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POROELASTIC BEHAVIOR OF CRACKED ROCKS AS HOMOGENIZED MEDIA

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Abstract. *The formulation of macroscopic poroelastic behavior of a cracked rock is investigated within the framework of a micro-macro approach. The micro-cracks are modeled as interfaces and their behavior is modeled by means of generalized poroelastic state equations. Starting from Hill´s lemma extended for a medium with cracks and extending the concept of strain concentration to relate the crack displacement jump to macroscopic strain, the overall poroelastic constitutive equations for the cracked rock are formulated. The analysis emphasizes the main differences and similarities of the resulting behavior with respect to that characterizing ordinary porous media. It is shown that, unlike ordinary porous media, conditions on the poroelastic parameters of cracks are required for the macroscopic drained stiffness to entirely define the poroelastic behavior. This is achieved, for instance, if the crack network is characterized by a unique Biot coefficient. Extension of the analysis to non-linear poroelasticity is also outlined. Finally, the theoretical formulation is applied to a particular case of cracked rock for which explicit expressions of the overall poroelastic parameters are derived.*

Keywords: *Cracked rock, Poroelasticity, Micromechanics.*

1. INTRODUCTION

Discontinuities are frequently present at different scales in rock masses and represent a fundamental component of rock deformation and transport of fluid or contaminants through rock masses. Usually referred to as joints, they correspond to zones of small thickness along which the mechanical and physical properties of rock matrix degrade. The presence of joints constitutes the key weak point for stability and safety of many engineering works, such as dam foundations, excavation of tunnels and caverns, oil and gas production, geothermal energy plants, repositories for toxic waste, etc. From transport properties viewpoint, joints within rock masses represent preferential channels for fluid flow and such, may be contributors to rapid transport of fluid and contaminants through rock masses, particularly when the permeability of the rock matrix is low.

As a consequence, comprehensive constitutive modeling of rocks requires accounting for the poromechanics coupling which occurs at the scale of joints and its implication at the scale of the rock structure. Primarily focus should be on the behavior modeling of the rock material as a porous medium with specific treatment for the coupled hydromechanical coupling governing the joint deformation.

Strength, deformation and permeability coupling of rock joints have been widely investigated during the previous decades, and a large amount of experimental works and models are available in the literature. Representative works include references [2,4,5,11,18,20], to cite a few.

Most research concerning hydromechanical coupling in rock joints has been, however, focused on the connection between normal and/or shear loading and unloading and their effect on joint permeability. As a matter of fact, the classical models, which are based on the cubic law and related modified forms, account for the poromechanical coupling only by incorporating the aperture variation of joints induced by applied stresses in the calculation of permeability. The effect of pressure of fluid in the interstitial space of rock joints seems to be traditionally neglected or not properly considered. A fully poromechanical modeling of joints behavior is seldom reported in the literature [3,8].

Several works have recently been devoted to jointed rock deformation in poroelasticity, nevertheless, these works are in their majority macroscopic-scale approaches that consider the joints as particular geometry pores and not as generalized poroelastic media, like it is done in the present analysis.

In this context, the main purpose of this paper is to clarify the formulation of the linear poroelastic behavior of a jointed rock regarded as a homogenized medium. Starting from the local behavior of rock matrix and joints modeled as generalized porous media, the upscaling procedure aims at analyzing the effect of fluid pressure in the interstitial space of rock joints on the overall behavior of the jointed rock. Emphasis shall be put on the fundamental specificity of the homogenized porous medium with respect to ordinary porous media. In particular, the question related to connection between the macroscopic drained stiffness and macroscopic Biot coefficient and Biot tensor, will be discussed.

The paper is concerned with the formulation of the macroscopic state equations in poroelasticity. In this respect, the non-linear aspects related to the irreversible behavior of joints that are fundamental for the analysis of rock masses deformation are disregarded.

2. HILL'S LEMMA FOR THE JOINTED MEDIUM

Let Ω (resp. Ω_0) denote the representative elementary volume (REV) in the current (resp. initial) configuration of a homogeneous rock matrix cut by a discrete distribution of joints $\omega = \bigcup_i \omega_i$. The REV is chosen so as to be statistically representative of the rock medium; in particular, the characteristic size, say d of the heterogeneities (joints) is supposed to be small with respect to the dimension, say l, of the REV, which in turn is supposed to be small as compared to the wavelength λ of the macroscopic solicitation. Moreover, l must be sufficiently smaller than the characteristic dimensions L of the whole rock body. The previous scale separation conditions may be summarized as $d \ll l \ll L$, $l \ll \lambda$.

The network of joints present within the rock medium is formed by long joints or short joints. The adjective ‘long’ characterizes joints crosscutting the REV, while adjective ‘short’ refers to joints with small extension when compared to the size of the REV. More precisely, short joints are in fact microfractures (or microcracks) that are able to transfer stresses.
Regarding the situation of short joints, one of the purposes of the present paper is to extend the classical mechanical model of microcracks in which no stresses are transferred across the microcrack. It is worth noting that the concept of REV implies the scale separation between its characteristic length and those of joints, namely the size of short joints or the average spacing between long joints.

The rock matrix fills the domain $\Omega \setminus \omega$, where symbol $\setminus$ stands for the set difference. Note that strains and stresses within the rock medium are defined on the rock matrix domain $\Omega \setminus \omega$ only, and not on the whole REV. Throughout the paper, symbol $\langle . \rangle$ denotes the volume average over the rock matrix:

$$\langle . \rangle = \frac{1}{|\Omega_0|} \int_{\Omega \setminus \omega} . \; dV \tag{1}$$

At the scale of the REV (microscopic scale), each joint is modeled as an interface, geometrically described by a surface ω_i, whose orientation is defined by a normal unit vector $\underline{n}_i$. At a smaller scale than the microscopic one, the joint ω_i would be represented by a volume of finite thickness with distinct upper and lower boundaries (surfaces) located at ω_i^+ and ω_i^-, respectively (Fig. 1). The boundary of the rock matrix comprises that of the REV, i.e. $\partial\Omega$, as well as the upper and lower boundaries of each joint, i.e. $\partial\omega_i = \omega_i^+ \cup \omega_i^-$.
Let $\underline{x}$ be the position of a point of ω_i at the scale of the REV. The displacement at point $\underline{x}$ is not defined in a unique way because of the relative displacement of the surfaces ω_i^- and ω_i^+. At the scale below the microscopic one, $\underline{x}$ is replaced by a segment parallel to $\underline{n}_i = \underline{n}_i(\underline{x})$ whose bounds are $\underline{x}^- \in \omega_i^-$ and $\underline{x}^+ \in \omega_i^+$. The displacement jump $[\underline{\xi}(\underline{x})]$ is defined as:

$$[\underline{\xi}(\underline{x})] = \underline{\xi}(\underline{x}^+) - \underline{\xi}(\underline{x}^-) = \underline{\xi}^+(\underline{x}) - \underline{\xi}^-(\underline{x}) \tag{2}$$

