Blucher Mechanical Engineering Proceedings
May 2014, vol. 1 , num. 1
www.proceedings.blucher.com.br/evento/10wccm

DETECTION OF ARBITRARILY ORIENTED MULTIPLE CRACKS IN LONG SHAFTS

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Abstract. *A method has been developed for detection of arbitrarily oriented multiple open cracks in a slender shaft or beam of circular section. The method is based on changes in the natural frequencies of torsion vibrations. It has been verified based on finite element data for shafts with two, three and four cracks. Case studies are presented for shafts with fixed-free and free-free end conditions. Maximum error in the prediction of location is 2 % and size is ~23%. The method can be useful for detection of arbitrarily oriented multiple cracks.*

Keywords: *Multiple crack detection in long shaft, Detection of arbitrarily oriented cracks in long shaft, Vibration of long shaft with multiple cracks.*

1. INTRODUCTION

Detection of crack in shafts and beams has been the focus of number of researchers in the last few decades [1-21]. Changes in mechanical behaviour of a component due to presence of a crack facilitate its detection. Among the various methods available in the literature, vibration based non-destructive techniques have been shown to have potential for practical applications. Most of the methods based on vibration response consider the component as one dimensional and change in natural frequency [2 - 9] or mode shape [14,17] as the basis. Out of the two, the first one is more widely considered. Ruotolo and Surace [18] have used weighted combination of these two in developing an optimization algorithm for the detection. Some investigators [10,11] have used change in forced vibration responses for the same purpose.

A review of the methods for detection of multiple cracks is presented by Sekhar [19]. Liang et al. [9], Hu and Liang [4] and Patil and Maiti [16] indicate that the problem of detection of multiple cracks of the same orientation in rectangular beams can be handled in two stages. In the first stage the focus is on localisation of a crack; in the second stage, the determination of exact location and size are addressed. Xiaoqing et al. [20] and Khiem and Lien [6] have formulated the forward problem of finding frequency of beams with multiple cracks by transfer matrix method. For solving the inverse problem an optimization method is given by Khiem and Lien. In methods based on sensitivity analysis [3,7], sensitivity of the frequency to the unknown crack parameters is first established. The crack details are predicted subsequently through an inverse analysis. Of late, researchers [1, 10] have reported usefulness of wavelet based transformation of mode shapes for detection of multiple cracks.

With an increase in number of cracks, number of unknown parameters increases. This adds to the complexity of the problem. Although multiple crack problems are widely encountered in practice, relatively few methods of solution are available. If these cracks have different orientation angles the problem gets further complicated; bending, torsion and axial vibrations get coupled [12,15]. Chasalevris and Papadopoulos [1] have addressed such problems with two arbitrarily oriented breathing cracks in a continuously rotating shaft. They have used wavelet based transformation of deflected shape of the rotating shaft with cracks to identify the crack locations. They have employed frequencies of vibration to predict further the depths and orientations of the individual cracks. Detection of such cracks for components other than rotating shafts has not yet been addressed. In the present study, detection of arbitrarily oriented multiple open cracks in slender circular shafts (with length/diameter ratio > 12) is examined.

2. METHOD OF DETECTION

A slender shaft with two arbitrarily oriented cracks with straight front is considered (Figure 1). Cracks are assumed to remain always open during a cycle of vibration.

