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REDUCED ORDER MODEL OF MISTUNED BLADED DISKS USING THE KRYLOV-SUBSPACE COMBINED WITH THE CRAIG-BAMTON REDUCTION TECHNIQUE

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Abstract. *In order to analyze the effects of mistuning of blisks, the frequency response function (FRF) has to be calculated as quickly as possible and with high accuracy. Therefore, commonly used finite element models with more than* 10^6 *degrees of freedom (DOF) have to be reduced to a model with only a few DOF. A usual way in literature is using reduced order models (ROM) like a modal reduction. This is either slow, inaccurate, or the possible extension to non-linearity is difficult. In this paper a new method of model reduction for mistuned turbine blades is introduced and the benefits are shown. The model of the blisk is separated into individually mistuned blades and a tuned disk where the latter is analyzed as a cyclic system. Analog to the Craig-Bampton reduction technique the system is divided into master- and slave-DOF. The introduced technique is the reduction based on the Krylov-Subspace-Method which performs better in terms of accuracy than the modal reduction if the Two-Side-Arnoldi algorithm is used. Afterwards, the reduced tuned model is mistuned by a variation of Youngs modulus which is the most popular way to introduce mistuning. This reduced system can be further described by modal reduction to decouple the equation of motion and describes the full system with as few DOF as possible. Finally, the paper gives a comparison of the standard Craig-Bampton-reduction and the combined Krylov-Craig-Bampton-method in terms of computational accuracy and efficiency to show the benefits of the new method.*

Keywords: *Mistuning, Krylov-Subspace, Craig-Bampton.*

Introduction

Current research focuses on the optimization of turbine blades with respect to the fluid behavior and the efficiency of the turbine. As a result the turbine blades become thinner and more fragile. Therefore, the blades are prone to vibrations and forced response. Due to the cyclic structure of a turbine the loads are assumed to be equal at every blade. In case of a perfectly tuned system all blades show the same vibration amplitude exited by the loads. Small variations of the blades could effect energy localization and increase amplitudes of single blades [1]. This effect is called mistuning and has a big influence on the lifetime of the turbine.

To analyze the influence of the mistuning the force response function (FRF) is calculated for every single blade. On the basis of the FRF the amplitude of the vibration can be calculated for different mistuning cases. Bladed disks are usually described by a finite element model (FEM) including more than 10^6 degrees of freedom (DOF). Such huge models need a lot of computation time to calculate the FRF. This is not effective if the mistuning should be analyzed by Monte-Carlo simulations or optimizing methods. So the number of DOF has to be reduced as much as possible to calculate the FRF with acceptable computing time.

This paper is about a two step reduction method for mistuned blisks. It contains an overview of the current reduction methods applied to mistuned bladed disks. The next part deals with the reduction of a tuned cyclic model with the Craig-Bampton method. Afterwards this method is extended with the Krylov-Subspace method and the mistuning of the blades is introduced. In the third chapter a case study is presented and the benefits and the limitations of the method are shown. Finally a conclusion is given.

During the last years a lot of papers have introduced new reduction methods for mistuned turbine states. The common way is the transformation to some kind of modal coordinates. The modal basis realizes the system behavior without the whole numbers of DOF. One possibility to choose this basis is using the modes of one segment with one blade and a part of the disk (see Figure 1). At the Subset of Nominal Modes (SNM) the nominal modes of one tuned segment are used and the mistuning is taken into account in a second step [2]. An extension of the SNM is the Fundamental Model of Mistuning (FMM) [3]. This method requires that the strain energy is concentrated in the blade and only one mode shape is of particular interest. This means the mode shape of the full system is quite similar to the mode shape of one fixed blade. This allows some simplification of the algorithm to increase its efficiency.