The objective of this section is to adapt Hill's lemma to the situation of a jointed medium. The loading applied to the REV is defined by homogeneous strain type boundary conditions on the boundary $\partial\Omega_0$ [21]:

$$\underline{\xi}(\underline{x}) = \underline{\underline{\in}} \cdot \underline{x} \quad \forall \underline{x} \in \partial\Omega_0 \tag{3}$$

where $\underline{\underline{\in}}$ represents the macroscopic strain. We introduce the set $\mathcal{C}$ of displacement fields which are kinematically admissible with $\underline{\underline{\in}}$. By definition, it is the set of displacements fields $\underline{\xi}'$ continuous and differentiable on $\Omega \setminus \omega$ and complying with the boundary condition (2). Likewise, $\mathbb{S}$ denotes the set of statically admissible stress fields $\underline{\underline{\sigma}}'$. Defined in Ω, they satisfy the local momentum balance equation $\operatorname{div}\underline{\underline{\sigma}}' = 0$ and the continuity of stress vector $\underline{\underline{\sigma}}' \cdot \underline{n}_i$ when crossing joint ω_i.

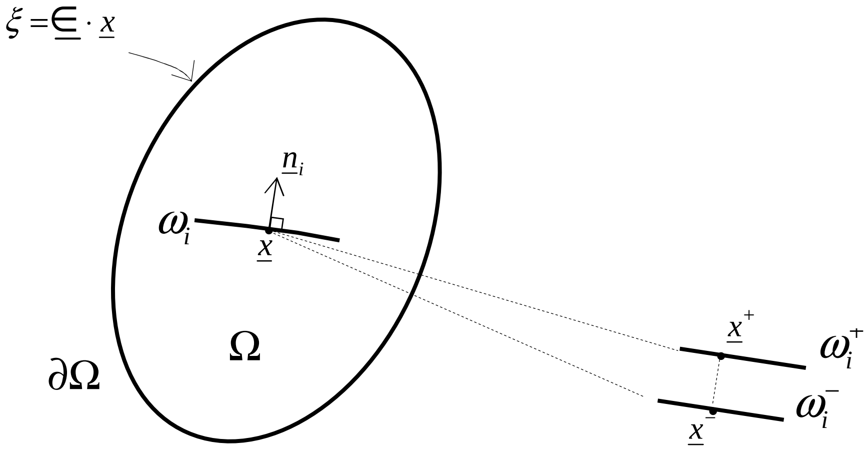


Figure 1. REV of a jointed rock and loading conditions

For all $\underline{\underline{\sigma}}'$ of set $\mathbb{S}$ and $\underline{\xi}'$ of set $\mathbb{S}$, it is readily obtained from integration by part

$$|\Omega_0|\,\langle \underline{\underline{\sigma}}' : \underline{\underline{\varepsilon}}' \rangle = \int_{\Omega\setminus\omega} \underline{\underline{\sigma}}' : \underline{\underline{\varepsilon}}' \;dV = \int_{\partial\Omega} \underline{\xi}' \cdot \underline{\underline{\sigma}}' \cdot \underline{n} \;dV + \int_{\partial\omega} \underline{T}' \cdot \underline{\xi}' \;dS \qquad (4)$$

where $\underline{\underline{\varepsilon}}'$ is the symmetric part of grad $\underline{\xi}'$. The stress vector $\underline{T}'$ is defined by

$$\underline{T}' = \begin{cases} \underline{T}'^{+} = -\underline{\underline{\sigma}}' \cdot \underline{n}_i & \text{along } \omega_i^+ \\ \underline{T}'^{-} = \underline{\underline{\sigma}}' \cdot \underline{n}_i & \text{along } \omega_i^- \end{cases} \qquad (5)$$

Taking (3) into account as well as the continuity of the stress vector $\underline{\underline{\sigma}}' \cdot \underline{n}_i$ when crossing ω_i, Hill's lemma in its classical form shows that

$$\int_{\partial\Omega} \underline{\xi}' \cdot \underline{\underline{\sigma}}' \cdot \underline{n} \;dV = |\Omega_0|\,\langle \underline{\underline{\sigma}}' \rangle : \underline{\underline{\in}} \qquad (6)$$

Moreover, one has

$$\int_{\partial\omega_i} \underline{T}' \cdot \underline{\xi}' \;dS = \int_{\omega_i^+} \underline{T}'^{+} \cdot \underline{\xi}' \;dS + \int_{\omega_i^-} \underline{T}'^{-} \cdot \underline{\xi}' \;dS = \int_{\omega_i^+} \underline{T}'^{+} \cdot [\underline{\xi}'] \;dS \qquad (7)$$

Identifying at the scale of the REV the surface ω_i with ω_i^+ (i.e. $\omega_i \equiv \omega_i^+$), and introducing the notation $\underline{T}'^{n_i} = \underline{T}'^{-} = -\underline{T}'^{+} = \underline{\underline{\sigma}}' \cdot \underline{n}_i$, one may write

$$\int_{\partial\omega} \underline{T}' \cdot \underline{\xi}' \;dS = \int_{\omega} \underline{T}'^{n} \cdot [\underline{\xi}'] \;dS \qquad (8)$$

where $\underline{n} = \underline{n}_i$ along ω_i. Combination of (4), (6) and (8) leads to the following equality:

$$\langle \underline{\underline{\sigma}}' \rangle : \underline{\underline{\in}} = \langle \underline{\underline{\sigma}}' : \underline{\underline{\varepsilon}}' \rangle + \frac{1}{|\Omega_0|} \int_{\omega} \underline{T}'^{n} \cdot [\underline{\xi}'] \ dS \tag{9}$$

which is the form of Hill's lemma extended to jointed rocks. It is noted that $\underline{\underline{\Sigma}}' = \langle \underline{\underline{\sigma}}' \rangle$ represents the macroscopic stress equilibrated by the microscopic stress field $\underline{\underline{\sigma}}'$ of $\mathbb{S}$. Hence, the work of the macroscopic stress $\underline{\underline{\sigma}}'$ in the macroscopic strain $\underline{\underline{\in}}$ comprises the contribution of the internal forces within the rock matrix as well as the work developed by the stress vector acting on the joint in the relative displacement of its boundaries.

