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Nonlinear Concurrent Multiscale Modeling of Concrete

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Abstract

In this work a concurrent multiscale analysis of concrete is presented, in which two distinct scales are considered: the mesoscale, where the concrete is modeled as a heterogeneous material and the macroscale that treats the concrete as a homogeneous material. The mesostructure heterogeneities are idealized as three phase materials composed by the coarse aggregates, mortar matrix and the interfacial transition zone (ITZ). Special continuum finite elements with high aspect ratio with a damage constitutive model are used to describe the complex nonlinear behavior due to propagation of cracks, which is conducted by crack initiation in the ITZ and propagation to the mortar matrix until macro-crack formation. The numerical simulation of an L-shaped panel is performed to show the ability of the proposed method to predict the behavior of cracks initiation and propagation in the tensile region of the concrete. The numerical results are compared with the experimental ones.

Keywords: multiscale analysis, interface finite element, continuum damage model, cracking, concrete

1. Introduction

The process of initiation and propagation of cracks in heterogeneous materials like concrete is a multiscale phenomenon. By seeking to understand more accurately this phenomenon, multiscale models have gained great prominence in this area of study [1-3]. In these models the solution of the macroscopic problem is controlled by mechanical and phenomenological internal structure of the material behavior, such as mesostructure or the microstructure, which leads the process of degradation observed at the macroscale translated in the form of loss of stiffness and energy dissipation of the material.

In the proposed concurrent multiscale modeling of concrete the macroscale is considered a homogeneous material where no internal structure can be observed. Mesoscopically the concrete is treated as a heterogeneous material. Its mesostructure heterogeneities are idealized as three phase materials composed by the coarse aggregates and the mortar matrix, which are considered homogeneous materials, and the interfacial transition zone (ITZ) which is treated as the weakest phase. In order to generate and place the coarse aggregates into the matrix, an efficient method based upon the Monte Carlo simulation technique is used; aggregate sizes are generated from a grading curve and placed into the mortar matrix randomly, so that no intersection between aggregates is observed [4].

In order to model the crack initiation in the ITZ and propagation through the matrix, instead of using zero-thickness interface elements with discrete (cohesive) constitutive models, relating relative displacement with interface stresses [5, 6], this paper presents a new technique to model these interfaces by means of degenerated solid finite elements (three-node triangular), i.e., elements with a very high aspect ratio (ratio of the largest to the smallest dimension), with the smallest dimension corresponding to the thickness of the interface region [7]. It is shown that, as the aspect ratio increases, the element strains also increase, approaching the kinematics of the strong discontinuity, as is the case of the Continuous Strong Discontinuity Approach (CSDA) [8, 9]. Therefore, based on the same principles as those of CSDA, it can be stated that bounded stresses can be obtained from unbounded strains by means of a continuum constitutive relation, which tends toward

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a discrete constitutive relation as the aspect ratio increases. A tensile damage constitutive relation between strains and stresses, compatible with the CSDA, is proposed to describe the nonlinear behavior of the interfaces. With this model the interface elements can transfer normal (compressive) stresses without overlapping. With this technique the analyses are performed integrally in the context of the continuum mechanics and complex crack patterns involving crack face bridges can be simulated.

2. Interface solid finite element

Let us consider the three-node triangular finite element, with height h given by the distance between node 1 and its projection on the element base, formed by the other nodes, as illustrated in figure 1.

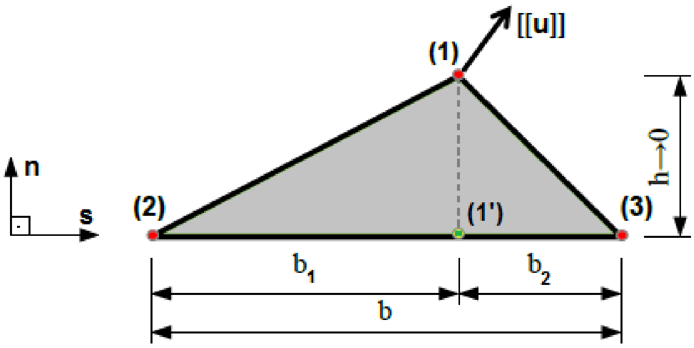


Figure 1: Triangular finite element with high aspect ratio

Following the standard finite element approximations, the strain tensor for any point of the element can be approximated by:

$$\boldsymbol{\varepsilon} = \tilde{\boldsymbol{\varepsilon}} + \hat{\boldsymbol{\varepsilon}} = \frac{1}{b}\begin{bmatrix} 0 & \frac{1}{2}\left(u_n^{(3)} - u_n^{(2)}\right) & 0 \\ \frac{1}{2}\left(u_n^{(3)} - u_n^{(2)}\right) & \left(u_s^{(3)} - u_s^{(2)}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{1}{h}\begin{bmatrix} [[u]]_n & \frac{1}{2}[[u]]_s & 0 \\ \frac{1}{2}[[u]]_s & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (1)$$

where the part $\hat{\boldsymbol{\varepsilon}}$ collects all components of the strain tensor which depends on h and $\tilde{\boldsymbol{\varepsilon}}$ contains the rest of the components. $u_n^{(i)}$ and $u_s^{(i)}$ are the displacement components of node (i), considering the coordinate system (**n**,**s**). $[[u]]_n$ and $[[u]]_s$ are the components of the relative displacement between node 1 and the point corresponding to its projection on the element base (1')