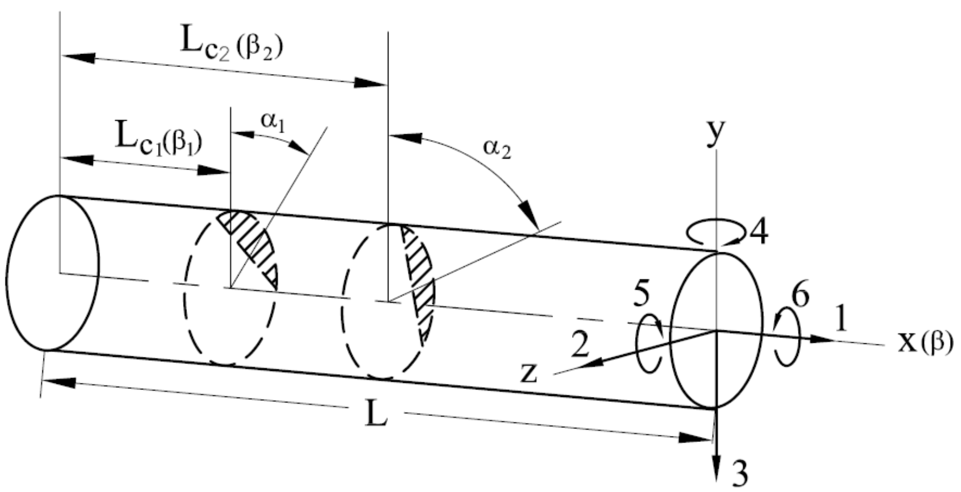


Figure 1. Schematic of a slender shaft with arbitrarily oriented two cracks.

Papadopoulos and Dimarogonas [15] and Naik and Maiti [12] have shown that for short (i.e. Timoshenko) shafts or beams with arbitrarily oriented cracks, flexural vibrations in two mutually perpendicular directions are coupled. Further the torsion and longitudinal vibrations are also coupled with the flexural vibrations. However, for slender (i.e. Euler-Bernoulli) shafts or beams the shear deformations are negligible. Therefore, though the flexural vibrations in the two orthogonal planes are coupled, the torsion vibration remains uncoupled with the flexural vibrations. This issue is utilized here to detect arbitrarily oriented multiple cracks. In the procedure followed, the cracks are localized in the first stage of solution, and, in the second stage, location and size of individual crack are determined.

2.1. Damage Localization

For the analysis, a given component is virtually split into a convenient number of segments of equal size. In the presence of cracks, the potential energy of each segment reduces, which depends on the crack size. This reduction is represented by a dimensionless parameter, whose value varies from 0 to 1. Value 0 corresponds to no crack and 1 corresponds to full separation of the segment. Through the procedure given by Patil and Maiti [16] it is possible to relate under purely linear elastic situation these damage parameters linearly to the change in natural frequency in a given mode. Knowing the changes in a set of natural frequencies these parameters can be solved for. In turn, the unknown crack location and size corresponding to each of the damage parameters can be determined. The linear relationships between the changes in natural frequencies and the damage parameters are obtained as follows.

The natural frequency ω_n of mode n of a crack-free shaft is given by Rayleigh quotient:

$$\omega_n^2 = \frac{U_n}{D_n} = \frac{\int_0^L \frac{1}{2} GJ \left(\frac{d\theta_n}{dx} \right)^2 dx}{\int_0^L \frac{1}{2} I_{mx} \theta_n^2 dx}. \tag{1}$$

θ_n is n^{th} torsion mode shape of the shaft, G is rigidity modulus, J is polar moment of inertia of cross-section, I_{mx} is the mass moment of inertia of a unit length of the shaft about the bending axis and L is the shaft length. In Eqn.(1), U_n represents maximum potential energy of the crack-free shaft in the natural mode n and D_n represents the maximum kinetic energy divided by ω_n^2 in the same mode. Change in ω_n due to crack/cracks in mode n, $\Delta\omega_n$, is given by

$$\frac{\Delta\omega_n}{\omega_n} = \frac{1}{2}\frac{\Delta U_n}{U_n} - \frac{1}{2}\frac{\Delta D_n}{D_n}. \tag{2}$$

It can be assumed that there is negligible difference between overall torsion mode shapes of the component with and without cracks. Further, the kinetic energy of the shaft in a particular mode does not change appreciably due to presence of a crack or cracks. Therefore

$$\frac{\Delta\omega_n}{\omega_n} = \frac{1}{2}\frac{\Delta U_n}{U_n}. \tag{3}$$

ΔU_n represents the reduction in potential energy due to all the cracks. Following Patil and Maiti [16], the shaft is virtually divided into m number of segments. Each of these segments may have a crack. Therefore the maximum number of cracks that can be handled simultaneously by this method is m. If a shaft or beam segment has a crack, its energy reduces from the level corresponding to the crack-free geometry. Representing this reduction by a fraction S_i for segment i, Eqn. (3) is rewritten as follows.

$$\frac{\Delta\omega_n}{\omega_n} = \frac{1}{2}\sum_{i=1}^{m}\frac{\left(U_{ni}S_i\right)}{U_n}. \tag{4}$$

U_{ni} is the potential energy of shaft segment i when it is crack-free.