For a given second order symmetric tensor $\underline{\underline{a}}$, the uniform stress field $\underline{\underline{\sigma}}' = \underline{\underline{a}}$ is obviously statically admissible, that is, belongs to $\mathbb{S}$. Use of (9) for any value of $\underline{\underline{a}}$ yields

$$\underline{\underline{\in}} = \langle \underline{\underline{\varepsilon}}' \rangle + \frac{1}{|\Omega_0|} \int_{\omega} [\underline{\xi}'] \overset{s}{\otimes} \underline{n} \ dS \tag{10}$$

where symbol $\overset{s}{\otimes}$ stands for the symmetric part of dyadic product: $\left(\underline{u} \overset{s}{\otimes} \underline{v} \right)_{ij} = (u_i v_j + v_i u_j)/2$.

Identity (10) physically means that the macroscopic strain $\underline{\underline{\in}}$ is the sum of two contributions, namely that of rock matrix strains and that of displacement jump along the joints.

Let us now comment on the assumption related to the homogeneity of the rock matrix. The latter should be understood as follows. It is assumed that the scale of the REV (i.e., microscopic scale) is such that the joints represent the only heterogeneities considered for the medium. Accordingly, we shall designate by the term *rock matrix* the composite material made of intact rock phase including possible micro-heterogeneities whose characteristic size is smaller than the dimension of joints. Hence, the rock matrix can be regarded at the scale of the REV, as a homogenized material whose behavior results from a preliminary homogenization procedure accounting for the presence of micro-cracks within the intact rock (see for instance references [7,13).

Remark. Even homogeneous strain boundary conditions (as adopted in the present analysis) are not in principle equivalent to homogeneous stress boundary conditions; this equivalence is implicitly assumed to be so when d/l tends to zero [12,15].

3. FORMULATION OF THE MACROSCOPIC BEHAVIOR IN THE DRY CASE

3.1. Behavior of the jointed rock constituents

We examine in this section the formulation of the macroscopic elastic behavior in absence of interstitial fluid. The rock matrix is assumed to be linearly elastic with fourth-order stiffness tensor $\mathbb{C}^s$. As regards the individual behavior of joints, it is assumed that the corresponding elastic domain in $\mathbb{R}^3$ does not reduce to vector $\underline{T} = 0$. Inside the latter domain, the elastic behavior of joints is assumed to remain linear, at least within the range of considered joint strains. The stiffness of joint ω_i, relating the stress vector to the displacement jump, is denoted by $\underline{\underline{k}}^i$:

$$\begin{cases} \underline{\underline{\sigma}} = \mathbb{C}^s : \underline{\underline{\varepsilon}} & \text{in } \Omega \setminus \omega \\ \underline{T} = \underline{\underline{\sigma}} \cdot \underline{n} = \underline{\underline{k}} : [\underline{\xi}] & \text{along } \omega \end{cases} \tag{11}$$

with $\underline{n} = \underline{n}_i$ and $\underline{\underline{k}} = \underline{\underline{k}}^i$ along ω_i.

At the scale adopted for the modeling, both long and short joints are handled within the same framework. They are modeled as interfaces and the associated deformation is described through a phenomenological law $\underline{T} = \underline{\underline{k}}.[\underline{\xi}]$ linking the stress vector and the displacement jump. In this context, the joint stiffness $\underline{\underline{k}}$ is traditionally evaluated from laboratory tests performed on rock specimen with a single joint. By nature, this phenomenological approach relates the joint stiffness to the geometry and elastic properties of the joint only in a global manner, which can be considered as a major limitation of the approach.
Aimed to formulate $\underline{\underline{k}}$ explicitly from the geometry and elastic properties of the joint, conceptual models have been developed in the literature. At a scale smaller than that of the REV, joints are regarded as rough surfaces in contact at some locations. Basically, the geometry characteristics of the joints such as the peak heights of asperities are described by statistical distributions and the rock fracture surface is treated as rough elastic surface. Hertzian contact theory is therefore used to analyze the deformation of the contacting asperities. The main limitation of such approaches lies in the difficulty to estimate in practice the joint parameters involved in the modeling.

Since the early works of Budiansky and O'Connell [6], a number of papers have been devoted to the micromechanical approach to damage induced by microcracks. On the one hand, it enables to predict how the effective properties are affected by a set of microcracks, including non linear effects associated with progressive cracks closure (see, e.g., [7]). On the other hand, it also provides a quantitative approach to the damage evolution related to the propagation of microcracks (see for instance [9,19]). By contrast, macroscopic damage models can only deal with these two issues in a phenomenological manner. One of the purposes of the present paper is to extend the classical mechanical model of microcracks in which no stresses are transferred across the microcrack. As stated by the state equation of the joint, the latter is able to transfer normal as well as tangential stresses. Nonetheless, large strains in the joint take place and are represented by a displacement jump.

3.2. Behavior of the jointed rock constituents

$\underline{\underline{\in}}$ being prescribed, we consider the elastic problem defined on the REV subjected to the loading defined by the boundary conditions (3). The solution to this problem is the couple $\left(\underline{\underline{\sigma}}, \underline{\xi}\right)$ in $\mathbb{S} \times \mathcal{C}$ and complying with (11). Clearly enough, $\underline{\underline{\sigma}}$ and $\underline{\xi}$ linearly depend on the loading parameter $\underline{\underline{\in}}$. This property is usually expressed through the concept of strain concentration tensor, denoted here by the fourth-order tensor $\mathbb{A}$. By definition, the term $\mathbb{A}(\underline{x}):\underline{\underline{\in}}$ represents the strain tensor $\underline{\underline{\varepsilon}}$ at point $\underline{x}$ corresponding to the load defined by (3). In other words, $\mathbb{A}(\underline{x})$ is the link between the local strain $\underline{\underline{\varepsilon}}(\underline{x})$ in the rock matrix to the macroscopic strain $\underline{\underline{\in}}$ applied to the REV. Besides, the strain concentration tensor also relates the local stress $\underline{\underline{\sigma}}$ to the macroscopic strain:

$$\underline{\underline{\sigma}} = \mathbb{C}^s : \mathbb{A} : \underline{\underline{\in}} \tag{12}$$

The macroscopic stress $\underline{\underline{\Sigma}}$ being defined as the average $\langle \underline{\underline{\sigma}} \rangle$, (12) yields :

$$\underline{\underline{\Sigma}} = \mathbb{C}^{\text{hom}} : \underline{\underline{\in}} \qquad \textit{with} \qquad \mathbb{C}^{\text{hom}} = \langle \mathbb{C}^s : \mathbb{A} \rangle \tag{13}$$

Likewise, concentration tensors are introduced in the following way to relate the components of the displacement jump to the loading $\underline{\underline{\in}}$. If the couple of vectors $\left(\underline{t}_i, \underline{t}_i^{'}\right)$ constitute an orthonormal frame of the plane tangent to ω_i at point $\underline{x}$ (Fig. 2), the normal and tangential components of $\underline{\xi}$ are expressed as:

$$[\underline{\xi}] = \left(\underline{\underline{a}}^n : \underline{\underline{\in}}\right)\underline{n} + \left(\underline{\underline{a}}^t : \underline{\underline{\in}}\right)\underline{t} + \left(\underline{\underline{a}}^{t'} : \underline{\underline{\in}}\right)\underline{t}' \qquad \text{along } \omega \tag{14}$$

with $\underline{n} = \underline{n}_i$, $\underline{t} = \underline{t}_i$ and $\underline{t}' = \underline{t}_i'$ along ω_i. Tensors $\underline{\underline{a}}^n$, $\underline{\underline{a}}^t$ and $\underline{\underline{a}}^{t'}$ are respectively the concentration tensors for normal and tangential displacement jumps of $\underline{\xi}$.

