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"NUMERICAL TOOLS AND MATHEMATICAL CONCEPTS OF FATIGUE AND FRACTURE MECHANICS APPLIED TO THE STRUCTURAL INTEGRITY EVALUATION OF MECHANICAL COMPONENTS"

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Abstract. *There are a lot of numerical approaches related to fatigue (HCF or LCF) and fracture mechanics which are followed by the different industries. The aerospace industry is especially concerned with the "damage tolerant approaches". The automotive industry prefers SN based methodologies. The offshore industry uses a mix of both SN and EN approaches. Despite the preferences, an issue comes first when FE analysis is employed: which stress(es) and/or strain(s) value(s) should be regarded in the calculations? Which amount is physical and which one is mathematical singularity? Many standards try to answer that question or, at least, overcome the side effects of the numerical tools, by linearization and related procedures. There is, however, a better way to address this problem, which has becoming increasingly important nowadays: "The Critical Distance Methods". In some circumstances, it's not necessary to completely avoid cracks. If we can determine correctly if such crack will grow or not, we can step forward and work with higher stress(es) values until we get that point where the crack will safely just not propagate. This can get us savings in material costs and weight. And all we need to do is to recall the concepts of transition length, from fracture mechanics, and turn it into the critical distance one, which stands for the position, away from the notch, where our measurements were supposed to be performed. At this distance, the stress(es) values can be taken and combined as needed. So, the present paper intends to show the state-of-the-art of the procedures employed to deal with the finite element results and apply it to the structural integrity evaluation of mechanical components. .*

Keywords: *Theory of Critical Distance, Stress Gradients, Fatigue assessment, Fracture Mechanics.*

1. INTRODUCTION

It is widely accepted that numerical methods (*FEM, DEM, BEM, etc.*) have become essential tools in research and development in any area of knowledge. Softwares and hardwares are becoming faster, more and more friendly, reliable and useful. There are some side effects, however, as any other tool or modern technology. The finite element softwares, for instance, can't prevent users from misunderstanding the results or setting up incorrect boundary conditions, which don't match the real phenomena. Another common source of error regards the material properties. The codes, unfortunately, can't make up for the lack of knowledge of material properties. We can't help using estimates one time or another. But the users should be aware about the numerical errors involved, so that conclusions can be established under a reasonable confidence level.

For durability evaluation purpose, the numerical errors must be lower than the changes in the stress/strain magnitudes. As an example, a difference of 10% in stress, according to the well-known relationship (*the so called Wohler equation*) described by equation (1) below (*with K ranging from 8 to 12*), can lead to a life 2.14 times higher or lower. In other words, accepting an input error of 10% will result in an output error of 114%.

$$\sigma^K N = C \tag{1}$$

The mentioned errors have many sources: the mesh employed to represent the physical domain, the uncertainties in the material properties or the input loadings, as well as the assumed boundary conditions or the constitutive laws applied to simulate the phenomena. The finite element method itself can lead to unrealistic stress concentrations (*see fig.1*), usually located at the surface sharp notches. Thus, besides the referred sources of error, the analyst has also the challenging task of figuring out how much of the post-processed results are real and how much is spurious.