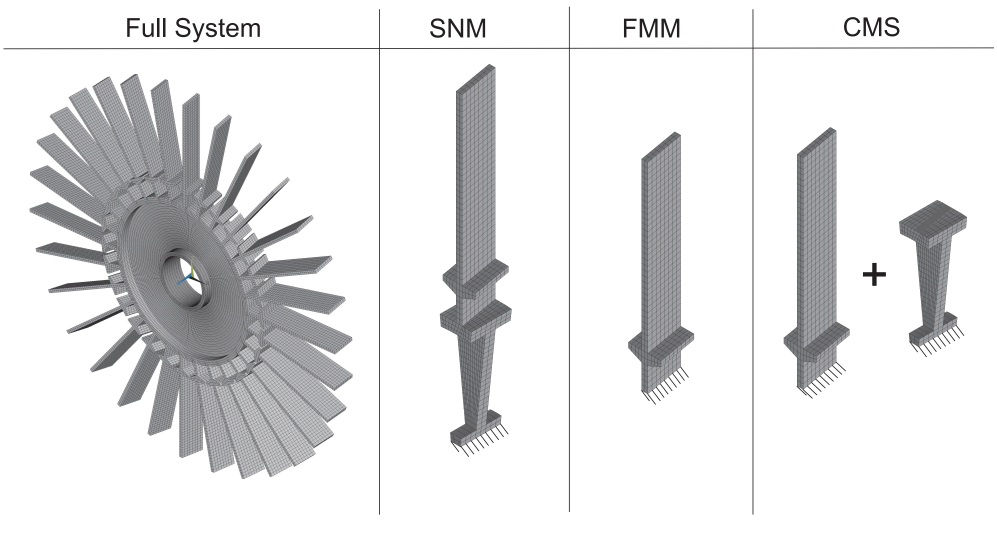

Figure 1. Full finite element model and submodels of different reduction methods.

In an alternative way the segment is divided into the blades and the disk and each component can be reduced independently of each other. This way is called Component Mode Synthesis (CMS). The disk could be handled as a cyclic symmetry model and the model of the blades is reduced only one time. After the reduction of the disk and the blades the full system could be analyzed [4]. There are many other ideas to reduce the mistuned systems like [5],[6],[7]. All of this reductions deal with the modal reduction.

Another approximation of FEMs is the Krylov-Subspace method [8]. The basic idea of this method is to fit the transfer function of the system with the Laplace transformation. This opens up the possibility to reduce the model around one chosen frequency. Also an error estimation for the reduced model is given in [8].

Modal reduction

This section introduces the reduction of a FEM with the Craig-Bampton method. Assuming small deformation and neglecting friction damping like underplatform damping or shroud coupling we can suppose a linear system with the equation of motion (EQM):

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t). \tag{1}$$

The matrices $\mathbf{M}, \mathbf{D}, \mathbf{K}$ denote the mass, the damping and the stiffness matrix. The vector $\mathbf{x}(t)$ presents the displacement of every single node of the FEM. Two different types of linear damping are taken into account. On the one hand the Rayleighs damping hypothesis $\mathbf{D}_\text{R} = \alpha\mathbf{M} + \beta\mathbf{K}$ determines the viscous material damping [9]. On the other hand there is the structural damping model $\mathbf{D}_\text{S} = \eta\mathbf{K}\text{i}$ with η as the hysteretic damping ratio. In the following the damping matrix is integrated in the mass and the stiffness matrix. Assuming the frequency of the force $\mathbf{f}(t)$ is the same at every node, the FRF can be calculated. The force can be described by a harmonic function $\mathbf{f}(t) = \hat{\mathbf{f}}\sin(\Omega t)$ with the amplitude $\hat{\mathbf{f}}$ and the frequency Ω.