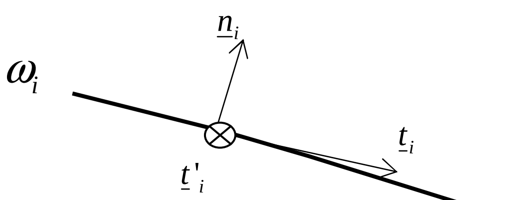


Figure 2. Local frame for joint ω_i.

Due to the presence of the joints the average rule $\underline{\underline{\in}} = \langle \underline{\underline{\varepsilon}} \rangle$ is not valid in the jointed REV as indicated by (10). Accordingly, the average $\langle \mathbb{A} \rangle$ of the strain concentration tensor over the rock matrix is not equal to the fourth order identity tensor $\mathbb{I}$ and thus, $\mathbb{C}^{\text{hom}}$ is not equal to matrix stiffness of rock matrix $\mathbb{C}^s$. More precisely, using (14), we first obtain

$$[\underline{\xi}]\overset{s}{\otimes}\underline{n}=\left(\underline{n}\otimes\underline{n}\otimes\underline{\underline{a}}^{n}+\underline{t}\overset{s}{\otimes}\underline{n}\otimes\underline{\underline{a}}^{t}+\underline{t}'\overset{s}{\otimes}\underline{n}\otimes\underline{\underline{a}}^{t'}\right):\underline{\underline{\in}} \tag{15}$$

Then, combining the average rule (10) and (15) leads to the equality

$$\langle\mathbb{A}\rangle=\mathbb{I}-\frac{1}{|\Omega_0|}\int_{\omega}\left[\underline{n}\otimes\underline{n}\otimes\underline{\underline{a}}^{n}+\underline{t}\overset{s}{\otimes}\underline{n}\otimes\underline{\underline{a}}^{t}+\underline{t}'\overset{s}{\otimes}\underline{n}\otimes\underline{\underline{a}}^{t'}\right]\ dS \tag{16}$$

As emphasized by (16), the discrepancy between $\mathbb{A}$ and $\mathbb{I}$ is due to the fraction of the macroscopic strain which localizes in the joints. The anisotropy of the elastic properties of a jointed rock is directly taken into account through the joints orientation.

4. MACROSCOPIC STATE EQUATIONS IN THE CASE OF SATURATED JOINT NETWORK

We now consider the situation where the connected joint network is saturated by a fluid at pressure p which is assumed to be uniform in the REV. With respect to the dry case, the elastic behavior of the rock matrix is the same as before: $\underline{\underline{\sigma}}=\mathbb{C}^{s}:\underline{\underline{\varepsilon}}$ in $\Omega\setminus\omega$. The behavior of the joints is replaced by a poroelastic formulation in order to account for the effect of the fluid pressure on the relationship between the stress vector acting on the joint and the corresponding relative displacement. The poroelastic state equations for the joints are written in the following form [3,8]

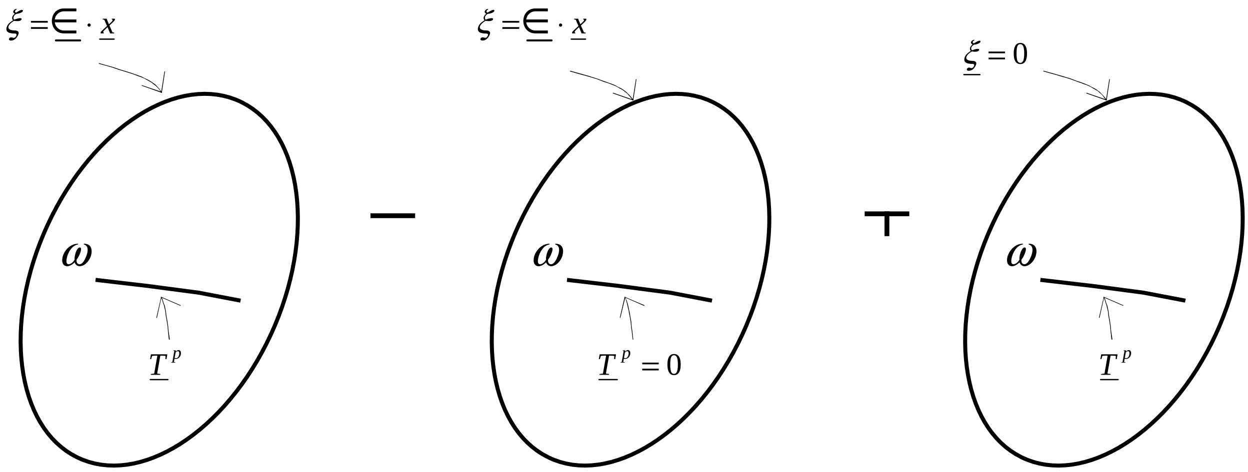
$$\begin{cases}\underline{T}^{n}=\underline{\underline{\sigma}}\cdot\underline{n}=\underline{\underline{k}}\cdot[\underline{\xi}]+\underline{T}^{p}\\ \varphi=\dfrac{p}{m}+\alpha\,[\underline{\xi}]\cdot\underline{n}\end{cases}\quad \text{along } \omega=\bigcup_i\omega_i \tag{17}$$

where

$$\alpha=\alpha_i\quad,\quad m=m_i\quad,\quad \underline{T}^{p}=-\alpha_i\,p\,\underline{n}_i\quad \text{along } \omega_i \tag{18}$$

Scalar α_i has the significance of a Biot coefficient for the joint ω_i modeled as a generalized porous medium. This means that the displacement jump $[\underline{\xi}]$ which represents the joint deformation is controlled by the effective stress vector $\underline{T}^{n}+\alpha p\ \underline{n}$. As regards the second state equation in (17) of the joint, it relates the joint pore change per unit joint surface φ to the fluid pressure p and the joint displacement jump $[\underline{\xi}]$. Scalar m_i represents the Biot modulus for joint ω_i. Physical interpretation as well as identification procedures of the above parameters from appropriate laboratory tests are outlined in [3].