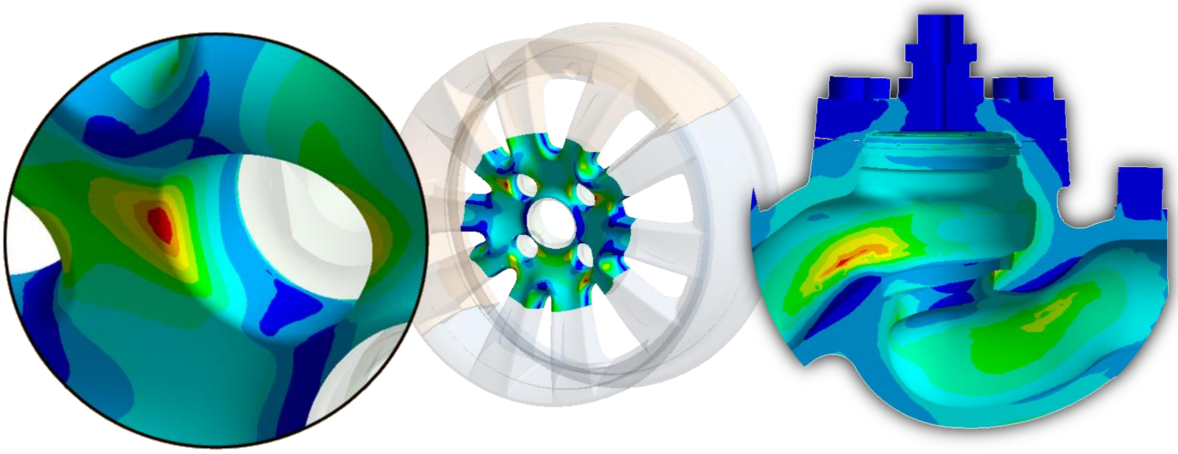


Figure 1. Stress Concentration areas

Many methodologies (*embedded in some engineering standards*) have been designed (*stress classification and linearization*) to deal with stress concentrations, taking into account the nature of such stresses (*bending, membrane, primary, secondary, etc.*) and how they are distributed along the classification line, as it is often called, allowing us to understand the stress gradients and their effect in our calculations.

1. TRANSITION CRACK LENGTH

Crack growth requires energy. Naming this energy δW (*per unit thickness*), and the equivalent crack extension δa, it's possible to write:

$$\delta W = \frac{\sigma^2}{E}\pi a \cdot \delta a \tag{2}$$

And also:

$$G_C = \frac{\delta W}{\delta a} \tag{3a}$$

$$\sigma_f = \sqrt{\frac{G_C E}{\pi \cdot a}} \tag{3b}$$

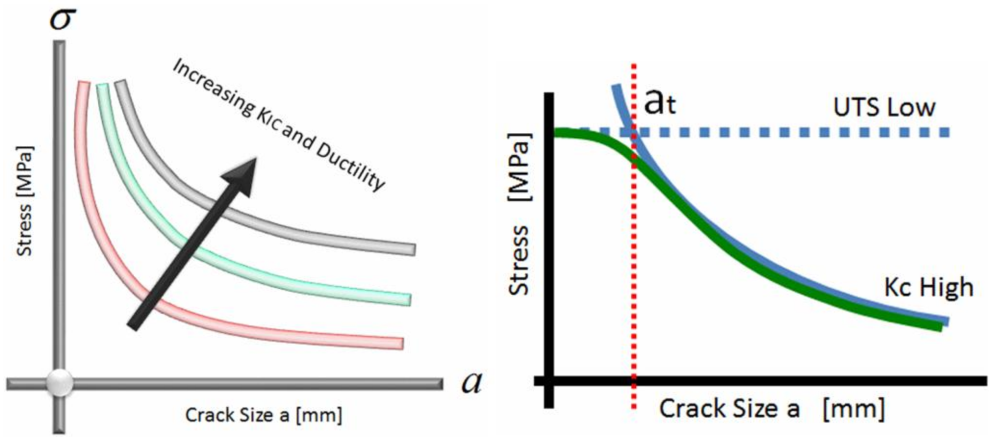
σ_f is defined as the brittle fracture strength, associated to the energy stored in the body to drive crack propagation. We also can combine G_C and *E*, resulting:

$$\sigma_f = \frac{K_C}{\sqrt{\pi \cdot a}} \tag{4}$$

Rearranging the terms, as a function of a_C :

$$a_C = \frac{1}{\pi}\left(\frac{K_C}{FS_{\max}}\right)^2 \tag{5}$$

And F is a factor that depends on the geometry. The equation (5) is particularly important because it tells how sensitive a material can be to a certain crack of size a_C . So, a higher fracture toughness material can withstand larger cracks. Higher toughness also means higher ductility and, consequently, lower UTS (*ultimate tensile strength, see fig.2a*).



(a) (b)

Figure 2. (a) Relationship among stress, crack size and ductility (b) Transition Crack length for a high-strength high-toughness material

This means, basically, that fatigue and fracture mechanics can be often at opposite sides. Improving fatigue strength means increasing UTS (*until a certain limit*), but this also means decreasing ductility as well as the critical crack size a_C , so that smaller cracks would be able to cause brittle fracture. This was precisely the problem with the liberty ships (*during the II world war*), besides the fact they were exposed to temperatures falling below a critical point, changing the mechanism of failure from ductile to brittle, so that the ship hull could fracture rather easily. The large number of failures between 1950 and 1960 [3] is due to the development of new high-strength materials for the aerospace industry, possessing sufficiently low fracture toughness so that they were sensitive to rather small cracks.