The damage parameters are independent of the mode of torsion vibrations and are dependent only on crack size. Considering N number of modes, a set of simultaneous equations is obtained.

$$\left\{\frac{\Delta\omega_n}{\omega_n}\right\}_{N\times 1} = [H]_{N\times m}\left\{S_i\right\}_{m\times 1}. \tag{5}$$

It may be noted here that changes in frequencies of the number of modes (N) to be provided as input is a function of the number of cracks to be detected. For any crack its location and size are the two unknowns, if the orientation is ignored. For detecting therefore, n_c number of cracks, changes in minimum ($2n_c$+1) number of frequencies are required as input. The number of segments in the shaft for such a case should at least be n_c, because each segment can have at the most one crack.

A typical element h_{ni} of matrix [H] corresponding to mode n and segment i of the shaft is as follows.

$$h_{ni} = \frac{\frac{1}{2}\int_{L_{i-1}}^{L_i}\frac{1}{2}GJ\left(\frac{d\theta_n}{dx}\right)^2 dx}{\int_{0}^{L}\frac{1}{2}GJ\left(\frac{d\theta_n}{dx}\right)^2 dx}. \tag{6}$$

The integration limits L_i and L_{i-1} are the end coordinates along the shaft axis of segment i.

Natural frequencies obtained by either experiment or numerical simulations are the basis to obtain the left hand side of the set of Eqn.(5). In the present study, the natural frequencies of shafts with and without cracks were obtained by finite element analysis using ANSYS10 and the shafts were split into ten segments for all the cases reported here. The coefficients of [H] matrix were evaluated through Eqn.(6). The set of equations obtained through Eqn.(5) were solved to obtain the damage parameters. The number of nonzero damage parameters thus obtained indicates the number of cracks present.

2.2. Prediction of Crack Location and Depth

To find out the crack location and size of a crack corresponding to a non-zero damage parameter, in the second stage of solution, the changes in natural frequencies are obtained from Eqn.(5) by setting all other damage parameters except the one under consideration as 0. The given shaft is now considered with a single crack. Vibration of each segment is governed by a second order partial differential equation [12].

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} - \frac{\rho}{G}\frac{\partial^2 \psi(x,t)}{\partial t^2} = 0. \quad (7)$$

ρ and G is density and rigidity modulus of shaft or beam material respectively. Its solution can be written in the following form.

$$\psi(x,t) = (A\sin\lambda x + B\cos\lambda x)(C\sin\omega t + D\cos\omega t). \quad (8)$$

$\lambda = \omega\sqrt{\frac{\rho}{G}}$, ω is circular natural frequency, A, B, C and D are arbitrary constants. The expression within the first set of brackets represents the mode shape. The mode shapes for the two segments of a shaft with a crack (Figure 2) can be represented separately as follows.

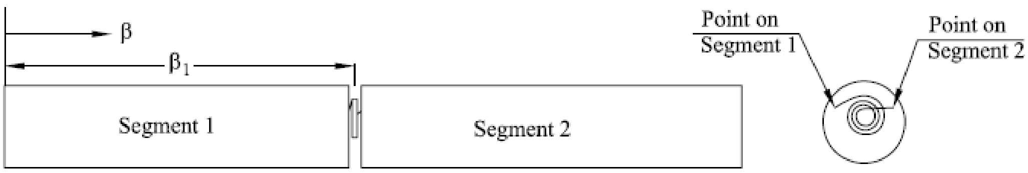


Figure 2. Representation of crack by equivalent rotational spring

$$\begin{aligned} \theta_1 &= A_1\sin(\lambda_{cn}\beta) + B_1\cos(\lambda_{cn}\beta) \ldots\ldots\ldots\ldots 0 \le \beta \le \beta_1 \\ \theta_2 &= A_2\sin(\lambda_{cn}\beta) + B_2\cos(\lambda_{cn}\beta) \ldots\ldots\ldots\ldots \beta_1 \le \beta \le 1 \end{aligned} \quad (9)$$