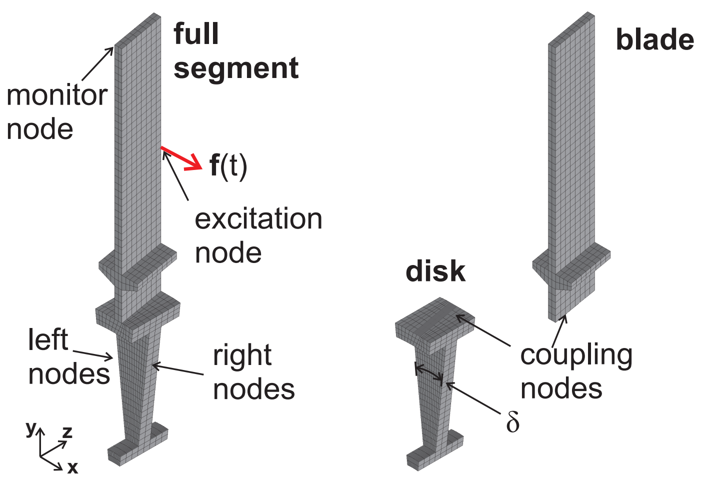

Figure 2. One segment of the bladed disk divided into blade and disk.

One sector of the disk can describe the whole disk cause of the cyclic symmetry. The nodes at the right side have the same displacement as the nodes at the left side except for a phase difference $\delta = \frac{2\pi}{n}$. This phase difference depends on the nodal diameter. The state vector is sorted in order that the nodes of the left side $\mathbf{x}_\text{l}$ are at the top and the nodes of the right $\mathbf{x}_\text{r}$ side are at the bottom of the state vector. The nodes $\mathbf{x}_\text{c}$ involve the rest of the nodes. Therewith the state vector can be written as

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_\text{l} \\ \mathbf{x}_\text{c} \\ \mathbf{x}_\text{r} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{0} & \text{e}^{-\text{i}\delta} \\ \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \mathbf{E} \end{bmatrix}}_{\mathbf{T}_\text{d,k}} \begin{bmatrix} \mathbf{x}_\text{c} \\ \mathbf{x}_\text{r} \end{bmatrix}. \tag{2}$$

The matrix $\mathbf{T}_\text{d,i}$ transforms the mass matrix and the stiffness matrix to

$$\left.\begin{aligned} \mathbf{M}_\text{d,k} &= \mathbf{T}_\text{d,k}^\text{H} \mathbf{M}_\text{d,1} \mathbf{T}_\text{d,k} \\ \mathbf{K}_\text{d,k} &= \mathbf{T}_\text{d,k}^\text{H} \mathbf{K}_\text{d,1} \mathbf{T}_\text{d,k} \end{aligned}\right\} \qquad \text{for k=1..n.} \tag{3}$$

This transformation of the disk segment reduces the problem of the whole disk to n small models for every nodal diameter. This models are divided into the blades and the disks. Each of them can be analyzed separatly and added together afterwards. Both subsystems include the coupling nodes and are arranged to the couple nodes $\mathbf{x}_\text{m}$ and the slave nodes $\mathbf{x}_\text{s}$. This results in $\mathbf{x} = \begin{bmatrix} \mathbf{x}_\text{m} & \mathbf{x}_\text{s} \end{bmatrix}^\text{T}$. The system matrices read as

$$\mathbf{M}_\text{d} = \begin{bmatrix} \mathbf{M}_\text{d,mm} & \mathbf{M}_\text{d,ms} \\ \mathbf{M}_\text{d,sm} & \mathbf{M}_\text{d,ss} \end{bmatrix}, \ \mathbf{K}_\text{d} = \begin{bmatrix} \mathbf{K}_\text{d,mm} & \mathbf{K}_\text{d,ms} \\ \mathbf{K}_\text{d,sm} & \mathbf{K}_\text{d,ss} \end{bmatrix}. \tag{4}$$

To reduce the model with m DOF to a small numbers of DOF p, the state vector $\mathbf{x} \in \mathbb{R}^m$ is multiplying with a matrix $\mathbf{T}_\text{R} \in \mathbb{R}^{m \times p}$

$$\mathbf{x} = \mathbf{T}_\text{R} \mathbf{x}_\text{R}. \tag{5}$$

This yields the state reduced vector $\mathbf{x}_R \in \mathbb{R}^p$ with $p \ll m$. Pre-multipliying the equation of motion by another matrix $\mathbf{T}_\text{L}^\text{H}$ reads as

