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CONTACT STIFFNESS OF BODIES WITH FRACTAL ROUGHNESS: COMPARISON OF 3D BEM RESULTS AND REDUCTION METHOD

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Abstract. *Using the linear elastic Boundary Element Method, we calculated the normal contact stiffness of fractal rough surfaces by means of the differential quotient of normal force and indentation and found a power-law dependence on the applied load. Exponents vary from 0.5 to 0.85 depending on the fractal dimension in contrast to Persson's theory, which predicts a linear dependency. For very high forces, saturation is reached, corresponding to full material contact of an equivalent smooth indenter. Efficient algorithms allowed for statistical evaluation after just a few days of calculation with grid sizes of 2049x2049 on a standard PC. The fractal behavior of the surface roughness was applied all the way from the sample size down to the shortest wavelength that could be represented on the chosen grid.*
The same cases were investigated using a reduction method proposed by one of the authors, which maps the 3D-contact onto a 1D-rough line having the same mechanical properties. Results were obtained with dramatically less investments in CPU time. As this approach allows for a much higher resolution up to 2^23, we found the power law to be valid in the asymptotic behavior for small normal forces, given that the surface has fractal-like roughness in the corresponding small length scales.

Keywords: *fractal roughness, reduction method, contact stiffness, constriction resistance, Hurst exponent.*

1. INTRODUCTION

The roughness of surfaces has a great influence on many physical phenomena such as friction, wear, sealing, adhesion, and electrical as well as thermal conductivity. While most investigations after the 1950's and 60's focused on the contact area between rough elastic surfaces, another important contact quantity was much less investigated. The contact length

$$L = \frac{1}{E^*}\frac{\partial F_n}{\partial d} \tag{1}$$

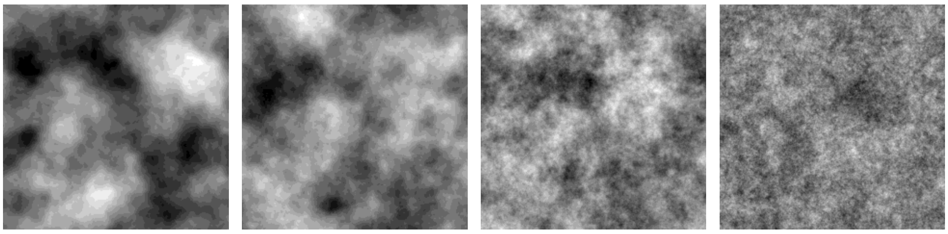
determines many practically important properties, such as the electrical and thermal conductivity. These quantities are all connected with each other by exact analytical relations. For example, the electrical contact conductance Λ is linearly proportional to the incremental stiffness [1].

Recently, the contact stiffness was studied numerically with the help of molecular dynamics [2] as well as with the boundary element method and analytically in the frame of Persson's contact theory [3]. According to this theory, an exact proportionality exists between the normal force and the contact stiffness. Initially, numerical simulations carried out in [2] and [3] as well as experiments seemed to support this conclusion. Results of our calculations, however, show a different behavior.

For self-affine fractal surfaces, the spectral density has a power law dependency on the wave vector

$$C_{2D}(q) = const \cdot \left(\frac{q}{q_0}\right)^{-2(H+1)} \tag{2}$$

wherein H is the Hurst exponent, ranging from 0 to 1. It is directly related to the fractal dimension: $D_f = 3 - H$. Fig. 1 shows typical graphical representations of surfaces generated according to this law.



$D_f = 2.0$ $D_f = 2.2$ $D_f = 2.6$ $D_f = 3.0$

Fig 1. Graphical representation of fractal surfaces having different fractal dimensions. Darker colors denote higher peaks in topography. Data has been scaled for giving optimal contrast in each picture.

2. STUDY OF THE 3D PROBLEM USING THE BOUNDARY ELEMENT METHOD

Rough surfaces were generated on a square A_0 with an equidistant discretization of 2049×2049 points. We applied the boundary element method with an iterative multi-level algorithm to obtain the pressure distribution and stiffness for a series of dimensionless normal forces comprising 8 orders of magnitude. All values were obtained by an ensemble averaging over 60 surface realizations having the same power spectrum. We defined the dimensionless stiffness

$$\bar{k} = k \Big/ \left(1.1419 \cdot E^* \sqrt{A_0}\right) \tag{3}$$

in order to obtain ‘1’ at saturated contact and our dimensionless normal force was chosen as

$$\bar{F} = F \Big/ \left(E^* h \sqrt{A_0}\right). \tag{4}$$

The calculated dependencies of the contact stiffness on normal force for six fractal dimensions are shown in Fig. 2 (right hand side). For low to medium forces, the stiffness is a power function of the normal force: $\bar{k} = const \cdot \bar{F}_N^{\alpha}$. The power α can be fitted as $\alpha \approx 0.266 \mathrm{D}_f$. Finally we obtain

$$\frac{k}{E^* \sqrt{A_0}} = \frac{D_f}{10} \left(\frac{F}{E^* h \sqrt{A_0}} \right)^{0.2567 D_f} . \tag{5}$$