The loading is now characterized by two parameters, namely the macroscopic strain $\underline{\underline{\in}}$ and the fluid pressure p. The solution in $\Omega\setminus\omega$ to this problem defined by the loading mode $\left(\underline{\underline{\in}}\ ,\ p\right)$ and denoted by (P), is the stress field $\underline{\underline{\sigma}}$ in $\mathbb{S}$ and the displacement field $\underline{\xi}$ in $\mathcal{C}$ related by the state equations of the medium constituents $\underline{\underline{\sigma}} = \mathbb{c}^s : \underline{\underline{\varepsilon}}$ in $\Omega \setminus \omega$ and (17). Due to the linearity of the material behavior expressed in rate form, the superposition principle can be used to decompose problem (P) into two elementary problems (P1) and (P2) respectively defined by the loading $\left(\underline{\underline{\in}}\ ,\ p=0\right)$ and $\left(\underline{\underline{\in}}=0\ ,\ p\right)$ as shown in Figure 3. (P1) corresponds to the dry case analyzed in section **3**, whereas (P2) corresponds to pressurized joint network and prevented macroscopic strain.



(P) (P1) (P2)

Figure 3. Decomposition of problem (P) into two elementary problems (P1) and (P2).

Let us designate by $\underline{\xi}_1$, $\underline{\underline{\varepsilon}}_1$ and $\underline{\underline{\sigma}}_1$ the displacement, strain and stress fields in the REV corresponding to problem (P1) and by $\underline{\xi}_2$, $\underline{\underline{\varepsilon}}_2$ and $\underline{\underline{\sigma}}_2$ the displacement, strain and stress fields in the REV corresponding to problem (P2). The fields solution to problem (P) can simply be obtained as $\underline{\xi} = \underline{\xi}_1 + \underline{\xi}_2$, $\underline{\underline{\varepsilon}} = \underline{\underline{\varepsilon}}_1 + \underline{\underline{\varepsilon}}_2$ and $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}_1 + \underline{\underline{\sigma}}_2$.

4.1. First state equation

(P1) being the problem analyzed in section **3**, the following relationships thus holds

$$\underline{\underline{\Sigma}}_1 = \langle\, \underline{\underline{\sigma}}_1 \,\rangle = \mathbb{C}^{\mathrm{hom}} : \underline{\underline{\in}} \qquad \textit{with} \qquad \mathbb{C}^{\mathrm{hom}} = \langle\, \mathbb{c}^s : \mathbb{A} \,\rangle \tag{19}$$

where the strain concentration tensor $\mathbb{A}$ relates $\underline{\underline{\varepsilon}}_1$ to the loading parameter $\underline{\underline{\in}}$ in problem (P)

$$\underline{\underline{\varepsilon}}_1 = \mathbb{A}(\underline{x}) : \underline{\underline{\in}} \tag{20}$$

Regarding problem (P2), $\underline{\underline{\Sigma}}_2 = \langle\, \underline{\underline{\sigma}}_2 \,\rangle$ represents the macroscopic stress associated with joint interstitial fluid pressure p which is required to prevent the appearance of any macroscopic strain. In order to evaluate $\underline{\underline{\Sigma}}_2$, Hill's lemma (9) is used twice. First, it is applied with the couple $\left(\underline{\underline{\sigma}}' = \underline{\underline{\sigma}}_2 \,,\; \underline{\xi}' = \underline{\xi}_1 \right)$:

$$\underbrace{\langle \underline{\underline{\sigma}}_2 \rangle}_{\underline{\underline{\Sigma}}_2} : \underline{\underline{\in}} = \langle \underline{\underline{\sigma}}_2 : \underline{\underline{\varepsilon}}_1 \rangle + \frac{1}{|\Omega_0|} \int_\omega [\underline{\xi}_1] \cdot \left(\underline{\underline{k}} \cdot [\underline{\xi}_2] + \underline{T}^p \right) \; dS \tag{21}$$

Hill's lemma is then applied with the couple $\left(\underline{\underline{\sigma}}' = \underline{\underline{\sigma}}_1 \,,\; \underline{\xi}' = \underline{\xi}_2 \right)$:

$$0 = \langle \underline{\underline{\sigma}}_1 : \underline{\underline{\varepsilon}}_2 \rangle + \frac{1}{|\Omega_0|} \int_\omega [\underline{\xi}_2] \cdot \underline{\underline{k}} \cdot [\underline{\xi}_1] \; dS \tag{22}$$

since the displacement field $\underline{\xi}_2$ in problem (P2) is kinematically admissible with the macroscopic strain $\underline{\underline{\in}} = 0$.

It follows from the state equation of the rock matrix that $\underline{\underline{\sigma}}_1 = \mathbb{C}^s : \underline{\underline{\varepsilon}}_1$ and $\underline{\underline{\sigma}}_2 = \mathbb{C}^s : \underline{\underline{\varepsilon}}_2$, which in turn ensure equality

$$\langle \underline{\underline{\sigma}}_2 : \underline{\underline{\varepsilon}}_1 \rangle = \langle \underline{\underline{\sigma}}_1 : \underline{\underline{\varepsilon}}_2 \rangle \tag{23}$$

Combination of (21), (22) and (23) yield

$$\underline{\underline{\Sigma}}_2 : \underline{\underline{\in}} = \frac{1}{|\Omega_0|} \int_\omega \underline{T}^p \cdot [\underline{\xi}_1] \; dS \tag{24}$$

Recalling that $\underline{T}^p = -\alpha\, p\, \underline{n}$ along ω, and substituting into (21) the displacement $[\underline{\xi}_1]$ in the dry problem (P1) by its expression (14), one obtains:

$$\underline{\underline{\Sigma}}_2 : \underline{\underline{\in}} = -p \; \frac{1}{|\Omega_0|} \int_\omega \alpha \, \underline{\underline{a}}^n \; dS \; : \underline{\underline{\in}} \tag{25}$$

and since the macroscopic stress in problem (P2) is independent of $\underline{\underline{\in}}$, (25) finally reads

$$\underline{\underline{\Sigma}}_2 = -p \, \underline{\underline{B}} \tag{26}$$

with

$$\underline{\underline{B}} = \frac{1}{|\Omega_0|} \int_\omega \alpha \, \underline{\underline{a}}^n \; dS \tag{27}$$

The first macroscopic state equation is obtained from (19) and (26), by superposition

$$\underline{\underline{\Sigma}} = \underline{\underline{\Sigma}}_1 + \underline{\underline{\Sigma}}_2 = \mathbb{C}^{\text{hom}} : \underline{\underline{\in}} - p \, \underline{\underline{B}} \tag{28}$$

Similarly to ordinary porous media, the macroscopic strain $\underline{\underline{\in}}$ is controlled in poroelasticity by an effective Biot stress $\underline{\underline{\Sigma}} + p\,\underline{\underline{B}}$. The tensor $\underline{\underline{B}}$ defined in Eq. (27) can be interpreted as the tensor of Biot coefficients for the jointed medium. The anisotropy introduced by the joint orientation is captured through that of the normal concentration tensor $\underline{\underline{a}}^n$.
The limit case of closed joints can be characterized by expressing that the normal component of the relative displacement $[\underline{\xi}]$ vanishes, which implies that $\underline{\underline{a}}^n \to 0$. In such a situation, the joint fluid pressure has no effect (i.e. $\underline{\underline{B}} \to 0$) on the relationship between the macroscopic strain and stress within the elastic domain.