θ_1 and θ_2 represent the modal rotation amplitudes of the two segments, $\lambda_{cn} = \omega_{cn}\sqrt{\frac{\rho}{G}}$, and ω_{cn} stands for circular natural frequency of shaft with a crack in mode n. A_1, A_2, B_1 and B_2 are arbitrary constants. By ensuring that the two mode shapes satisfy the boundary conditions and the continuity and compatibility conditions at the crack section the characteristic equation of vibration of the shaft with single crack is obtained. The continuity of torque and compatibility of rotation at the crack location is given by the following relations.

$$\left.\frac{d\theta_1}{d\beta}\right|_{\beta=\beta_1} = \left.\frac{d\theta_2}{d\beta}\right|_{\beta=\beta_1},$$
$$GJ\left.\frac{d\theta_1}{d\beta}\right|_{\beta=\beta_1} = k\left(\theta_2 - \theta_1\right)\Big|_{\beta=\beta_1}. \tag{10}$$

β_1 is the crack location and k is the spring stiffness representing the crack. The characteristic equation of a shaft with a single crack, for example, with fixed-free end conditions, has the following form.

$$K = f\left(\lambda_{cn}, \beta_1\right) = 0.5\lambda_{cn}[2\tan\lambda_{cn}\cos^2(\lambda_{cn}\beta_1) - 2\sin(2\lambda_{cn}\beta_1). \tag{11}$$

$$K = \frac{kL}{GJ}. \tag{12}$$

Eqn.(11) can be employed to plot the variation of K with β_1 for at least three natural frequencies calculated using changes in the damage parameter. The intersection of three curves gives the required crack location and the corresponding spring stiffness K. To obtain the crack size from K, the following relation based on the compliance coefficients given by Naik and Maiti [12] is used.

$$\left(\frac{a}{d}\right) = 0.3866\left(\frac{8}{(1-\nu)}\left(\frac{L}{d}\right)\left(\frac{1}{K}\right)\right)^{0.3987}. \tag{13}$$

ν is Poisson's ratio. The proposed method has been tested for shafts with two, three and four cracks and fixed-free and free-free boundary conditions.

3. CASE STUDIES

3.1. Fixed-free End Conditions

A steel cantilever shaft with Young's modulus 210GPa and Poisson's ratio 0.33 is considered for the numerical experiments. Length of the shaft is 0.8m and diameter is 0.04m. These shaft dimensions correspond to a slender configuration. Cases with two, three and four cracks have been studied. The case details and the input natural frequencies, obtained by finite element analysis (ANSYS 11), are given in Table 1.

For a fixed-free shaft, matrix [H] is obtained noting that its typical element

$$h_{ni} = \int_{\beta_l}^{\beta_r}\left(\cos\left(\frac{(2n-1)}{2}\pi\beta\right)\right)^2 d\beta. \tag{14}$$

β_l and β_r are the dimensionless coordinates of the left and right ends respectively of the segment i. Using the finite element data, Eqn.(5) is solved for the damage parameters S_i using

Table 1. Input data for prediction of of multiple cracks in fixed–free shaft.