$$\begin{aligned} \mathbf{T}_\text{L}^\text{H} \mathbf{M} \mathbf{T}_\text{R} \ddot{\mathbf{x}}_\text{R} + \mathbf{T}_\text{L}^\text{H} \mathbf{D} \mathbf{T}_\text{R} \dot{\mathbf{x}}_\text{R} + \mathbf{T}_\text{L}^\text{H} \mathbf{K} \mathbf{T}_\text{R} \mathbf{x}_\text{R} &= \mathbf{T}_\text{L} \mathbf{f}(t) \\ \Leftrightarrow \tilde{\mathbf{M}} \ddot{\mathbf{x}}_\text{R} + \tilde{\mathbf{D}} \dot{\mathbf{x}}_\text{R} + (\tilde{\mathbf{K}}) \mathbf{x}_\text{R} &= \tilde{\mathbf{f}}(t). \end{aligned} \tag{6}$$

The matrices $\mathbf{T}_\text{L}$ and $\mathbf{T}_\text{R}$ have to be bases of the system. The well known Craig-Bampton reduction [11] deals with two different kinds of approaches. There is on the one hand a static displacement of the master nodes, on the other hand there is a modal reduction of the slave nodes. When the nodes of the system are divided into master and slave nodes like in equation (4), the transformation is

$$\begin{bmatrix} \mathbf{x}_\text{m} \\ \mathbf{x}_\text{s} \end{bmatrix} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{\Psi} & \mathbf{\Phi} \end{bmatrix} \begin{bmatrix} \mathbf{x}_\text{m} \\ \eta \end{bmatrix}. \tag{7}$$

The time depend behavior of the master nodes is unchanged during the transformation. The motion of the slave nodes depends on the modes and the static displacement of the master nodes $-\mathbf{K}_{\text{ss}}^{-1}\mathbf{K}_{\text{sm}}\mathbf{x}_{\text{m}} = \boldsymbol{\Psi}\mathbf{x}_{\text{m}}$. The matrix $\boldsymbol{\Phi}$ is the modal matrix combined of the first p eigenvectors of $\mathbf{M}_{\text{ss}}^{-1}\mathbf{K}_{\text{ss}}$. After the reduction the master nodes still have the characteristics of a cyclic system. The Fourier matrix

$$\mathbf{F} = [f_{\text{ij}}] \quad \text{with } f_{\text{ij}} = \frac{1}{\sqrt{n}}\text{e}^{-\text{i}\delta(i-1)(j-1)} \tag{8}$$

converts the master nodes of one segment to the nodes of the whole system. That means the systems matrices arises to:

$$\begin{aligned}
\tilde{\mathbf{M}}_{\text{d,mm}} &= (\mathbf{F}\otimes\mathbf{E})^{\text{H}}\underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{d,mm,k}})(\mathbf{F}\otimes\mathbf{E})\\
\tilde{\mathbf{M}}_{\text{d,ms}} &= (\mathbf{F}\otimes\mathbf{E})^{\text{H}}\underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{d,ms,k}})\\
\tilde{\mathbf{M}}_{\text{d,sm}} &= \underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{d,sm,k}})(\mathbf{F}\otimes\mathbf{E}).
\end{aligned} \tag{9}$$

The matrices $\mathbf{bdiag}(\tilde{\mathbf{M}}_{\text{d,mm,k}})$ are diagonal submatrices of the mass matrices sorted by the nodal diameter k. The Fourier matrix expands the stiffness matrix analog. Therewith the system matrix of the disk is full described. The master nodes of the disk have the same displacements like the master nodes of the blade. That means that the disk and the blade can be put together as:

$$\mathbf{x}_{\text{ges}} = \begin{bmatrix}\eta_{\text{d}} & \eta_{\text{b,1}} & \dots & \eta_{\text{b,n}} & \mathbf{x}_{\text{m}}\end{bmatrix}^{\text{T}}. \tag{10}$$

