deliver a constant torque to the system. Regarding the rotations inputs, for Prius, ω_{ICE} is considered 0 until the vehicle reaches 40 km/h and varies linearly onwards starting from 900 rpm. In Volt, the input is the rotation ω_{MG2} in the ring gear. This entrance is considered to be proportional to the carrier rotation ω_O.

All the other parameters are obtained by previous relations shown in this paper. Using the example in Fig. 4, it is possible to assemble torque diagrams shown in Fig. 8 for Toyota Prius (a) and Chevrolet Volt (b), respectively.

The diagrams provide equations similar to Eq.6-15 that can be assembled in matrixes to be solved in MATLAB®. Tab. 3 show the Prius and the Volt basic data used in the simulation.

Table 3 – Prius and Volt basic data in simulation.

Basic data in simulation	**Value [Unit]**
Prius	
Reduction to wheels (i_W)	3.47370
Planetary Basic Ratio (i_1)	2.60000
MG2 reduction (i_2)	3.12000
Tire Radius (R_p) [m]	0.28194
Volt	
Reduction to wheels (i_W)	2.16070
Planetary Basic Ratio (i_1)	2.24320
Tire Radius (R_p) [m]	0.28194

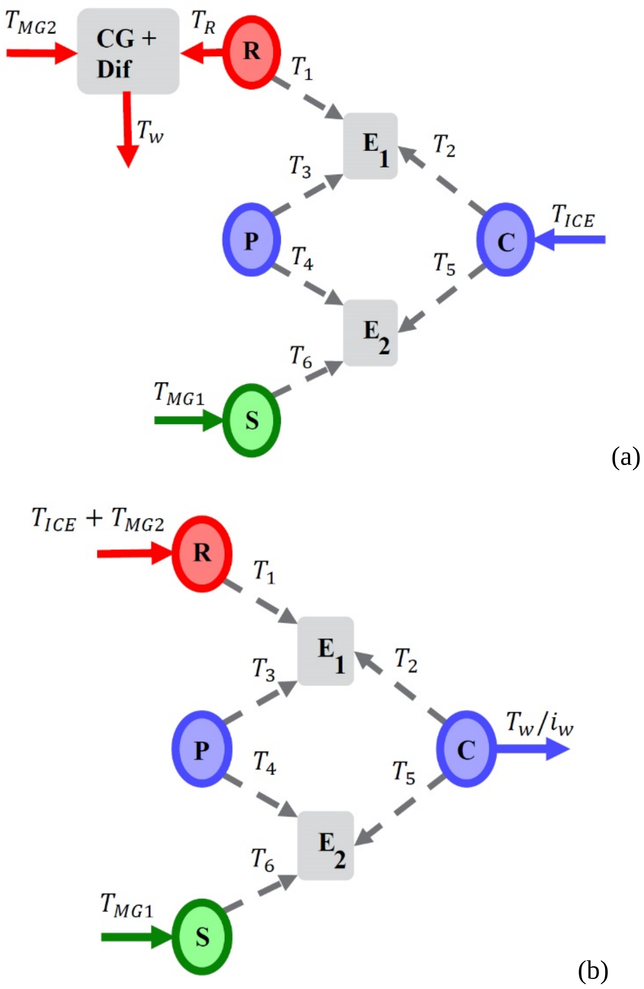

Figure 8 – Torque diagram for (a) Toyota Prius (b) Chevrolet Volt.

RESULTS

Using the Tab. 3 ratios of the Prius applied in Eq. 22 whereas ω_{ICE} is known, it is possible to obtain Fig. 9 indicating all rotations progression.

In Fig. 9 is possible to observe the change in direction caused by the start of ICE. In previous generations, MG2 was coupled directly to the ring gear, which makes MG1 rotates in a very high negative value, close to the motor limit. Thus, the presence of a reduction, between the MG2 and the ring gear, reduces the necessity of higher rotation ranges in the EMs. This reduction can be achieved by another PG, as done in 3th Gen Prius.