2. SMITH-MILLER DIAGRAM

Elastic finite element analysis often show very high stress gradients at notches. The standard fatigue methodologies might fail to predict lifetime under this condition. Essentially because a fatigue crack can arrest the propagation at a certain distance from the notch, and the additional energy needed to keep the propagation is no longer available.

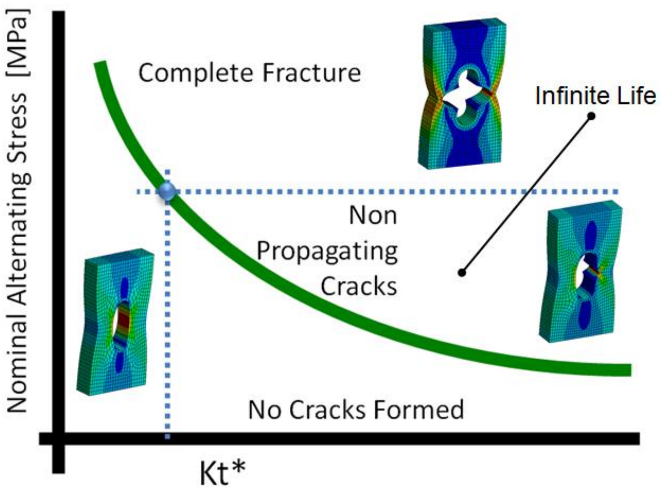


Figure 3. Smith-Miller Diagram

Fig. 3 above uses a central notch component to illustrate the smith-miller diagram. The lower the Kt (*elastic notch coefficient*), the higher the alternating stresses. The area below the green line is safe against crack initiation. There's a Kt*, however, which is worth mentioning, since it intercepts the green line at an allowable nominal alternating stress below which the formed cracks will just not propagate (*infinite life*). This is especially interesting, because the life of a component can be as infinite as if a crack had never arisen.

With this in mind, another methodology has been studied nowadays, gaining followers around the world. This is the critical distance method (*CDM*). In CDM, a small crack of size a_O is placed at the notch, allowing us to calculate the correspondent fracture toughness (K_C), and to adopt a fracture mechanics approach (eq.6) to the component design.

3. CRITICAL DISTANCE

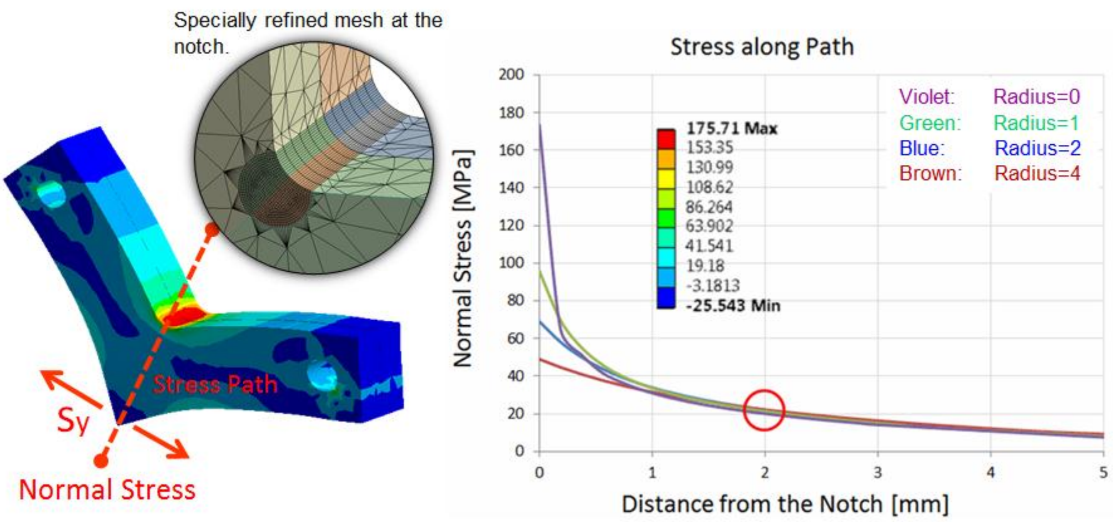
Equation (6) comes from the so called point method. As discussed by Susmel (*2009*) [2] in this book entitled "Multiaxial Notch Fatigue", TCD (*Theory of Critical Distance*) can be formalized in many ways. It's worth mentioning that, besides PM (*point method*), we have also LM (*line method*), AM (*area method*) and VM (*volume method*). The early ideas of our PM have started with Peterson (1959) and Neuber (1958).