As a consequence the mass matrix is composed to:

$$\tilde{\mathbf{M}}_{\text{ges}} = \begin{bmatrix}
\tilde{\mathbf{M}}_{\text{d,ss}} & \mathbf{0} & \tilde{\mathbf{M}}_{\text{d,ms}}\\
\mathbf{0} & \underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{b,ss,k}}) & \underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{b,ms,k}})\\
\tilde{\mathbf{M}}_{\text{d,sm}} & \underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{b,sm,k}}) & \underset{k=1..n}{\mathbf{bdiag}}(\tilde{\mathbf{M}}_{\text{b,mm,k}}) + \tilde{\mathbf{M}}_{\text{d,mm}}
\end{bmatrix}. \tag{11}$$

The force on the tuned blades excites one of the nodal diameters depending on the engine order (EO). Every EO effects a different phase shift of the single forces of the blades

$$\tilde{\mathbf{f}}_{\text{k}}(t) = \tilde{\mathbf{f}}_1(t)\text{e}^{-\text{iEO}(k-1)\delta}. \tag{12}$$

A second modal reduction could bring the model down to a small number of DOF. The minimal number of DOF depends on the numbers of mode shapes which should be analyzed.

Krylov-Subspace method

The modal reduction deals with the modal matrix $\boldsymbol{\Phi}$ as the matrices $\mathbf{T}_{\text{L}}$ and $\mathbf{T}_{\text{R}}$ in Equation (6). For non-symmetric systems they are chosen as the left-side and the right-side eigenvectors. In the Krylov-Subspace method the matrix $\mathbf{T}_{\text{L}}$ and the matrix $\mathbf{T}_{\text{R}}$ are the output and input subspace. Every second order system like Equation (1) could be written in a first order system:

$$
\begin{aligned}
\underbrace{\begin{bmatrix}\dot{\mathbf{x}}\\ \ddot{\mathbf{x}}\end{bmatrix}}_{\dot{\mathbf{z}}} &= \underbrace{\begin{bmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{M}^{-1}\mathbf{K} & \mathbf{M}^{-1}\mathbf{D}\end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix}}_{\mathbf{z}} + \underbrace{\begin{bmatrix}\mathbf{0}\\ \hat{\mathbf{f}}\end{bmatrix}}_{\mathbf{B}} \underbrace{\sin \Omega t}_{\mathbf{u}}\\
\mathbf{y} &= \mathbf{C}\mathbf{z}.
\end{aligned} \tag{13}
$$

The matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are the state, the input and the output matrix. The vector $\mathbf{y}$ contains the monitoring nodes. Equation (13) is converted with the Laplace transformation defined as:

$$
F_{\mathrm{L}}(s) = \int_0^\infty \mathrm{e}^{-st} f_{\mathrm{L}}(t)\mathrm{d}t. \tag{14}
$$

Therewith Equation (13) can be stated as

$$
\mathbf{y} = \underbrace{\mathbf{C}[s\mathbf{E} - \mathbf{A}]^{-1}\mathbf{B}}_{\mathbf{H}}\,\mathbf{u} \tag{15}
$$

with $\mathbf{E}$ as the identity matrix. The matrix $\mathbf{H}$ is called transfer function or impulse response function of the Equation (13). This equation developed in the Taylor series gives:

$$
\mathbf{H}(s - s_0) \approx \sum_{k=0}^{n} \underbrace{\left.\frac{\partial^k \mathbf{H}(s)}{k!\partial s^k}\right|_{s_0}}_{\mathbf{h}(k)} (s - s_0). \tag{16}
$$

For $s_0 = 0$ the coefficients of the function $\mathbf{h}(k)$ are called the moments of $\mathbf{H}$ and Markow parameter for $s_0 = \infty$. From Equation (13) the coefficients of the Taylor series are

$$
\mathbf{h}(k) = \left.\frac{\partial^k \mathbf{H}(s)}{k!\partial s^k}\right|_{s_0} = \mathbf{C}\mathbf{A}^{-(k+1)}\mathbf{B}. \tag{17}
$$