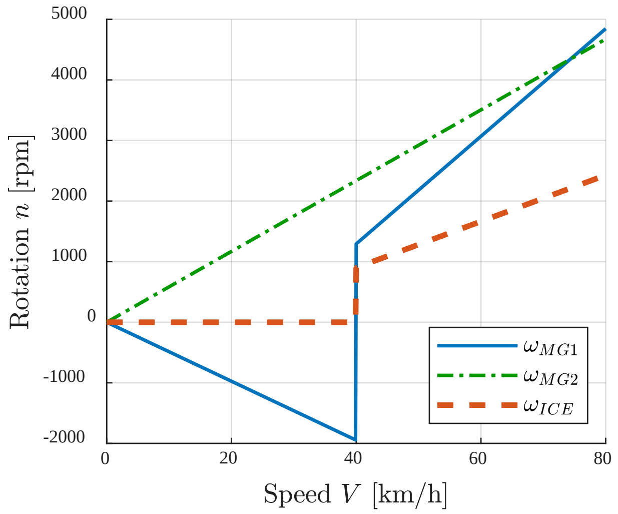

Figure 9 – ICE and MGs rotations vs Prius velocity.

Also, through torque diagram matrix considering T_{MG1} as a constant 30 N.m torque applied and a known T_{ICE}, is calculated T_{MG2}. This result, based on kinematic relations, are compared with T_{MG2} calculated by Eq. 24, using power conservation. Both are shown in Fig. 10.

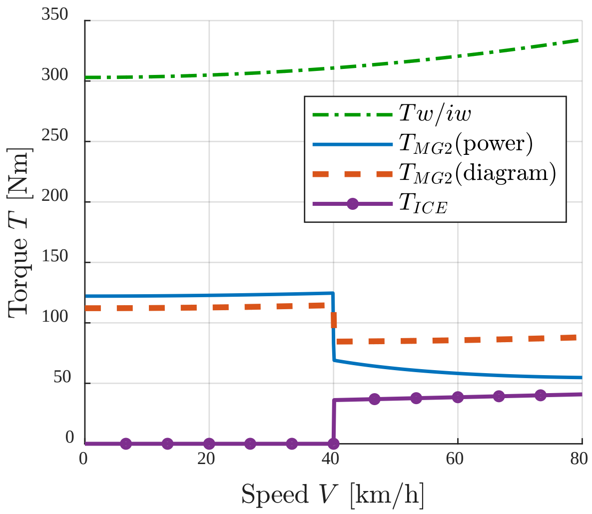

Figure 10 – ICE and MGs torques vs Prius velocity.

Regarding the torques in Fig. 10, the fits to TMG2 calculated by conservation of power and torque diagram does not match exactly. This occurs due to kinematics relations are based only in PG geometric relations and does not consider torque and rotation relationship, although the diagram is suitable to describe consistently application of torques in static conditions, when associated with rotations it differs from torque calculated in equation of power conservation. A great advantage in this method is to also describe the torques applied between components, that can be very convenient in stress analysis and suitable for gear design.

Using the Tab. 3 ratios of the Volt applied in Eq. 28, whereas $\omega_{MG2} = 1.6\,\omega_w$ is known, it is possible to obtain Fig. 11 indicating all rotations progression, where ω_o represents the planetary output rotation in the carrier.

Also, through torque diagram matrix considering T_{MG2} as a constant 40 N.m torque applied and a known T_{ICE}, T_{MG1} is calculated. This result, based on kinematic relations, are compared with the T_{MG1} calculated by Eq. 24, using power conservation. Both are shown in Fig. 12.

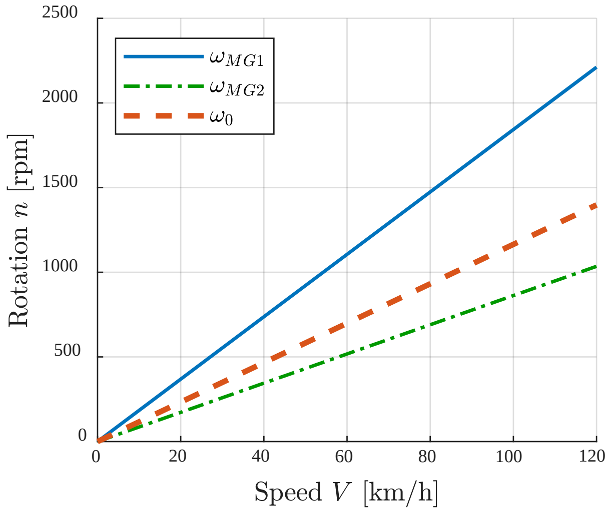

Figure 11 – ICE and MGs rotations vs Volt velocity.