$$a_O = \frac{1}{\pi}\left(\frac{K_C}{S_O}\right)^2 \tag{6}$$

In an equivalent manner, knowing ΔK (*threshold stress intensity factor amplitude*) allows us to estimate the acceptable length for an equivalent defect, that one which will just not propagate, the way stated by Taylor, D. (2007) [1]:

$$L = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta \sigma_O}\right)^2 \tag{7}$$

In the chapter 5 of his book (*Theory of Critical Distance*), David Taylor [1] comes up with a curious example, which we ended up reproducing as well. Fig.4 shows an L-shaped specimen, subjected to a displacement at the ends, leading us to the stress field (*normal stress SY*) shown below. The experiment consisted in changing the notch radius (*R= 0, 1, 2, 4*), evaluating the normal stresses along the dashed line which crosses the root and follows the steepest stress gradient. In the book, David has used a nuclear graphite material and came to a different results as the ones we're showing here.



(a) (b)

Figure 4. Stresses along a path line for several notch radius

Despite the differences, however, we're able to observe the same effect, that is the intersection of all curves (*for every fillet radius*) at a distance d=L/2. Whatever the peak stresses are, on the notch surface, 2mm away they end up converging to a much smaller stress value, in the middle of the critical distance.

3. NOTCH EFFECT

Besides "Critical Distance Theory", a plenty of other methodologies populate the literature. Most of them are based on Neuber's (1937), Siebel's (1955) and Peterson's (1959) ideas. The stress averaging approach was first proposed by Neuber. The critical distance approach is found in the early works of Peterson [4]. And the stress gradient approach is attributed to Siebel and Stieler [5].

According to Neuber, it's possible to link the so called fatigue strength reduction factor, Kf, and the stress concentration factor, K_t [2]:

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{\frac{a_N}{r_N}}} \tag{8}$$

Where:

$$K_t = \frac{\sigma_{ep}}{\sigma_{net}} \tag{9}$$

a_N is a constant that depends on the ultimate tensile strength and r_N is the root radius of the notch of the component to be assessed. According to Dowling (2007) [3], such constant a_N

can be calculated by equation (10):

$$a_N = 10^{-\frac{\sigma_{UTS}-134}{586}} \tag{10}$$

It's implicit in the equation (8) the so-called "notch sensitivity factor" q:

$$q = \frac{1}{1+\sqrt{\frac{a_N}{r_N}}} \tag{11}$$

According to Siebel and Stieler, K_t and K_f can be related this way:

$$K_t = \frac{K_f}{n_\chi} \tag{12}$$

$$n_\chi = 1 + \sqrt{\chi} \cdot 10^{-\left(0.33+\frac{\mathrm{Re}}{712}\right)} \tag{13}$$

$$\chi = \frac{1}{\sigma}\left(\frac{d\sigma_y}{dx}\right)_{x=0} \tag{14}$$

Where:

Re: Material yield stress

χ : Relative stress gradient

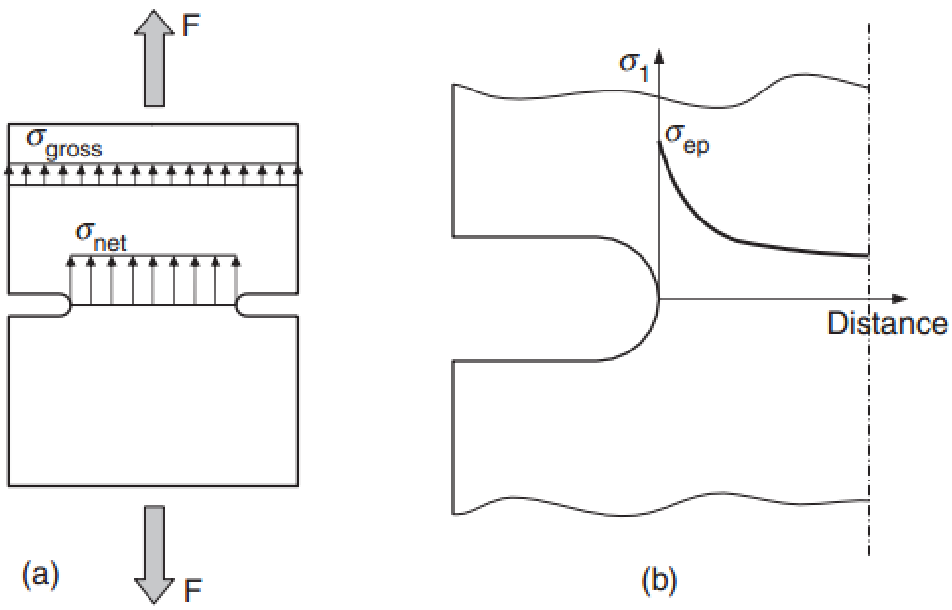


Figure 5. Net nominal stress (a) and linear-elastic peak stress at the tip of a notch (b)