Crack parameters			Torsion natural frequencies (Hz)								
Location (β/L)	Depth (a/h)	Orientation with vertical	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
No crack			981.44	2944.33	4907.25	6870.22	8833.26	10796.41	12759.64	14723.03	16686.57
0.15 0.25	0.3 0.25	0 0	970.78	2930.22	4898.85	6842.00	8780.53				
0.15 0.25	0.3 0.25	0 45	970.88	2930.26	4898.92	6842.71	8780.97				
0.15 0.25	0.3 0.25	0 90	970.84	2930.27	4898.92	6842.20	8779.68				
0.15 0.25 0.35	0.3 0.25 0.2	0 45 0	969.14	2930.42	4888.08	6833.35	8779.74	10704.88	12638.82		
0.15 0.25 0.35	0.3 0.25 0.2	0 90 0	968.97	2930.25	4888.11	6832.28	8777.61	10704.43	12637.62		
0.15 0.25 0.35 0.45	0.3 0.25 0.2 0.2	0 90 0 0	967.61	2928.27	4877.23	6831.42	8754.77	10704.70	12606.67	14555.89	16488.43

'*lsqnoneg*' command of MATLAB. Number of nonzero damage parameters represents number of cracks present in the shaft. The total number of input frequencies is kept as ($2n_c$+1), when n_c is the number of unknown cracks. Corresponding to a damage parameter the first three natural frequencies of the shaft with crack are obtained through Eqn.(5). Employing one frequency at a time, λ_{cn} is obtained and it is given as input to the characteristic equation Eqn.(11) to plot a variation of K *vs.* β_1 (Figure 3).

Three such variations corresponding to the three frequencies intersect at a point to give the crack location and the spring stiffness K. If the three curves do not intersect at a point, the centre of gravity of the smallest triangle formed by the three paired intersections is taken as the approximate intersection point [12, 13]. The chance of getting the intersection or the smallest triangle improves through the zero setting [13]. The crack size is obtained using K and Eqn.(13). Similar exercise is carried to obtain the details of the other cracks.

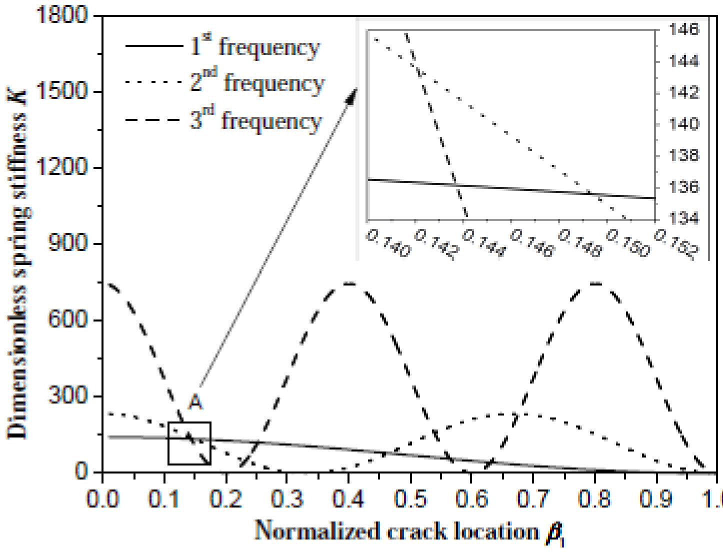
Zero Setting

The natural frequencies, which are given as input for the detection of single crack details to Eqn.(11) are principally based on finite element analysis. Since the changes in the frequencies are mainly due to the cracks in the shaft, the frequencies of the corresponding crack-free shaft obtained by the finite element analysis should tally with the theoretical values obtained from Eqn.(11) using K=∞. In most cases this does not happen. To settle this discrepancy, correction is applied to the frequencies obtained by the finite element analysis through the shear modulus. This is termed as zero setting [13]. The modified shear modulus utilized to calculate the dimensionless natural frequency parameter λ_{cn}, which goes as input in the second stage to Eqn.(11), corresponding to a case with a crack in a particular mode n, is obtained as follows.