The goal of the reduction with the Krylov-Subspace is to find a reduced model which matches the first moments of the FRF. Therefore the transformation of Equation (6) can be applied to $\mathbf{T}_{\mathrm{R}}$ and $\mathbf{T}_{\mathrm{L}}$ as two side Krylov-Subspace matrices. They are called input and output subspaces $\mathbf{V}$ and $\mathbf{W}$ and are defined as

$$
\begin{aligned}
\mathbf{V} &= \mathcal{K}_{\mathrm{q}}(\mathbf{A}^{-1}, \mathbf{B}) = \mathrm{span}\left\{\mathbf{B}, \mathbf{A}^{-1}\mathbf{B}, \dots, \mathbf{A}^{-(q-1)}\mathbf{B}\right\}\\
\mathbf{W} &= \mathcal{K}_{\mathrm{p}}(\mathbf{A}^{-\mathrm{H}}, \mathbf{C}^{\mathrm{H}}) = \mathrm{span}\left\{\mathbf{C}^{-\mathrm{H}}, \mathbf{A}^{-\mathrm{H}}\mathbf{C}^{\mathrm{H}}, \dots, (\mathbf{A}^{-\mathrm{H}})^{p-1}\mathbf{C}^{\mathrm{H}}\right\}.
\end{aligned} \tag{18}
$$

$\mathbf{A}^{\mathrm{H}}$ means the Hermitian matrix of $\mathbf{A}$ and $\mathbf{A}^{\text{-H}}$ the Hermitian matrix of the inverse of $\mathbf{A}$. For a second order system like Equation (1) the subspaces can be formulated (see e.g. [10]):

$$
\begin{aligned}
\mathbf{V} &= \mathcal{K}_{\mathrm{q}}(-\mathbf{K}^{-1}\mathbf{D}, -\mathbf{K}^{-1}\mathbf{M}, -\mathbf{K}^{-1}\mathbf{B})\\
\mathbf{W} &= \mathcal{K}_{\mathrm{p}}(-\mathbf{K}^{-\mathrm{H}}\mathbf{D}^{\mathrm{H}}, -\mathbf{K}^{-\mathrm{H}}\mathbf{M}^{\mathrm{H}}, -\mathbf{K}^{-\mathrm{H}}\mathbf{C}).
\end{aligned} \tag{19}
$$

The Krylov-Subspace can be found by the two-side Arnoldi algorithm or the Lanczos algorithm like [8] shows. The static displacement of the Craig-Bampton method is applied to couple disk and blades. The input matrix reads as:

$$\begin{bmatrix}\mathbf{x}_{\text{m}}\\ \mathbf{x}_{\text{s}}\end{bmatrix} = \begin{bmatrix}\mathbf{E} & \mathbf{0}\\ \mathbf{\Psi} & \mathbf{V}\end{bmatrix}\begin{bmatrix}\mathbf{x}_{\text{m}}\\ \xi\end{bmatrix} = \mathbf{T}_V\begin{bmatrix}\mathbf{x}_{\text{m}}\\ \xi\end{bmatrix}. \quad (20)$$

Analog the left side transformation matrix $\mathbf{T}_{\text{W}}$ is created from the output subspace. The reduce model matrices $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{K}}$ results in:

$$\tilde{\mathbf{M}} = \mathbf{T}_{\text{W}}^{\text{H}}\begin{bmatrix}\mathbf{M}_{\text{mm}} & \mathbf{M}_{\text{ms}}\\ \mathbf{M}_{\text{sm}} & \mathbf{M}_{\text{ss}}\end{bmatrix}\mathbf{T}_{\text{V}}. \quad (21)$$

Therewith the model of the full model can be created like equation (11). The new stat vector can be stated as

$$\mathbf{x}_{\text{ges}} = \begin{bmatrix}\xi_{\text{d}} & \xi_{\text{b,1}} & \dots & \xi_{\text{b,n}} & \mathbf{x}_{\text{m}}\end{bmatrix}^{\text{T}}. \quad (22)$$

The reduced model can be reduced twice down to the final number of DOF.