In 2002, Eichlseder [7] has proposed the equation (15) as a way of automate fatigue assessment, by finite element results, of notched specimens. The stress gradient χ' is the input parameter for estimating σ_f (*fatigue limit*). Two values are required: (1) the fatigue limit for an unnotched specimen under tensile loading (*stress gradient* $\chi' = 0$) and (2) the fatigue limit for a bending specimen (*stress gradient* $\chi' = 2/b$*; b=specimen thickness*). When χ' has the same value as χ' of a bending specimen, the fatigue limit converges to σ_{bf}. When χ' has the same value as χ' of a pure tensile specimen, the fatigue limit approaches σ_{tf}.

$$\sigma_f = \sigma_{tf}\left(1+\left(\frac{\sigma_{bf}}{\sigma_{tf}}-1\right)\left(\frac{\chi}{(2/b)}\right)^{K_D}\right) \tag{15}$$

Fig.6 shows the fatigue limit versus stress gradient for a heat treatable steel Ck45 (AISI 1042). The squares represent performed bench tests, whilst the continuous line is given by finite element analysis.

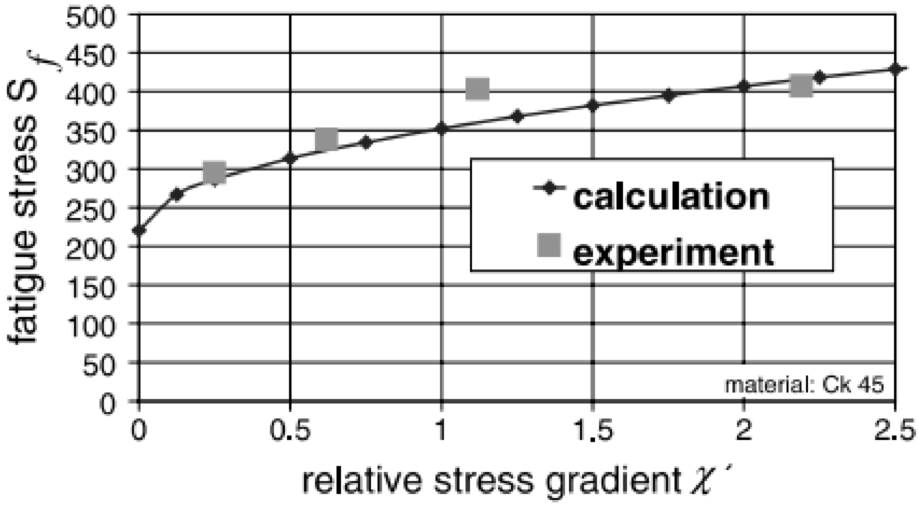


Figure 6. Fatigue limit versus stress gradient

Some standards derive the well-known "hot spot stresses", as in the "Recommended Practice DNV-RP-C203" [6], developed by Det Norske Veritas (*DNV*). Recommended stress evaluation points are located at distances 0.5t and 1.5t away from the hot spot, where t is the plate thickness at the weld toe. These locations are also denoted as stress read out points (fig.7).

The Design of Pressure Vessels can count on methodologies such as "Leak-Before-Break", which is also based on fracture mechanics concepts. An arisen crack on the surface of a vessel wall can be extremely dangerous, causing a sudden brittle fracture prior the vessel leaking.