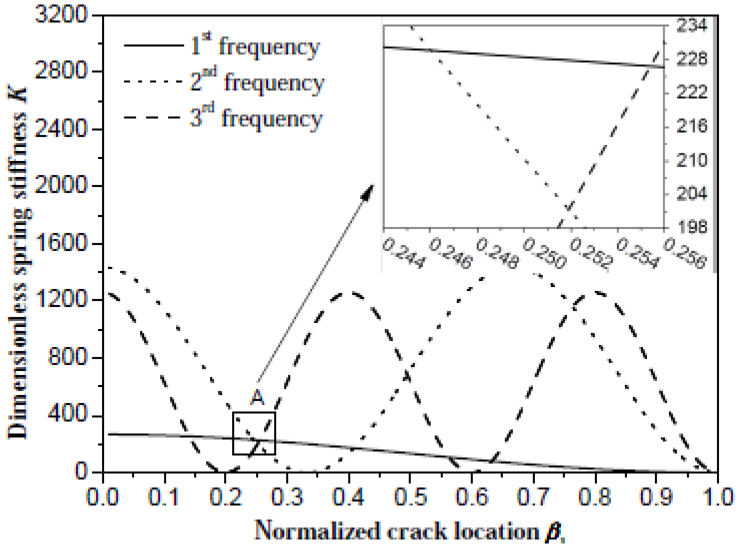
$$G_{cor,n} = G\left(\frac{\omega_{fea,n}^{uncrack}}{\omega_{theor,n}^{uncrack}}\right)^2$$
$$\lambda_{cn} = \omega_{cal,n}^{crack} L\sqrt{\frac{\rho}{G_{cor,n}}} \qquad (15)$$

$\omega_{fea,n}^{uncrack}$ = natural frequency of crack-free shaft in mode n obtained by finite element analysis, $\omega_{theor,n}^{uncrack}$ = theoretical natural frequency of crack-free shaft in mode n calculated from Eqn.(11) using K=∞, $\omega_{cal,n}^{crack}$ = natural frequency of shaft with crack in mode n obtained using (S_1=0, S_2=0,.....,S_i≠0, S_{i+1}=0, ...) and Eqn.(5), and $G_{cor,n}$ = corrected modulus of rigidity G. This zero setting must be applied at the second stage of locating an individual crack.

Table 2 lists results for six cases. The absolute error (~0.75%) in the prediction of location of a crack is higher than that (~19%) in its size. This observation is consistent with that reported by earlier investigators, e.g. Patil and Maiti [16].



(a)



(b)

Figure 3. K Vs β_l plots for (a) 1st crack and (b) 2nd crack for case 2 of Table 1.

3.2 Free-free End Conditions

Material and geometric properties have been taken the same as in the earlier case of fixed-free shaft or beam. Again, problems of two, three and four cracks were considered. Details of these cases are included in Table 3. In this case, typical element h_{ni} of matrix $[H]$ has the form

$$h_{ni} = \int_{\beta_l}^{\beta_r} \left(\sin\left(n\pi\beta\right)\right)^2 d\beta. \tag{16}$$

Table 2. Accuracy of prediction of location and size of multiple cracks in fixed–free shaft.

Crack parameters			Predicted crack parameters			
Location	Depth	Relative angle	Location		Depth	
			Value	% Error	Value	% Error
0.15	0.3	0	0.1465	-0.3520	0.4708	17.0811
0.25	0.25	0	0.2519	0.1903	0.3897	13.9677
0.15	0.3	0	0.1465	-0.3521	0.4707	17.0748
0.25	0.25	45	0.2519	0.1927	0.3856	13.5647
0.15	0.3	0	0.1465	-0.3502	0.4691	16.9083
0.25	0.25	90	0.2519	0.1897	0.3909	14.0940
0.15	0.3	0	0.1463	-0.3714	0.4897	18.9692
0.25	0.25	45	0.2522	0.2167	0.3493	9.9337
0.35	0.2	0	0.3575	**0.7542**	0.2937	9.3676
0.15	0.3	0	0.1463	-0.3720	0.4903	**19.0294**
0.25	0.25	90	0.2521	0.2084	0.3608	11.0806
0.35	0.2	0	0.3574	0.7444	0.2877	8.7658
0.15	0.3	0	0.1463	-0.3698	0.4880	18.8039
0.25	0.25	90	0.2520	0.2025	0.3696	11.9632
0.35	0.2	0	0.3563	0.6268	0.2404	4.0415
0.45	0.2	0	0.4518	0.1829	0.3172	11.7207

The integration limits β_l and β_r are the dimensionless coordinates of the left and right ends respectively of segment *i*.

