

# Optimum vehicle suspension damper design for structural endurance due to vertical impact during landing

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**Abstract:** Optimum damper design, for structural endurance, due to vehicle vertical suspension impact during jumping is proposed herein. Off-road vehicles should be designed to support severe irregular terrain. This type of competition vehicle must support intensive impact during landing due to jumps without any structural damage or fatigue. An analytic expression is derived from the single degree of freedom model that identifies the suspension force. From the vehicle basic parameters, the damping factor can be evaluated to produce the minimum impact peak force.

**Keywords:** dynamic, vehicle, impact, landing, optimum, damping.

## 1. Introduction

The vehicle suspension design can be guided by different goals. Depending on the vehicle application, comfort can be the main goal. For sportive application, safety should be valued. Another possibility is to improve vehicle driveability, ensuring maximum capacity of horizontal forces production in the tire contact with the ground. Each of the aspects mentioned has its own parametric formulation to quantify specific suspension properties. For instance passenger comfort (*ride*) is tailored by the suspension ability not to transmit the vibration due to the pavement roughness to the car body. In the other hand, vehicle safety is the ability to keep as stable as possible the tyre vertical contact force to get available the larger horizontal force (*handling*). Additionally for irregular pavement, the vehicle proneness to tyre contact loss is a function of the power spectral density (*PSD*) of the road vertical roughness content and vehicle travelling speed (Barbosa, 2011-a).

There is a particular type of design request due to the touchdown impact after a jump or during the landing. This impact is typical for competition vehicle like motocross and off-road vehicles, which must support severe vertical force without any structural damage or fatigue.



A comprehensive evaluation index for impact harshness (*IH*) is proposed by Yang (2006) based on studies with full model. Wu (2007) attempts to rank-order the relative sensitivity of each parameter and proposes approaches to improve ride quality. Also, the vehicle design to transpose hump profiles is studied (Baslamisli, 2009). The bump-stop effects during impacts were treated by Wu and Griffin (1998) revealing the importance of the damping contribution.

Detailed complex models (Aydin, 2012) are used to study vehicle suspension response to impact confirming the relevance of the main suspension components: the spring and the damper. Also, a complex model including the tyres is used to optimise suspension (Wang, 2013).



The aim of this work is to identify the vehicle suspension configuration for minimum body load due to the impact of vertical jump for damper optimisation design.

## 2. Fundamentals

During early vehicle development and preliminary concept design, some properties are adopted defining a particular vehicle class. For instance vehicle mass  $m$  and vertical suspension elasticity  $k$  define the main natural vehicle bounce frequency  $\omega_n$  and static settlement deformation  $\delta_{static}$  described as:

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{and} \quad \delta_{static} = \frac{mg}{k} \quad (1)$$

Another important design restriction is the physical space for wheel excursion. Considering the available suspension stroke ratio three times ( $SR = 3$ ) the static deformation, one gets de maximum deflexion prior to bump-stop actions:

$$\delta_{max} = \frac{SR \cdot g}{\omega_n^2} \quad \rightarrow \quad \delta_{max} = \frac{3g}{\omega_n^2} \quad (2)$$

The choice of the natural frequency for passenger vehicles points in general for low values, which better isolate the evenness of the pavement (Barbosa, 2011-b). But it is not the case for sports vehicles.

The last relevant component to be designed is the damper. Suspension optimum damping value  $c_{optimum}$  can be determined from the vehicle mass ( $m$ ) and wheel hub mass ( $m_{hub}$ ) respectively and suspension ( $k_s$ ) and tyre elastic ( $k_t$ ) characteristics. For a non-oscillating system ( $\zeta = 1$ ) one gets (Hill, 2007) the following expressions for comfort and for safety objectives:

$$c_{comfort}^{optimal} = 2\sqrt{k_s m} \quad \text{and} \quad c_{safety}^{optimal} = 2\sqrt{(k_s + k_t)m_{hub}} \quad (3)$$

Dixon (2007) proposed an optimum damping value regarded to power dissipation due to periodic excitation, which values seems to be extremely high. The following expression has been proposed (Dixon, 2007):

$$c_{power\_dissipation}^{optimal} = \frac{(k_s + k_t)}{\omega_n}$$

The suspension of an off-road vehicle should be designed to support besides its own weight, exceptional landing impact loads due to jumps that result form the free fall of a few meters. The velocity at the instant of the touchdown from the free fall height  $h$ , may be determined from the kinetic energy theorem as follows:

$$\Delta T = W \quad \rightarrow \quad \frac{1}{2} m \dot{z}^2 = mgh \quad \rightarrow \quad \dot{z} = \sqrt{2gh} \quad (4)$$

This value can be used as initial condition for starting the deflexion of stretched suspension spring ( $\dot{z}_{(t=0)}$  and  $z_{(t=0)} = 0$ ).