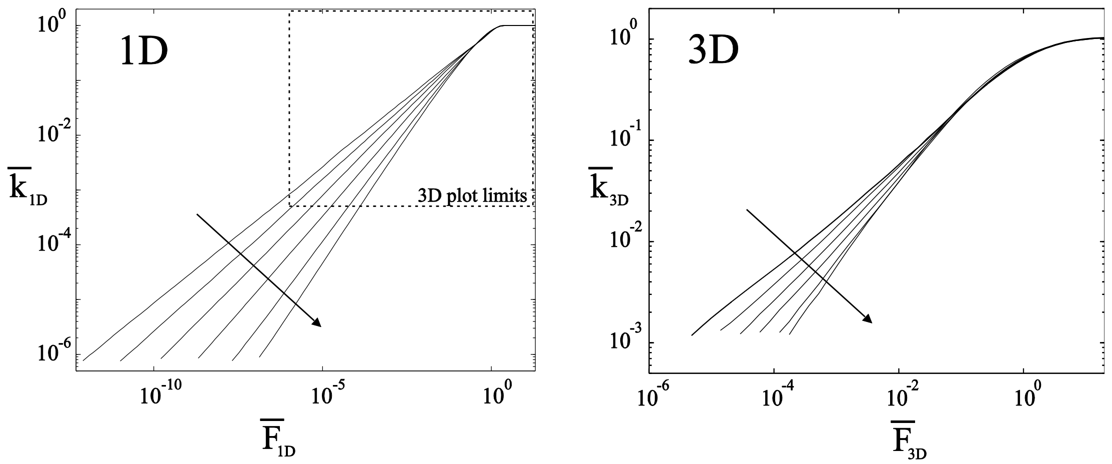


Fig 2. Dimensionless contact stiffness $\overline{k_{1D}}$ and $\overline{k_{3D}}$ as a function of the applied dimensionless load for 6 different values of the fractal dimension $D_f = 2.0,\ 2.2, 2.4, 2.6, 2.8, 3.0$ following the indicated arrow. Results are obtained using the 1D reduction method and the 3D boundary element method. Different definitions for the dimensionless force and stiffness apply:

$$\overline{k_{1D}} = \frac{\partial F}{\partial d} \left(E^* L \right)^{-1} \quad , \quad \overline{F_{1D}} = F \left(E^* h L \right)^{-1}$$

$$\overline{k_{3D}} = \frac{\partial F}{\partial d} \left(1.1419 E^* \sqrt{A_0} \right)^{-1} \quad , \quad \overline{F_{3D}} = F \left(E^* h \sqrt{A_0} \right)^{-1}$$

2. CALCULATION IN “REDUCED DIMENSIONALITY” APPROACH

The approach of reduced dimensionality was first demonstrated for single contacts and rough surfaces with a constant spectral density [4,5] and verified later in applications to surfaces with various fractal properties by a comparison of the results with the data for 3D models existing in literature.

According to this approach, a solution of an original 3D contact problem can be formally substituted by an equivalent one-dimensional study of an artificially constructed line having the following effective spectral density

$$C_{1D}(q) = \pi q C_{2D}\left(\left|\vec{q}\right|\right), \tag{6}$$

In the approach of reduced dimensionality, the normal contact force at given position z of an upper flexible plate can be easily accumulated as a sum of partial forces corresponding to a set of ***independent*** springs.

The dimensionless forces are normalized by a new combination $E^{*}L$, in which the length of the system L substitutes the square root of the apparent contact area $\sqrt{A_0}$ in the 3D approach (see [5] for details).

If our understanding of the problem is correct, the results obtained in two different approaches must give the same relation for the contact stiffness for all forces and all fractal dimensions after the same scaling transformation, with the only substitution $\sqrt{A_0} \to L$. Indeed, as can be seen from Fig. 2, the results coincide almost perfectly. For small to medium forces, a power law dependence is found.

We can also learn from the reduced model that the power law dependence remains valid even for very small stiffness values, at least down to 10^{-6}, if the surface is still fractal at very small wavelengths. Grid sizes in 3D that would allow for these small contact areas to be reproduced correctly, cannot be calculated within a reasonable time, not even with Multigrid techniques as we have been using.

3. REFERENCES

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