Because of the symmetric nature of mode shape of crack-free shaft or beam in this case, only one half virtually split into five segments is analysed. Again the MATLAB command '*lsqnonneg*' is used to obtain the values of the damage parameters. To obtain the locations and sizes of cracks, the following characteristic equation involving K, β_1 and λ_{cn} has been employed.

$$K = (\lambda_{cn})\left\{\cos(\lambda_{cn}) + \frac{\cos(\lambda_{cn}\beta_1 - \lambda_{cn})}{2\sin(\lambda_{cn})}\right\} \tag{17}$$

For this case too the input data for six cases were obtained by finite element analysis. These are given in Table 3. Typical variations of normalized spring stiffness with crack location obtained using Eqn.(17) in the second stage after the zero setting are shown in Figure 4. From such plots crack location and K are obtained. Crack size is then obtained using Eqn.(13). Table 4 presents the absolute error in prediction of crack location is again smaller (~2%) than that (~23%) in depth.

Table 3. Input data for prediction of multiple cracks in free–free shaft.

Crack parameters			Torsion natural frequencies (Hz)								
Location (β/L)	Depth (a/h)	Orientation with vertical	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
No Crack			981.44	2944.33	4907.25	6870.22	8833.26	10796.41	12759.64	14723.03	16686.57
0.15 0.25	0.3 0.25	0 0	970.78	2930.22	4898.85	6842.00	8780.53				
0.15 0.25	0.3 0.25	0 45	970.88	2930.26	4898.92	6842.71	8780.97				
0.15 0.25	0.3 0.25	0 90	970.84	2930.27	4898.92	6842.20	8779.68				
0.15 0.25 0.35	0.3 0.25 0.2	0 45 0	969.14	2930.42	4888.08	6833.35	8779.74	10704.88	12638.82		
0.15 0.25 0.35	0.3 0.25 0.2	0 90 0	968.97	2930.25	4888.11	6832.28	8777.61	10704.43	12637.62		
0.15 0.25 0.35 0.45	0.3 0.25 0.2 0.2	0 90 0 0	967.61	2928.27	4877.23	6831.42	8754.77	10704.70	12606.67	14555.89	16488.43

4. CONCLUSIONS

Presence of arbitrarily oriented multiple cracks in a component results in coupling of axial flexural and torsion vibrations. For slender components however, the torsion vibrations remain decoupled. This fact is utilized to develop a method based on changes in torsion natural frequency to detect arbitrarily oriented multiple open cracks in slender shafts. The approach can be applied to beams of circular cross-sections. The performance of the method proposed has been demonstrated through numerical studies in the case of fixed-free and free-free shafts involving two, three and four cracks simultaneously. More case studies are reported in [5]. The maximum error in prediction of location and size of crack is less than 2% and 23% respectively. It is recommended that the number of input frequencies be kept as ($2n_c$+1) for detecting n_c number of cracks. Since the torsion natural frequencies are not dependant on the orientations of the cracks, the method proposed cannot predict the crack orientations. Further studies are required to facilitate the predictions of individual crack orientations.