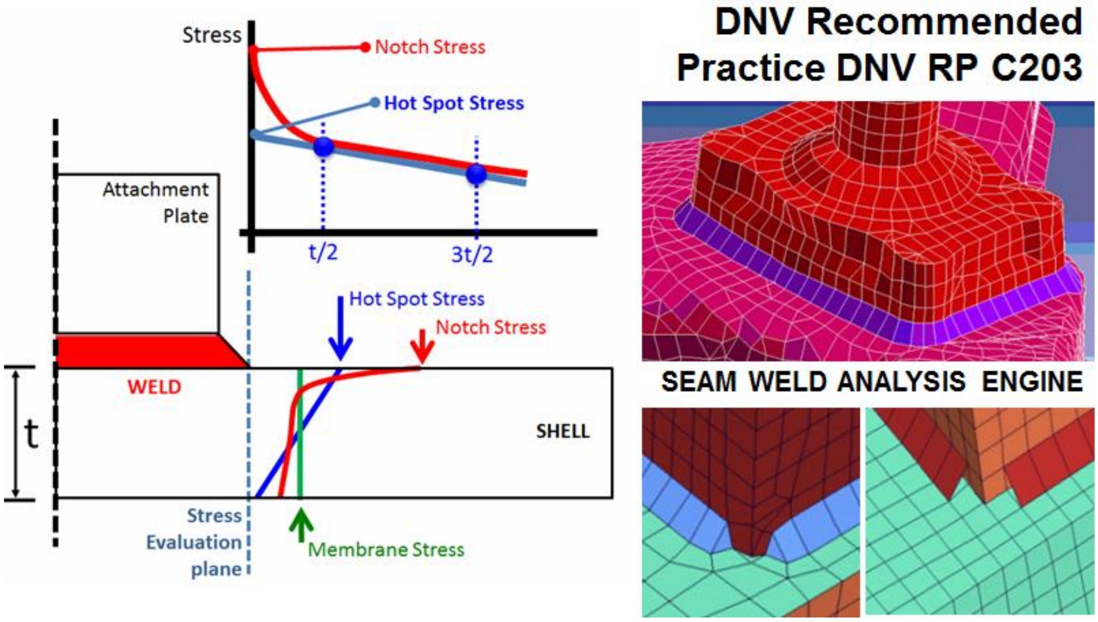


Figure 7. Derivation of effective hot spot stresses from FE analysis

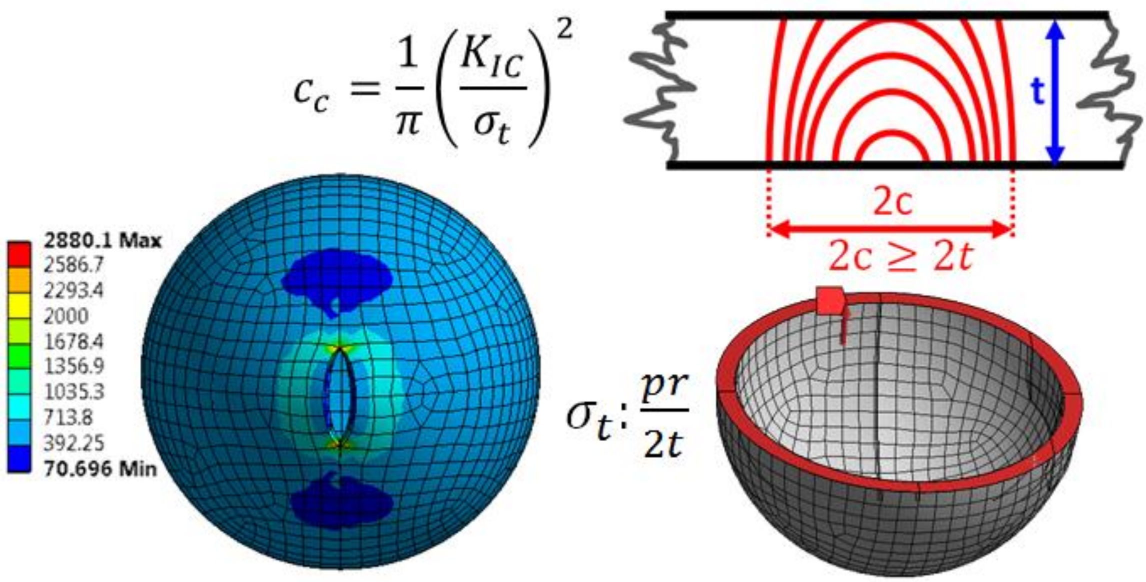


Figure 8. Leak-Before-Break Design

If no such brittle fracture happens, however, the afore mentioned crack may grow through the wall until a length *2C* that is approximately twice the thickness, *2t*, as shown in fig.8. Brittle fracture will not occur provided that the material has a fracture toughness to withstand a through-wall crack of the size $C_C \geq t$ (see fig.8). Equation (16) can be used to check if the “Leak-Before-Break” design condition is met.

$$C_C = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{16}$$

By knowing K_{IC} (*material property*) and the stress σ_t along the vessel wall, it’s possible to verify if a critical crack would be smaller or larger than the wall thickness. In case $C_C \geq t$, so the stress analyst can compute the J-Integral along the edges of a crack placed in a FE model, so that the stress intensity factor K is found and may be compared to K_{IC} resulting in a safety factor SF= K_{IC}/K .