The optimum damping from the structural point of view is the value that absorbs smoothly all the kinetic energy due to the impact, prior to touching the bump-stop, therefore producing the minimum landing structural reaction force. According to the *Minner's* linear cumulative fatigue life damage rule, the structural fatigue live is mainly affected by the number of occurrences of the most severe peak force.

The amount of work undertaken by the suspension after touchdown is calculated by:

$$W = \frac{1}{2} k z^2 + \int c \dot{z}(z) dz \quad (5)$$

from which the maximum suspension compression can be determined. This value should be less than the available suspension stroke range.

It is clear that the softer the suspension is (reduced  $k$ ), the larger the static deflexion and the lower the dynamic impact factor will be (reduced impact force). However this value has design limits due to the space restriction underneath vehicle body.

Yet to optimise the design of the suspension, taking into account the structural aspect, the forces during impact should be minimized. As the lift forces on the vehicle are the

outcome of the suspension forces, namely spring and damper forces, which depend on the out of phase displacement and velocity respectively, what is sought is an ideal combination of them. The reason for the existence of a minimum is exactly the delay between the peak maxima of each supporting forces.

### 3. Modelling

The traditional lumped single degree of freedom mass-spring-damper system as shown in Figure 1 will be used as a simplified linear model.

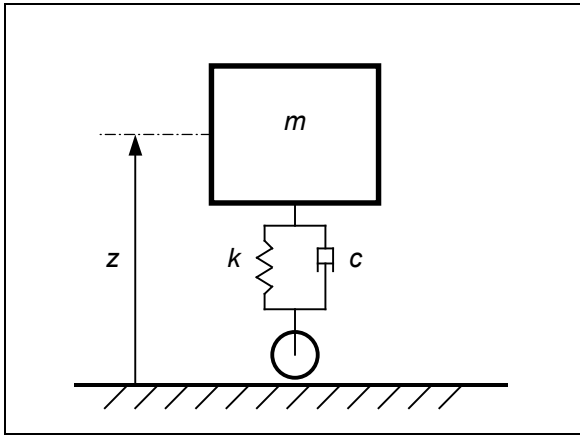


Figure 1 – Simplified Model

The model naturally contains all the relevant parameters confirmed by a complex model study conducted by Aydin (2012): the spring and the damper.

The assumptions used throughout this study are:

- there is movement only in the vertical direction;
- the spring and damper are linear and ideal, and their masses can be neglected;

The traditional second order differential equation with constant terms that describes the dynamics of the model shown in Figure 1 is:

$$m \ddot{z} + c \dot{z} + k z = Q \quad (6)$$

For a damped system, the damping factor is the ratio between the actual damping value ( $c$ ) over the critical damping value ( $c_{critic}$ ):

$$\zeta = \frac{c}{c_{critic}} \quad \text{or} \quad \zeta = \frac{c}{2m\omega_n} \quad (7)$$

where  $c_{critic}$  is the value of damping over which no oscillatory movement is observed and the system damped frequency will be:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \quad (8)$$

and equation 6 also can be written as:

$$\ddot{z} + 2\zeta\omega_n \dot{z} + \omega_n^2 z = -g \quad (9)$$

Considering underdamped system ( $\zeta < 1$ ) the analytical solution of the homogeneous ( $Q = 0$ ) equation with initial conditions different from zero ( $\dot{z}_0 \neq 0$ ) is (eg.: Vierk or Dixon):

$$z(t) = e^{-\zeta\omega_n t} [A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)] \quad (10)$$

which express a harmonic motion with a decaying amplitude. In fact the vehicle weight is an external constant action and must be treated as an additional particular solution.