The fundamental difference between the jointed rock and an ordinary porous medium arises when examining how the Biot tensor $\underline{\underline{B}}$ is connected to the macroscopic elastic tensor of drained moduli $\mathbb{C}^{\text{hom}}$.
For an ordinary porous medium, the classical relationship $\underline{\underline{B}} = \underline{\underline{1}} : \left(\mathbb{I} - (1-\phi)\, \mathbb{c}^{s^{-1}} : \mathbb{C}^{\text{hom}} \right)$ where ϕ is the porosity of the medium [40], shows that the macroscopic Biot tensor is entirely defined once the macroscopic tensor of elastic moduli is determined.
As regards the jointed medium, it readily follows from (16)

$$\frac{1}{|\Omega_0|} \int_{\omega} \underline{\underline{a}}^n \, dS = \underline{\underline{1}} : \left(\mathbb{I} - \langle \mathbb{A} \rangle \right) \tag{29}$$

Recalling that the concentration tensor $\mathbb{A}$ can be related to $\mathbb{C}^{\text{hom}}$ and $\mathbb{c}^s$ as in (19), the above equality (29) takes the alternative form

$$\frac{1}{|\Omega_0|} \int_{\omega} \underline{\underline{a}}^n \, dS = \underline{\underline{1}} : \left(\mathbb{I} - \mathbb{c}^{s^{-1}} : \mathbb{C}^{\text{hom}} \right) \tag{30}$$

Hence, two possibilities are to be considered:

Case 1. All the joints have the same Biot coefficient, i.e. $\forall i \;\; \alpha_i = \alpha$. In this situation, comparison of equalities (27) and (30) provides the following identity

$$\underline{\underline{B}} = \alpha \, \underline{\underline{1}} : \left(\mathbb{I} - \mathbb{c}^{s^{-1}} : \mathbb{C}^{\text{hom}} \right) \tag{31}$$

which extends to the case of a jointed medium the classical relationship relating the tensor of Biot coefficients to the elastic tensors of the porous medium and solid matrix.

Case 2. There exists at least two joints having distinct Biot coefficients, i.e. $\exists (i, j) \mid \;\; \alpha_i \neq \alpha_j$. Unlike the situation of ordinary porous medium, there is no direct connection between $\underline{\underline{B}}$, $\mathbb{C}^{\text{hom}}$ and the poroelastic properties of the rock matrix. In this rather general case, the determination of $\underline{\underline{B}}$ defined by (27) would require *a priori* the knowledge of the normal concentration tensor $\underline{\underline{a}}^n$.

4.2. Second state equation

The complete formulation of the overall poroelastic behavior for the jointed medium is achieved by providing the second macroscopic state equation. The second state equation for the macroscopic poroelastic behavior classically relates the pore volume change to the fluid pressure p and the macroscopic strain $\underline{\underline{\epsilon}}$. In the particular case under consideration, the pore volume change is exclusively due to the joint volume change. For this purpose, we introduce a dimensionless variable called lagrangian porosity change defined as:

$$\Phi = \frac{1}{|\Omega_0|}\int_\omega \varphi \; dS \tag{32}$$

which reads by virtue of (17)

$$\Phi = \frac{1}{|\Omega_0|}\int_\omega \left(\frac{p}{m}+\alpha\,[\underline{\xi}]\cdot\underline{n}\right)\, dS \tag{33}$$

Referring to the decomposition of $\underline{\xi}$ as $\underline{\xi}_1+\underline{\xi}_2$, (33) takes the form

$$\Phi = \frac{1}{|\Omega_0|}\int_\omega \frac{p}{m}\; dS \;+\; \frac{1}{|\Omega_0|}\int_\omega \alpha\,[\underline{\xi}_1]\cdot\underline{n}\;\, dS \;+\; \frac{1}{|\Omega_0|}\int_\omega \alpha\;[\underline{\xi}_2]\cdot\underline{n}\;\, dS \tag{34}$$

The first term of the right hand side in the above equality writes

$$\frac{1}{|\Omega_0|}\int_\omega \frac{p}{m}\; dS \;= \frac{p}{\bar{m}} \tag{35}$$

where the average Biot modulus $\bar{m}$ is given by

$$\frac{1}{\bar{m}} = \frac{1}{|\Omega_0|}\int_\omega \frac{1}{m}\; dS \;=\; \sum_i S_i\,\frac{1}{m_i} \tag{36}$$

S_i represents the specific area of joint ω_i .

From definition (14) relating the displacement jump $[\underline{\xi}_1]$ in the dry problem to the concentration tensors $(\underline{\underline{a}}^n, \underline{\underline{a}}^t, \underline{\underline{a}}^{t'})$ and macroscopic strain tensor $\underline{\underline{\epsilon}}$, it can readily be shown that the second term of the right hand side in (34) reads

$$\frac{1}{|\Omega_0|}\int_\omega \alpha\,[\underline{\xi}_1]\cdot\underline{n}\;\, dS \;=\; \underline{\underline{B}}:\underline{\underline{\epsilon}} \tag{37}$$

Finally, the last integral of the right hand side in (34) can be evaluated by invoking linearity arguments. Indeed, the response $\underline{\xi}_2$, and consequently the corresponding jump $[\underline{\xi}_2]$, associated with the loading $\left(\underline{\underline{\in}} = 0 \;,\; p\right)$ in problem (P2) is proportional to fluid pressure p. Thus, a scalar $\tilde{m}$ exists such that

$$\frac{1}{|\Omega_0|}\int_\omega \alpha\,[\underline{\xi}_2]\cdot\underline{n}\;dS \;=\; \frac{p}{\tilde{m}} \tag{38}$$

Finally, the conjunction of Equations (34), (35), (37) and (38) yields

$$\Phi \;= \frac{p}{M} + \underline{\underline{B}} : \underline{\underline{\in}} \tag{39}$$

Relationship (39) is the second state equation for jointed porous medium. It constitutes with (28) a set of two equations governing the response of the jointed porous medium.
The expression of the macroscopic Biot modulus is given by

$$\frac{1}{M} = \frac{1}{\bar{m}} + \frac{1}{\tilde{m}} \tag{40}$$

As regards the connection between the overall Biot Modulus M and the elastic properties of jointed medium $\mathbb{C}^{\mathrm{hom}}$, once again one should examine as in section **4.1** whether the value of the Biot coefficient is the same for all joints or not.