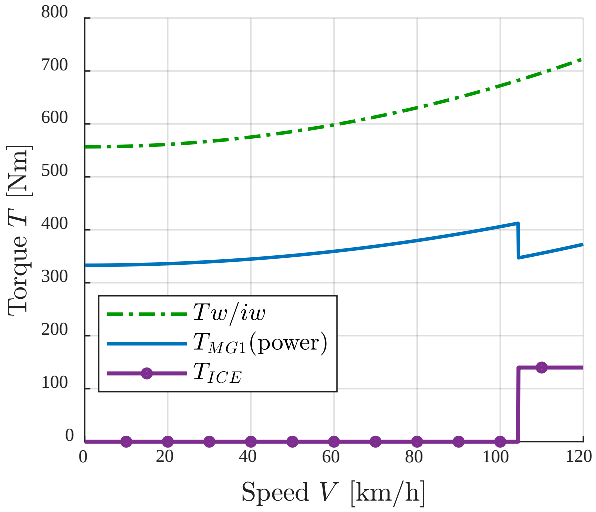

Figure 12 – ICE and MGs torques vs Volt velocity.

As discussed, Volt has 4 operating modes, two of them are presented in this simulation: the electric mode with two EMs operating, for ω_{MG2} values under or equal to 900 rpm, and the "mountain mode", using also ICE to obtain traction when ω_{MG2} overcomes 900 rpm. The ICE action is visible in Fig. 12 at the end of the simulation speed range, when it presents a step in the curve.

The Chevrolet Volt, even using the same concept to design a Power Split Device, delivers a different proposal as HEV in comparison to Prius. Firstly, comparing Prius' components rotations in Fig. 9 to Volt's components rotation in Fig. 11, it becomes evident that Volt, by positioning the ICE in the ring gear, gives preference to a series operating mode at low speeds, using in most situations only the EMs to provide torque. The ring gear has a lower rotation and, in this simulation, the vehicle barely reaches the minimum rotation to start the ICE. Different from Prius that even in urban velocities has to use ICE to avoid drops in EMs efficiency [3] (around 3000 rpm), Volt has a powerful torque from the MG1 and does not reach this limit in rotation. In Fig. 12 is also possible to notice that most of the car's torque is from MG1.

CONCLUSIONS

This paper provides a guide for analyzing the kinematic relationship of planetary gear systems, which can assist engineers in understanding the structural properties, constraints, and conditions of planetary sets. The basic epicyclic gearset and regular planetary gear were first discussed and then applied in HEVs. Also, it produces a review in several commercial car examples and shows the difference in the components configuration and its impact in the vehicle operation.

A comparison between the Chevrolet Volt and in the Toyota Prius transmissions was presented and the results showed the Volt prefers a series hybrid configuration for low speeds, using only the EM most time. On the other hand, Prius needs to use ICE to keep good efficiency even in an urban scenario.

These examples exhibit the flexibility that a PG associated with an HEV can provide, even for simple and more direct application. Moreover, the presented methodology can be applied to evaluate and compare other vehicle configurations, expanding the possible analyses.

ACKNOWLEDGEMENTS

This work was conducted during scholarships supported by the Brazilian Federal Agency for Support and Evaluation of Graduate Education (CAPES), grant number 88887.483349/2020-00, and the University of Campinas (UNICAMP).

REFERENCES

[1] M. Mace, "Mitigation Commitments under the Paris Agreement and the Way Forward," *Clim. Law*, vol. 6, pp. 21–39, Jan. 2016.

[2] C. A. Horowitz, "Paris Agreement," *Int. Leg. Mater.*, vol. 55, no. 4, pp. 740–755, 2016.

[3] Eckert, J. J., Silva, L. C. A., Costa, E. S., Santiciolli, F. M., Corrêa, F. C. & Dedini, F. G (2019). Optimization of electric propulsion system for a hybridized vehicle. Mechanics Based Design of Structures and Machines, 47:2, 175-200.

[4] Costa, E. S., Santiciolli, F. M., Eckert, J. J., Silva, L. C. A., Dionízio, H. J., Corrêa, F. C. & Dedini, F. J. (2014). Computational and Experimental Analysis of Fuel Consumption of a Hybridized Vehicle. SAE International, 2014-36-0385.