$$z(t) = z_A(t) + z_B(t) \quad (11)$$

Considering the initial conditions as de non-deformed suspension ( $z_0 = 0$ ) and the free fall speed  $\dot{z}_0 = \sqrt{2gh}$  the constant values are:

$$A_1 = z_0 \quad ; \quad A_2 = \frac{\dot{z}_0 + \zeta\omega_n z_0}{\omega_d} \quad (12)$$

$$z_B(t) = -mg/k \quad (13)$$

From the free body force diagram one can identify the force acting on the vehicle body as:

$$F(t) = -c \dot{z}(t) - k z(t) \quad (14)$$

The maximum peak force during the impact can be identified from the derivation of the force function at null value:

$$F_{\max} = \frac{d}{dt} F(t) = -c \ddot{z}(t) - k \dot{z}(t) = 0 \quad (15)$$

This require the derivative of the position equation, that is the velocity equation and the acceleration equation given by:

$$\dot{z}(t) = e^{-\zeta\omega_n t} [B_1 \cos(\omega_d t) - B_2 \sin(\omega_d t)] \quad (16)$$

where:

$$B_1 = A_2\omega_d - A_1\zeta\omega_n \quad ; \quad B_2 = A_1\omega_d + A_2\zeta\omega_n \quad (17)$$

and

$$\ddot{z}(t) = e^{-\zeta\omega_n t} [-C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)] \quad (18)$$

where:

$$C_1 = B_2\omega_d + B_1\zeta\omega_n \quad ; \quad C_2 = -B_1\omega_d + B_2\zeta\omega_n \quad (19)$$

The instant at the maximum force ( $F_{\max}$ ) obtained from the solution of equation 15, is:

$$t_{\max} = \arctan \left( \frac{k B_1 - c C_1}{k B_2 - c C_2} \right) / \omega_d \quad (20)$$

that can be used again in the force function equation 14 that will be only a function of the initial conditions and vehicle class:

$$F_{\max} = f(\dot{z}_0, \omega_n, \zeta) \quad (21)$$

which the minimum, for a given jump height ( $\dot{z}_0$ ), results in the optimum damping factor (see Figure 3) for a given vehicle class ( $\omega_n$ ).

## 4. Analytical Results

The dynamic of the impact with the floor is simulated by imposing a nonzero initial velocity in to the mass due to the freefall (as described in equation 6) and the initial position of the spring as totally distended. Therefore the initial conditions are  $z_o = 0$  and  $\dot{z}_o = \dot{z}(t_o)$ .

Integrating the acceleration equation 9 for a particular vehicle (mass, natural frequency and damping factor) with respect to time, one obtains the time history of the suspension displacement and velocity (see top and middle graph of Figure 2) and the body external force for  $\omega_n = 1.4$  Hz,  $\zeta = 0.38$  and  $h = 0.5$  meters (bottom graph of Figure 2).

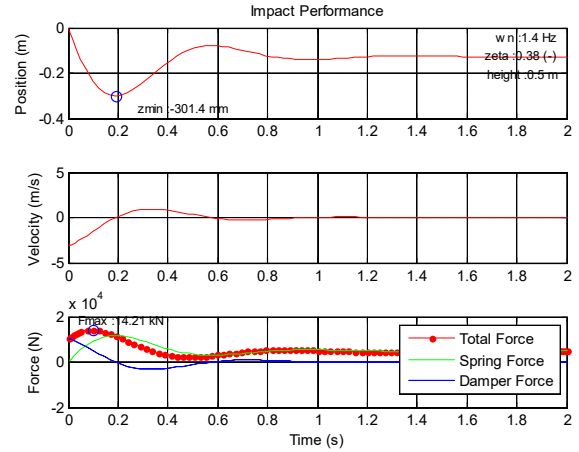


Figure 2 – Impact Temporal History

The bottom graph Figure 2 also allows observing the peak delay of the elastic spring force and dissipative damper force (continuous and dashed line presented separately), due to approximately  $\omega_h/4$  phase shift between them. The reduction of the peak of the total force (sum of each force – line with marks) due to this delay is also pointed out (circle at time around 0.1 seconds).

This analysis can be extended to various normalized jumping heights and different damping factors, producing the surface of the peak force. Figure 3 shows the map of the dynamic factor (normalized peak force over vehicle weight) as a function of the damping factor ( $\zeta$ ) and height factor.