It is possible to show that the strain part associated with h can be written as:

$$\hat{\boldsymbol{\varepsilon}} = \frac{1}{h}(\mathbf{n} \otimes [[\mathbf{u}]])^s \qquad (2)$$

where $(\bullet)^s$ refers to the symmetric part of $(\bullet)$, **n** is the unit vector normal to the element base, and $\otimes$ denotes a dyadic product and [[**u**]] is a vector of the components of the relative displacement. Therefore, the total strain tensor of equation (1) becomes:

$$\boldsymbol{\varepsilon} = \tilde{\boldsymbol{\varepsilon}} + \underbrace{\frac{1}{h}(\mathbf{n} \otimes [[\mathbf{u}]])^s}_{\hat{\boldsymbol{\varepsilon}}} \qquad (3)$$

When the height h tends to zero, the strain component $\tilde{\boldsymbol{\varepsilon}}$ remains bounded while the component $\hat{\boldsymbol{\varepsilon}}$ is no longer bounded. Therefore, in the limit situation of h tending to zero, the element strains are related almost exclusively to the relative displacement between node 1 and its projection on the element base, [[**u**]]. Note that, in the limit situation, node 1 and its projection tend to the same material point. As a consequence the relative displacement [[**u**]] becomes the measure of a displacement discontinuity (strong discontinuity). The structure of the strain field in equation (3) corresponds to the typical kinematics of the CSDA. Therefore, the same applications of the CSDA can be treated with triangular finite elements with high aspect ratio.

3. Tension damage model

3.1. Continuum /discrete constitutive relation of the degenerated finite element

Table 1 shows the continuum constitutive damage model and in the limit case, when h tends to zero, the resulting discrete constitutive equations of the tensile damage model.

Table 1. Continuum and discrete constitutive equations for the tensile damage model

Continuum/Discrete model	Continuum model	Resulting discrete model
Constitutive relation	$\boldsymbol{\sigma} = (1-d)\bar{\boldsymbol{\sigma}}$ $\bar{\boldsymbol{\sigma}} = \mathbf{C}:\boldsymbol{\varepsilon}$	$\mathbf{t} = (1-d)\frac{1}{h}E\begin{bmatrix}1 & 0\\ 0 & 1/2\end{bmatrix}$
Equivalent stress	$\bar{\tau} = \bar{\sigma}_{nn}$	$\bar{\tau} = \bar{\sigma}_{nn} = \frac{1}{h}E[\![u]\!]_n$
Damage criterion	$\bar{\varphi} = \bar{\tau} - r \le 0$	$\bar{\varphi} = \bar{\tau} - r \le 0$
Evolution law for the strain-type variable	$r(t) = r = \max[\bar{\tau}(s), f_t]$	$r(t) = r = \max[\bar{\tau}(s), f_t]$
Damage evolution	$d(r) = 1 - \frac{q(r)}{r}$	$d(r) = 1 - \frac{q(r)}{r}$
Hardening law	$q(r)$	$q(r)$

4. Results

4.1. L-shaped Panel

The panel tested experimentally by [10], at the University of Innsbruck, has become a popular benchmark for many authors who aim to validate their methodologies through comparisons with the crack pattern and the structural force-displacement curve. Figure 2 shows the test setup with the geometric properties and the boundary conditions for the experimental test (see figure 2a) and the assumed boundary conditions with the finite element mesh for the numerical simulation proposed (see figure 2b). The coarse aggregate was generated in a restricted region (see figure 2b) assuming the particle size distribution shown in table 2.

The crack initiation in the ITZ and its propagation through the matrix is simulated introducing the interface elements between all bulk matrix elements and between the matrix elements and the coarse aggregate elements (see figure 2c).

The table 3 shows the assumed properties as suggested by [1], which also simulated numerically the same panel.

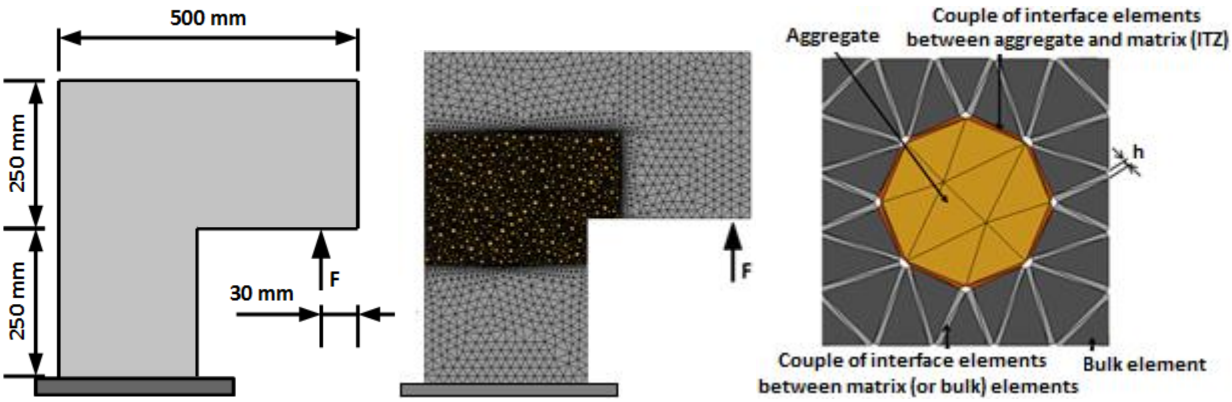
Figure 3 shows the numerical crack path compared with the experimental and numerical proposed by [1] (see figure 3a), the deformed configuration (see figure 3b) and the curves vertical load – vertical displacement (see figure 3c) of the panel. The results are in agreement with the experimental ones, which may indicate the robustness of the proposed model to predict the crack formation and propagation for composite materials like concrete.