Case 1. All the joints have the same Biot coefficient, i.e. $\forall i \;\; \alpha_i = \alpha$. As shown by relationship (31) established in section 4.1, the macroscopic tensor $\underline{\underline{B}}$ of coefficients is determined from the knowledge of $\mathbb{C}^{\mathrm{hom}}$ and the poroelastic properties of the jointed medium constituents. To provide the expression of M defined in (40), it is first observed that $\bar{m}$ is simply obtained by (36) from the specific areas and Biot coefficients of the individual joints.
On the other hand, the displacement field $\underline{\xi}_2$ in problem (P2) is kinematically admissible with the macroscopic strain $\underline{\underline{\in}}_2 = 0$, which leads by virtue of (10) to

$$\langle \underline{\underline{\varepsilon}}_2 \rangle \;+\; \frac{1}{|\Omega_0|}\int_\omega [\underline{\xi}_2] \overset{s}{\otimes} \underline{n}\;dS \;=\; 0 \tag{41}$$

which implies that

$$\frac{1}{|\Omega_0|}\int_\omega [\underline{\xi}_2]\cdot\underline{n}\;dS \;=\; -\underline{\underline{1}}:\langle \underline{\underline{\varepsilon}}_2 \rangle \tag{42}$$

The above equality simply expresses that the volumetric strain associated with the joint normal displacement jump is balanced by the volume change of the rock matrix, resulting in zero volume change of the REV.

The state equation of the rock matrix reads $\underline{\underline{\sigma}}_2 = \mathbb{C}^s : \underline{\underline{\varepsilon}}_2$, thus

$$\underline{\underline{1}}:\langle \underline{\underline{\varepsilon}}_2 \rangle \;=\; \underline{\underline{1}}:\left(\mathbb{C}^s\right)^{-1} : \langle \underline{\underline{\sigma}}_2 \rangle \tag{43}$$

Recalling that in the present situation defined by $\alpha_i = \alpha \ \ \forall i$, the stress average given by (26) is $\underline{\underline{\Sigma}}_2 = \langle \underline{\underline{\sigma}}_2 \rangle = -p \underline{\underline{B}}$, the following identity is therefore deduced from (42) and (43)

$$\frac{1}{|\Omega_0|}\int_{\omega} \alpha\, [\underline{\xi}_2]\cdot \underline{n} \ \ dS \ = \ \alpha\, p\, \underline{\underline{1}}: \left(\mathbb{c}^s\right)^{-1} : \underline{\underline{B}} \tag{44}$$

Comparison of (38) and (44) yields

$$\frac{1}{\tilde{m}} \ = \ \alpha \ \underline{\underline{1}}: \left(\mathbb{c}^s\right)^{-1} : \underline{\underline{B}} \tag{45}$$

Hence,

$$\frac{1}{M} \ = \ \frac{1}{\bar{m}} + \frac{1}{\tilde{m}} \ = \ \sum_i S_i \frac{1}{m_i} + \alpha \ \underline{\underline{1}}: \left(\mathbb{c}^s\right)^{-1} : \underline{\underline{B}} \tag{46}$$

Relationships (31) and (46) show that the overall properties M and $\underline{\underline{B}}$ are entirely known once the macroscopic tensor of elastic moduli has been determined. These relationships extend to the situation of jointed rock medium the classical relationships providing the Biot tensor and Biot modulus as functions of solid matrix elasticity $\mathbb{c}^s$ and dry porous medium elasticity $\mathbb{C}^{\text{hom}}$ [1].

Case 2. There exist at least two joints having distinct Biot coefficients, i.e. $\exists (i, j) \ | \ \ \alpha_i \neq \alpha_j$. As already mentioned in section **4.1**, there is no direct connection between $\underline{\underline{B}}$, $\mathbb{C}^{\text{hom}}$ and the poroelastic properties of the rock matrix. The same remark holds for the scalar $\tilde{m}$ and consequently for the macroscopic Biot modulus M. Actually, the determination of $\underline{\underline{B}}$ defined by (27) would require *a priori* the knowledge of the normal concentration tensor $\underline{\underline{a}}^n$, and that of $\tilde{m}$ defined by (38) would require *a priori* the knowledge of the displacement field $\underline{\xi}_2$ solution of problem (P2) .

5. APPLICATION TO CRACKED ROCK MEDIUM

We deal herein with the situation of a cracked rock. The only heterogeneities considered for the rock medium are short joints (i.e., cracks with load transfer). The analysis presented in the sequel is intended as an extension of classical results established in poroelasticity for cracks which do not transfer stresses.

A convenient way to represent cracks is in the form of oblate spheroids [10]. As made in section 3.1, we introduce for a crack an orthonormal frame $\left(\underline{t}\ , \underline{t}',\ \underline{n}\right)$, in which $\underline{n}$ denotes the normal to the crack plane (Fig. 4). The geometry of this oblate spheroid is defined by the crack radius a and the half opening of the crack c. The aspect ratio $X = c/a$ of such a penny-shaped crack is subjected to the condition $X \ll 1$. In the continuum micromechanics approach employed herein, a crack represents an inhomogeneity embedded within the rock matrix. We assume for simplicity that the latter is elastically isotropic:

$$\mathbb{C}^s = 3k^s \mathbb{J} + 2\mu^s \mathbb{K} \tag{47}$$

where k^s is the bulk modulus and μ^s is the shear modulus. The fourth-order tensors $\mathbb{J}$ and $\mathbb{K}$ are defined as

$$\mathbb{J} = \frac{1}{3} \underline{\underline{1}} \otimes \underline{\underline{1}} \quad ; \quad \mathbb{K} = \mathbb{I} - \mathbb{J} \tag{48}$$

The crack modeled as a short joint (crack with stress transfer) has a stiffness in the form

$$\underline{\underline{k}} = k_n \, \underline{n} \otimes \underline{n} + k_t \left(\underline{t} \otimes \underline{t} + \underline{t}' \otimes \underline{t}' \right) \tag{49}$$

where k_n and k_t denote respectively the normal stiffness and shear stiffness.

We consider the situation of a homogeneous rock with parallel cracks defined by the same radius a and crack aspect ratio X. The volume fraction of cracks present in the medium is denoted by f:

$$f = \frac{4}{3} \pi \varepsilon X \tag{50}$$

where $\varepsilon = \mathcal{N} a^2$ is the crack density parameter of the considered set of parallel cracks introduced by Budiansky and O'Connel [6], $\mathcal{N}$ being the number of cracks by unit volume.