Mistuning

One possible way to mistune the blades is a variation of the stiffness matrix of every single blade. Therewith the equation of motion is modified to

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{D}\dot{\mathbf{x}}(t) + (1 + c_{\text{k}})\mathbf{K}\mathbf{x}(t) = \mathbf{f}(t). \quad (23)$$

The factors c_{k} are the mistuning factors for every single blade k=1...n. That includes the eigenvalues of the single blades are afflicted with:

$$\lambda_{\text{k,mistuned}} = \sqrt{(1 + c_{\text{k}})}\lambda_{\text{tuned}}. \quad (24)$$

The mistuning is implemented during the assembly of the system matrices in equation (11). The submatrix is modified to

$$\underset{k=1..n}{\mathbf{bdiag}}((1 + c_{\text{k}})\mathbf{K}_{\text{b,ss,k}}). \quad (25)$$

Case study

In [8] the properties of the Krylov-Subspace method are given. The most important points are the requirement for memory and the simple algorithms. One drawback is loosing the orthogonality of the system. In this case study a simple example of a turbine stage will demonstrate the benefits and the limitations of the Krylov-Subspace method in the application of a tuned and a mistuned system.

The model consists of 30 blades on the disk. One segment is shown in Figure 2. The full system of the blade has more than $2 \cdot 10^5$ DOF. The force excites the system at one node in the middle of each blade. The observed node is at the top of the blades. Figure 3 shows the

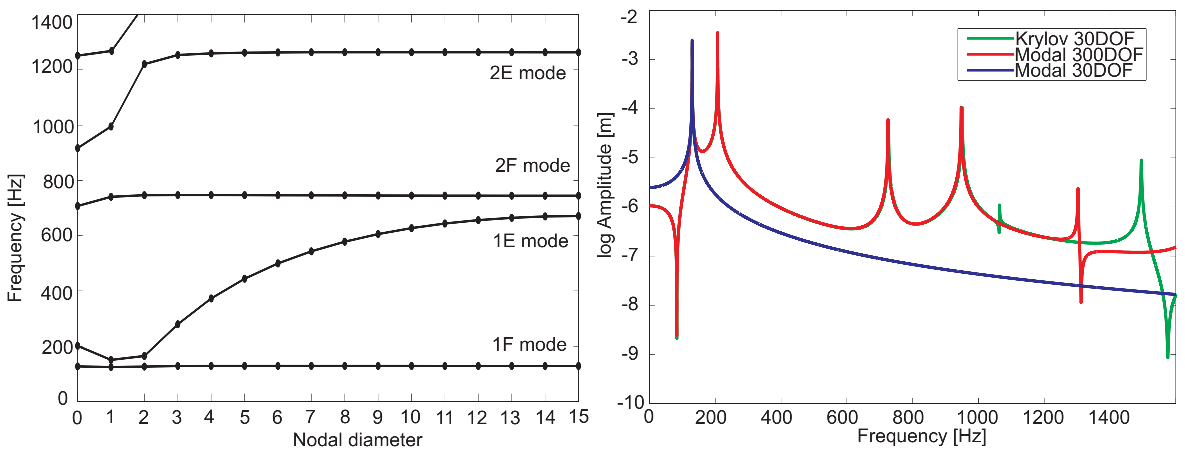

Figure 3. Nodal diameter diagram and FRF of the tuned model.

nodal diameter diagram with the first four mode shapes on the left side.

The first simulation calculates the FRF of the tuned system. Thereby the reference model is a modally reduced modell with 300 DOF. This reduction method was validated in previous work [4]. It can be assumed that this number of DOF is big enough to be close to the full system with all DOF. To compare the Krylov method and the modal reduction the model is reduced down to 30 DOF.

In Figure 3 the FRF of the tuned system is shown on the right side. As compared to a modal reduction with the same number of DOF the Krylov-Subspace is more accurate to the reference model. The first four mode shapes are well depicted. The modal reduction can only realize the first mode shape with 30 DOF. As shown in [8] the Krylov-Subspace method nullifies the orthogonality of the system. On the other hand the modal reduction decouples the EQM. This means that the Krylov-Subspace needs more time to calculate the FRF for one frequency in case of the same number of DOF.