Table 2. Particle size distribution

Diameter of the coarse aggregate (mm)	Volume fraction
0-2	0,483
2-4	0,172
4-8	0,035
Matrix	0,31

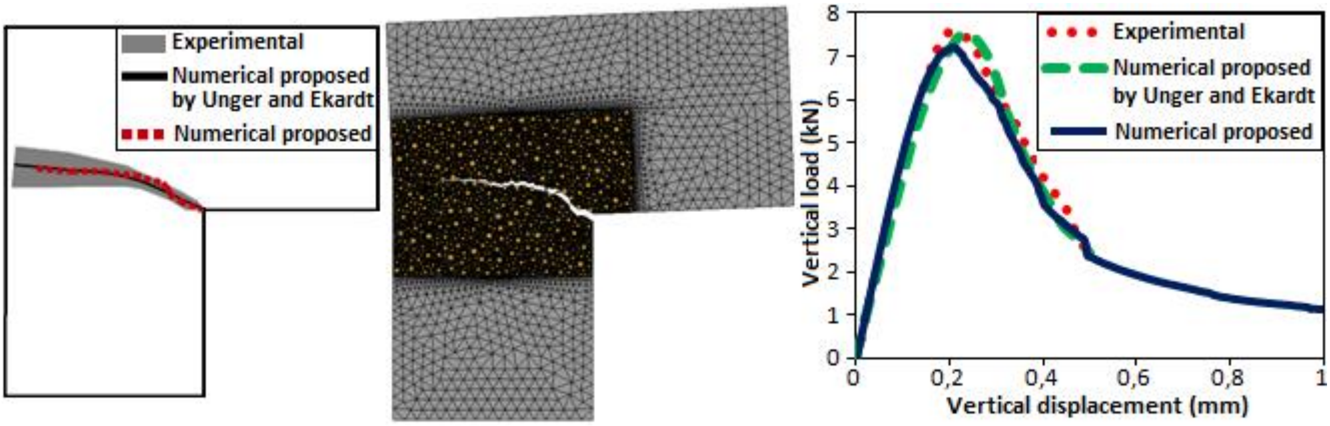
Table 3. Material parameters

Materials	Elastic modulus	Poisson ratio	Fracture energy	Tensile strength
Concrete	$E_c = 20{,}5$ GPa	$\nu_c = 0{,}18$	___	___
Aggregate	$E_c = 37{,}0$ GPa	$\nu_{Ag} = 0{,}18$	___	___
Matrix	$E_m = 18{,}5$ GPa	$\nu_m = 0{,}18$	___	___
Matrix interface	$E_{i/m} = 18{,}5$ GPa	$\nu_{i/m} = 0$	$G_{f_{i/m}} = 0{,}14$ N/mm	$f_{t_{i/m}} = 2{,}6$ MPa
ITZ	$E_i = 18{,}5$ GPa	$\nu_i = 0$	$G_{f_i} = 0{,}07$ N/mm	$f_{t_i} = 1{,}3$ MPa



(a) **(b)** **(c)**

Figure 2: L-shaped panel: a) Test setup, b) multiscale representation with the finite elements mesh and boundary condition and c) interface elements insertion technique



(a) **(b)** **(c)**

Figure 3: a) Experimental and numerical crack paths, b) numerical crack pattern and c) the structural load-displacement curves

5. Conclusions

The insertion of high aspect ratio triangular elements between all bulk matrix elements and between the matrix elements and aggregate elements (ITZ) is able to describe the kinematics of strong discontinuities in the context of the CSDA. As a consequence these elements can be used to represent the behavior of interfaces representing potential crack surfaces.

A simple continuum damage constitutive model based on the tensile stresses normal to the base of these elements is able to describe the cracks initiation in the ITZs, its propagation through the mortar matrix and its coalescence process.

The proposed mesh fragmentation technique, consisting of using solid elements with high aspect ratio in between all regular (bulk) elements, is shown to be appropriate to predict the propagation of cracks without the need of special schemes to track the crack paths during the analysis, This procedure would be very cumbersome for the prediction of a complex path crack propagation, involving crack face bridges, or multiple cracks like in reinforced concrete members.

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