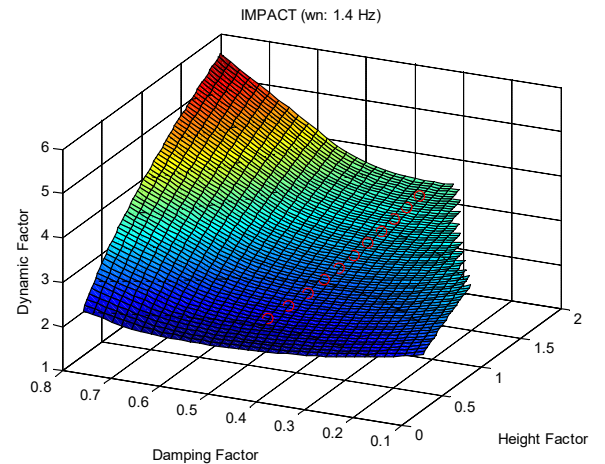


Figure 3 – Optimisation Map

The dynamic load factor is how many times the vehicle proper weight is magnified during the impact. The height factor is the normalization of the jumping height over nominal available suspension excursion. The optimum damping factor is pointed out in the Figure 3 (circles at minimum dynamic force for a given height factor).

It should be noticed that the suspension excursion is physically limited. Beyond this point, the bump-stop of the real suspension (not modelled here) enters in action. This limit is presented in Figure 4 as a displacement saturation at 380 millimetres of deflexion. This value is relevant for the suspension components design dimension. This effect is also observed on the right side of Figure 3, where the surplus values were omitted.

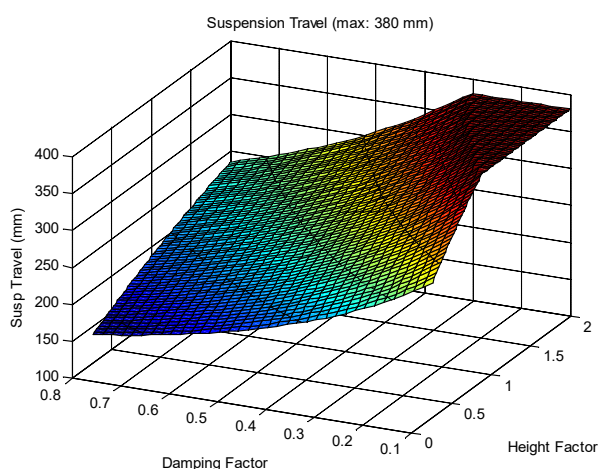


Figure 4 – Maximum suspension excursion

Table 1 – Indicative values for optimum damping factor

$\omega_n$ height	1.4 Hz	1.6 Hz	1.8 Hz	2.0 Hz	2.2 Hz
0.5 m	0.386	0.370	0.358	0.348	0.340
1.0 m	0.348	0.338	0.330	0.322	0.316
1.5 m	0.332	0.324	0.316	0.312	0.308
2.0 m	0.322	0.316	0.310	0.304	0.302

Obs.: provided enough suspension stroke available

Generally speaking, a basic rule of thumb can be produced for structural suspension design: the optimum damping factor ( $\zeta_{OPT}$ ) that produces the minimum impact force for a given vehicle class ( $\omega_n$ ) and jump height ( $h$ ) is according to the table 1, obtained from the solution of equation 15 using instant identified in equation 20. The sensitivity of the optimum damping factor is smaller values for higher natural frequency and larger jump height (eg.: 0.302 for  $\omega_n = 2.2$  Hz and  $h = 2.0$  meters).

## 5. Summary and Conclusions

An analytical formula for optimum design for off-road vehicles suspension is presented. This expression can determine the suspension optimum-damping factor for the minimum impact force during a free fall jumping. This allows complementing the existing vehicle design criteria, focusing on vehicle structural and fatigue issues.

Therefore, for a vehicle class (mass and natural frequency) and severity of use (jump height), one can calculate the optimum damping factor and the resulting dynamic load factor (Genta, 1997).

The optimum suspension design for minimum impact force during vehicle landing and therefore minimum acceleration will increase structural endurance and passenger comfort. The minimum occurs due to the shift delay between the peak of elastic force (spring) and dissipative force (damper). A table of typical values has been generated to the vehicle designer use.

It is clear that the analytical expressions proposed are not the only criterion available for the vehicle design. Also the expression proposed is linear and may be used only on the vehicle design first stage. For a complete vehicle design with non-linear suspension, further methods and tools should be employed.

Future research must increase the model representativity, including the body pitch movement and tyre elasticity, expanding the actual model to four degrees of freedom. Additionally, the eventual components non-linearity, spring pre-compression and bump-stop actions, must be included into the analysis.

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