In a second simulation a mistuned system is analyzed. The blades have a variance of $0,0001$ as Figure 5 shows. To compare the reduction method the FRF of one blade is calculated with every method. The differences between the Krylov-Subspace method and the modal reduction seem to be as small as at the tuned system. On the right side of Figure 4 an enlargement of

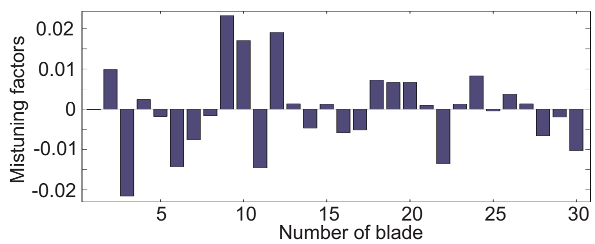

Figure 4. Mistuning factors of all 30 blades.

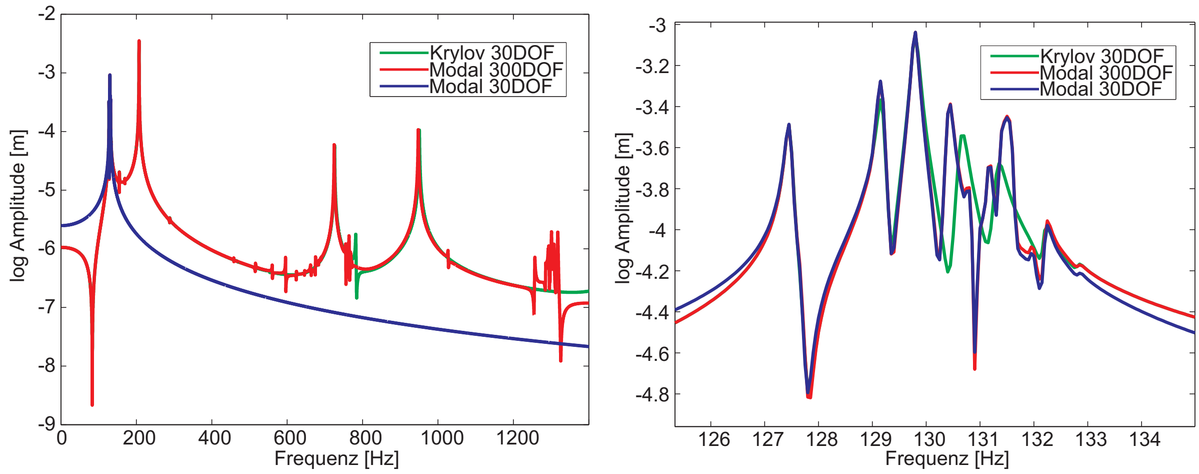

Figure 5. FRF of one blade with mistuning. Whole FRF on the left side, zoom into the first mode on the right side.

the FRF indicates a different behavior at the area around the first mode. The mistuning has the biggest influence at this passage and has to be solved as well as possible. The calculation with the Krylov-Subspace method shows a different behavior of the model in the area around the first mode. A calculation with the Krylov-Subspace method with more than 30 DOF approximates the FRF of the modal reduction. The way of adding DOF at the Krylov-Subspace method up to a good accuracy is at the expense of the computing time. A similar simulation offers the same error independent of the variance of the blades or the mode shape.

Conclusion

This paper deals with the combination of the Krylov-Subspace method and the Craig-Bampton method. In a limited frequency range the Krylov-Subspace method can be quicker and numerically better than the modal reduction. The error of the Krylov-Subspace grows up with the distance to the development point. In contrast the modal reduction approximates the FRF in the area around the eigenfrequency with a lower error. The Krylov-Subspace needs less DOF as the modal reduction in the case of a tuned turbine stage. It reduces FEMs with cyclic symmetry very efficiently. Also, the differentiation between the master and the slave nodes enables the extension to nonlinear systems. In the case of mistuning the method lost its benefits and shows its weakness. The error is independent of the variance of the mistuning and the mode shapes. The advantages and disadvantages need to be reconsidered for the application of mistuning.

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