[5] Cordera, R., dell'Olio, L., Ibeas, A., & Ortúzar, J. de D. (2019). Demand for environmentally friendly vehicles: A review and new evidence. International Journal of Sustainable Transportation, 13(3), 210–223.

[6] Chiaradia, C. (2015). Estudo da viabilidade da implantação de frotas de veículos elétricos e híbridos elétricos no atual cenário econômico, político, energético e ambiental brasileiro. Aleph.

[7] Du, M. & Yang, L. (2020). A basis for the computer-aided design of the topological structure of planetary gear trains. Mechanism and Machine Theory, 145 (2020) 103690.

[8] Liu, C., Yin, X., Liao, Y., Yi, Y., & Qin, D. (2020). Hybrid dynamic modeling and analysis of the electric vehicle planetary gear system. Mechanism and Machine Theory, 150 (2020) 103860.

[9] Pei, H., Hu, X., Yang, Y., Tang, X., Hou, C., Cao, D. (2018). Configuration optimization for improving fuel efficiency of power split hybrid powertrains with a single planetary gear. Applied Energy, 214 (2018) 103–116.

[10] Sin, C. K., Ng, W. C., Tan, L. W., Ong, J. Q., & Chew, B. C. (2012). PESTLE Analysis on Toyota Prius. International Conference on Technology Management and Technopreneurship (ICTMT), 1–9.

[11] Murphey, Y. L., Park, J., Chen, Z., Kuang, M. L., Masrur, M. A., & Phillips, A. M. (2012). Intelligent hybrid vehicle power control part I: Machine learning of optimal vehicle power. IEEE Transactions on Vehicular Technology, 61(8), 3519–3530.

[12] M. Ehsani, Y. Gao, and A. Emadi, *Modern Electric, Hybrid Electric, and Fuel Cell Vehicles.*, 3rd Editio. Boca Raton: CRC Press, 2018.

[13] Liu, J., & Peng, H. (2008). Modeling and control of a power-split hybrid vehicle. IEEE Transactions on Control Systems Technology, 16(6), 1242–1251.

[14] Zhang, X., Li, S. E., Peng, H., & Sun, J. (2016). Design of Multimode Power-Split Hybrid Vehicles - A Case Study on the Voltec Powertrain System. IEEE Transactions on Vehicular Technology, 65(6), 4790–4801.

[15] Wang, L., Cui, Y., Zhang, F., & Li, G. (2020). Architectures of planetary hybrid powertrain system: Review, classification and comparison. Energies. MDPI AG.

[16] Zhang, X., Li, C. T., Kum, D., & Peng, H. (2012). Prius + and volt -: Configuration analysis of power-split hybrid vehicles with a single planetary gear. IEEE Transactions on Vehicular Technology, 61(8), 3544–3552.

[17] Amaral, Danilo, *Methodology and Systematization of the Epicyclic Gear Trains Design With One and Two linked EGTs,* Campinas. School of Mechanical Engineering, State University of Campinas, 2000. 150p. Thesis (Ph.D.)

[18] Kabalan, B., Vinot, E., Yuan, C., Trigui, R., Dumand, C., & Hajji, T. E. (2019). Efficiency Improvement of a Series-Parallel Hybrid Electric Powertrain by Topology Modification. IEEE Transactions on Vehicular Technology, 68(12), 11523–11531.

[19] Petrescu, R. V. V., Aversa, R., Apicella, A., & Petrescu, F. I. T. (2016). About the gear efficiency to a simple planetary train. American Journal of Applied Sciences, 13(12), 1428–1436.

[20] Morais, Ricardo Ferrari de, *Study of the Dynamic os Mechanical Transmissions using Planetary Gear Trains,*Campinas. School of Mechanical Engineering, State University of Campinas, 2004. 99p. Dissertation.

[21] Kouroussis, G., Dehombreux, P., & Verlinden, O. (2015). Vehicle and powertrain dynamics analysis with an automatic gearbox. Mechanism and Machine Theory, 83, 109–124.

[22] Norton, R.L. *Projeto de máquinas: uma abordagem teórica*. 4th ed. Porto Alegre, 2013.

[23] Toyota do Brasil Indústria e Comércio Ltda, "Toyota Corolla 2020 Technical Sheet," 2020.