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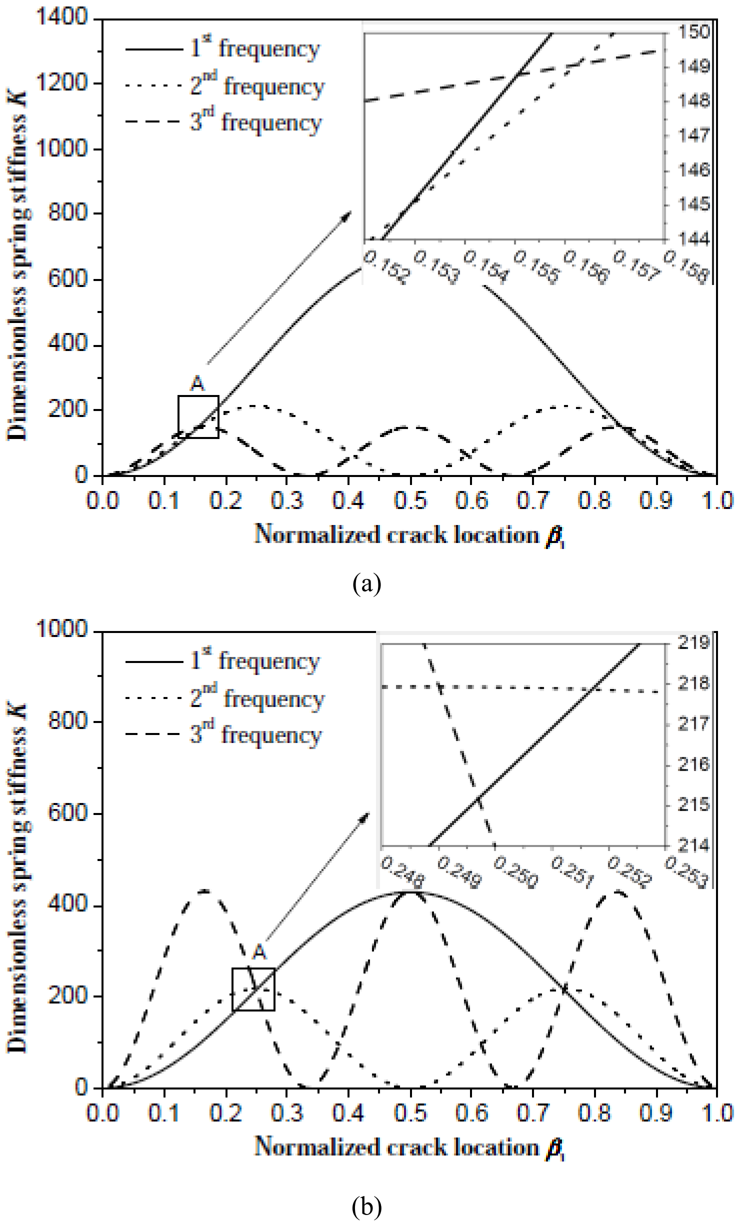

Figure 4. K Vs β_1 plots for (a) 1st crack (b) 2nd crack for case 2 discussed of Table 2.

Table 4 Accuracy of prediction of location and size of multiple cracks in free–free shaft.

Crack parameters			Predicted crack parameters			
Location *(β/L)*	Depth *(a/h)*	Relative angle (^{o})	Location		Depth	
			Value	% Error	Value	% Error
0.15	0.3	0	0.1549	0.4940	0.4583	15.8290
0.25	0.25	0	0.2501	0.0140	0.4001	15.0050
0.15	0.3	0	0.1550	0.4980	0.4601	16.0080
0.25	0.25	45	0.2502	0.0150	0.3948	14.4830
0.15	0.3	0	0.1550	0.5003	0.4611	16.1100
0.25	0.25	90	0.2501	0.0144	0.3976	14.7570
0.15	0.2	0	0.1299	**-2.0110**	0.2616	6.1560
0.25	0.25	45	0.2498	-0.0170	0.4811	**23.1110**
0.35	0.3	0	0.3584	0.8440	0.3991	9.9070
0.15	0.2	0	0.1338	-1.6200	0.2682	6.8200
0.25	0.25	90	0.2498	-0.0160	0.4794	22.9440
0.35	0.3	0	0.3584	0.8440	0.3990	9.8990
0.15	0.2	0	0.1443	-0.5700	0.3042	10.4240
0.25	0.2	90	0.2501	0.0090	0.2774	7.7370
0.35	0.25	0	0.3583	0.8250	0.3836	13.3550
0.45	0.3	0	0.4449	-0.5130	0.4967	19.6700

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