Equation (16) is quite similar to equation (6), showing that the concepts behind "Critical Distance Theory" is the trend for the most modern state-of-the-art structural integrity evaluation methodologies.

4. PRACTICAL APPLICATION OF CRITICAL DISTANCE

The shear coupling below (*fig.9*) will be taken in order to demonstrate the critical distance concept. Firstly the geometry (*CAD model*) was split into some domains, so that local refinement could be provided at the fillet radius shown. Also, splitting the model results in a pathline aver which the stresses will be retrieved as shown in fig.10. The gray cast iron employed in the simulation has an ultimate strength of Su=230MPa, yielding limit of Sy=200MPa, fatigue limit of Sf=65MPa, a stress intensity threshold amplitude of ΔKth=5.7MPa.m$^{1/2}$. These parameters result in a critical distance of a_t = 2.44mm (*fig.9*).

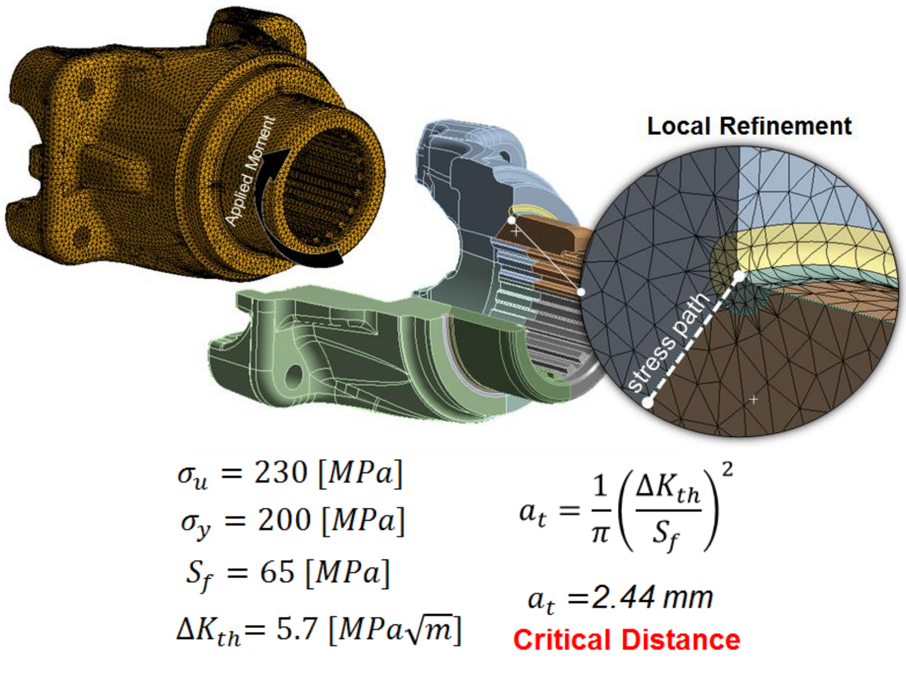
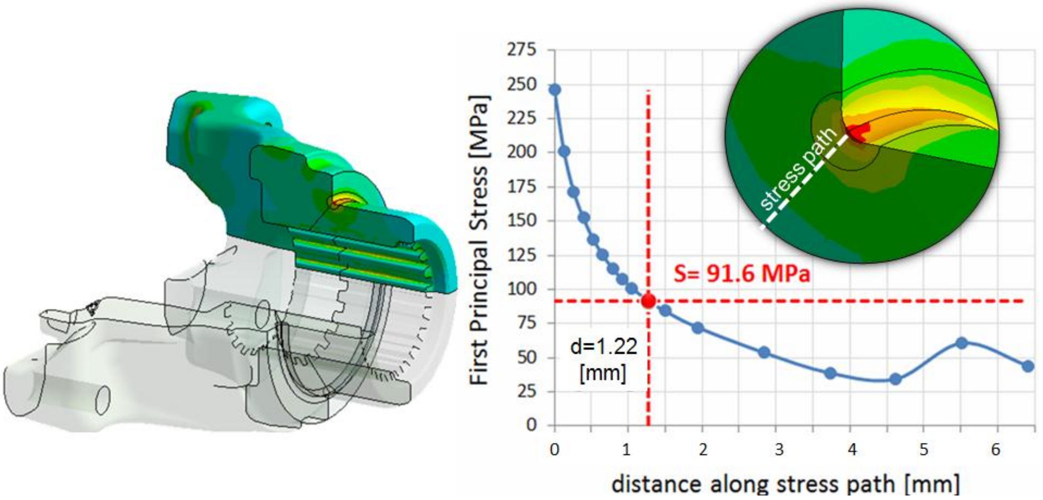