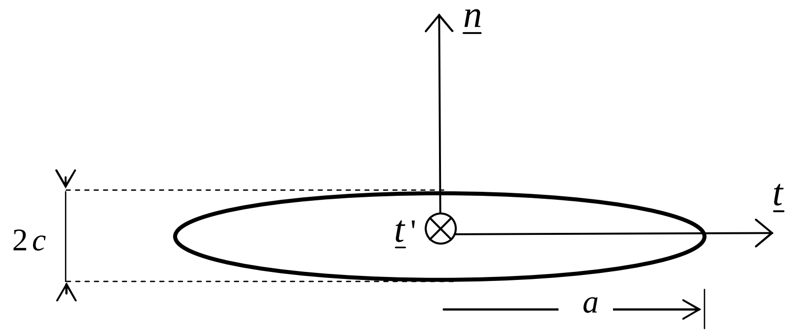


Figure 4. Crack as oblate spheroid.

Using a Mori-Tanaka scheme, the estimate of the tensor of drained moduli $\mathbb{C}^{\text{hom}}$ reads:

$$\mathbb{C}^{\text{hom}} = \lim_{X \to 0} \left(\mathbb{C}^s + f \mathbb{C}^j : \left(\mathbb{I} + \mathbb{P} : (\mathbb{C}^s - \mathbb{C}^j) \right)^{-1} \right) : \left(\mathbb{I} + f \left(\mathbb{I} + \mathbb{P} : (\mathbb{C}^s - \mathbb{C}^j) \right)^{-1} \right)^{-1} \tag{51}$$

where $\mathbb{P} = \mathbb{P}(X, \underline{n})$ is the Hill tensor associated with the considered crack family. It depends on the aspect ratio X of the oblate spheroid and its orientation $\underline{n}$. The components of the Hill tensor of an oblate spheroid can be found in Handbooks [16,17]. Tensor $\mathbb{C}^j$ is related to the crack stiffness

$$\mathbb{c}^j = 3X\,a\left(k_n - 4/3k_t\right)\mathbb{J} + 2X\,a\,k_t\,\mathbb{K} \tag{52}$$

The components of the Mori-Tanaka estimate of $\mathbb{C}^{\text{hom}}$ given by (51) are expressed analytically. Since all the cracks have the same poroelastic properties $\left(\underline{\underline{k}}, \alpha, m\right)$, the Mori-Tanaka estimate of the Biot tensor reads

$$\underline{\underline{B}} = \alpha \lim_{X\to 0} f\,\underline{\underline{1}}:\left(\mathbb{I}+\mathbb{P}:(\mathbb{c}^s-\mathbb{c}^j)\right)^{-1}:\left(\mathbb{I}+f\left(\mathbb{I}+\mathbb{P}:(\mathbb{c}^s-\mathbb{c}^j)\right)^{-1}\right)^{-1} \tag{53}$$

and the Biot modulus estimate can therefore be deduced from that of $\underline{\underline{B}}$

$$\frac{1}{M} = \frac{1}{m} + \alpha\,\underline{\underline{1}}:\left(\mathbb{c}^s\right)^{-1}:\underline{\underline{B}} \tag{54}$$

The components of $\mathbb{C}^{\text{hom}}$, $\underline{\underline{B}}$ and M are given below.

$$\begin{aligned}
C_{1111} = C_{2222} &= (3k^s+4\mu^s)\,\frac{\kappa_2+\pi(1+16/3\varepsilon)\,\kappa_1(1-\kappa_1)}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}\\
C_{3333} &= (3k^s+4\mu^s)\,\frac{\kappa_2+\pi\,\kappa_1(1-\kappa_1)}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}\\
C_{1122} = C_{2211} &= (3k^s-2\mu^s)\,\frac{\kappa_2+\pi(\kappa_1+8/3\varepsilon)(1-\kappa_1)}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}\\
C_{1133} = C_{3322} &= (3k^s-2\mu^s)\,\frac{\kappa_2+\pi\,\kappa_1(1-\kappa_1)}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}\\
C_{2323} = C_{3131} &= 2\mu^s\,\frac{4\kappa_3+\pi\,(1-\kappa_1)(1+2\kappa_1)}{4\kappa_3+16/3\pi\,\varepsilon\,(1-\kappa_1)+\pi(1+2\kappa_1)(1-\kappa_1)}\\
C_{1212} &= \mu^s
\end{aligned} \tag{55}$$

Only diagonal components of Biot tensor $\underline{\underline{B}}$ are not equal to zero:

$$\begin{aligned}
B_{11} = B_{22} &= 4\alpha\pi\,\varepsilon\,\frac{(4/3\kappa_1-1)\,\kappa_1-8/9(1-\kappa_1)^2}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}\\
B_{33} &= \frac{4\alpha\pi\,\varepsilon}{3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon}
\end{aligned} \tag{56}$$

Finally, the Biot modulus estimate reads

$$\frac{1}{M} = \frac{1}{m} + \frac{12\alpha^2\pi\,\varepsilon}{(3k^s+4\mu^s)(3\kappa_2+3\pi\,\kappa_1(1-\kappa_1)+4\pi\,\varepsilon)} \tag{57}$$

where the non-dimensional parameters κ_1, κ_2 and κ_3 are defined by

$$\kappa_1 = \frac{3k^s + \mu^s}{3k^s + 4\mu^s} \quad ; \quad \kappa_2 = \frac{3k_n\, a}{3k^s + 4\mu^s} \quad ; \quad \kappa_3 = \frac{3k_t\, a}{3k^s + 4\mu^s} \tag{65}$$

6. CONCLUSION

The micromechanical analysis of the behavior of rocks with fluid saturated joint network has been presented. Extending the concept of strain concentration tensor to jointed media, the reasoning relies upon the formulation of Hill lemma for such materials and the introduction of strain concentration tensors for the displacement jump along the joints, modeled as interfaces. The two state equations for the rock medium with a fluid saturated connected joint network have been formulated. They can be viewed as an extension of the poroelasticity Biot theory to such materials. The particular role played by the Biot coefficient α of joints is discussed in detail. In the situation when all the joints are characterized by the same Biot coefficient, it is established that the homogenized Biot coefficient $\underline{\underline{B}}$ and Biot modulus M are related to the homogenized tensor of drained moduli $\mathbb{C}^{\hom}$ by Eqs. (31) and (46), extending to the case of jointed rocks the classical relationships available for ordinary porous media. From a practical viewpoint, this means that the determination of poroelastic properties reduces to elastic homogenization in the dry case.

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