Figure 9. FE representation of a Shear Coupling

At half of this distance we can find a useful node where we're able to post-process the stress and strain tensors to be used in the fatigue calculation. Note that only 1.22mm away from the hot spot we have a maximum principal stress of 91.6[MPa], which is 36.7% of the maximum value found at the most external node. Under a zero-based loading condition (*R=0*), for instance, taking S=250MPa, would lead us to: Sm=125MPa (*mean stress*) and Sa=125MPa (*alternating stress*). For an ultimate strength of Su=230MPa, Goodman would predict an equivalent alternating stress of 273.8MPa, and a fatigue safety factor FF=0.24 (*Sf=65MPa*).

In other words, hot spot stresses drive us to a very conservative result. Under the same loading condition, but taking the principal stress at 1.22mm (*L/2*) away from the notch, would lead us to (*fig.11*): Sm=45.8MPa, Sa=45.8MPa. And the same Goodman would predict a very different equivalent alternating stress of 57.18[MPa], with a safety factor FF=1.14, almost five times (*4.75x*) higher!



(a) (b)

Figure 10. First principal stresses along the pathline

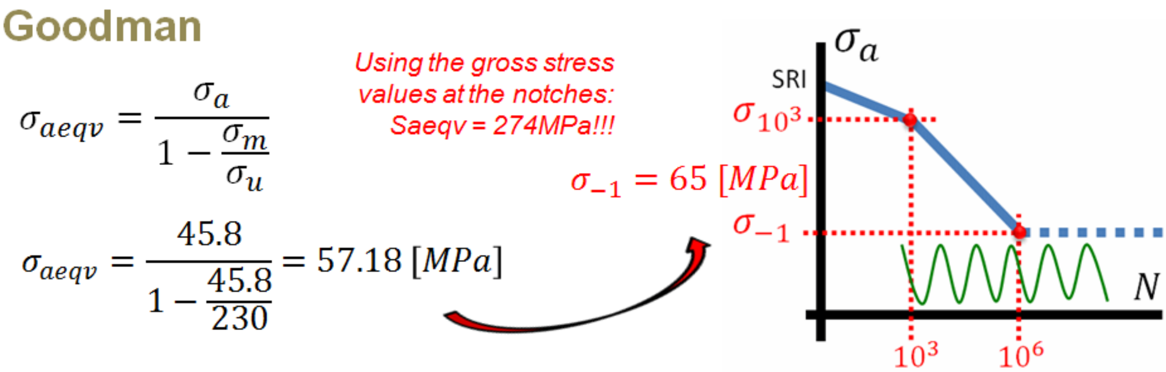


Figure 11. Fatigue Factor Assessment at the critical distance

5. SUMMARY

Critical Distance or Stress Gradient theories are very useful and practical t*ools (embedded in many fatigue softwares*) that can help engineers not only to deal with fatigue assessment, but also to deal with common questions as unrealistic stress concentrations or mathematical singularities found in numerical analysis.

Since the stresses at a critical distance (*and not at the apex*) are the most important variables, ignoring them in an optimization loop, for instance, could be misleading. As for the bracket shown in fig.4, taking hot spot stresses as a parameter could lead us to change the radius fillet (*reducing the maximum stresses*) without improving the endurance of the component, that is related to the equivalent alternating stresses at the critical distance.

Thus, mastering the aforementioned techniques is essential to a more confident, precise, effective and less conservative engineering judgment.

So, the present article intended to show that the basis of “fracture mechanics” can be applied efficiently to investigate the influence of notches as welding toes, fillet radius and high stress gradient areas neighboring surfaces under contact.

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