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INTERFACE ELEMENT FOR THE DYNAMIC ANALYSIS OF VEHICLE-STRUCTURE INTERACTION

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Abstract. *In current literature, there are several approaches for the numerical study of the vehicle-structure (e.g. train-bridge) interaction problem. One of the most prominent approaches combines both vehicle and structure systems into a single one through the use of special interaction elements. In this work it is proposed a different method that joins vehicle and structure by introducing an interface element. This element automatically makes forces and displacements compatible at contact nodes. The method is applicable for the study of interaction of concentrated loads, vehicles and train compositions. The method was applied to the study of a concentrated load moving on a bridge obtaining excellent results when compared to analytical ones. Finally, an analysis of a passing carriage over a bridge is made.*

Keywords: *Interface element, dynamics analysis, vehicle-structure interaction.*

1. INTRODUCTION

The study of interaction between a bridge and a train is a typical case of the vehicle-structure dynamic interaction problem. Often, most research is focused on the dynamics response of the bridge, the study of resonance [1][12][13] and the effects of track irregularities [1][8]. For the cases where only the bridge response is required, the vehicles are usually represented by a set of moving loads with constant magnitude. This approach is reflected in several studies (e.g. [4][8][13]) and international standards [5][10]. Several methodologies have emerged from these studies in order to simplify calculations such as the Decomposition of Excitation in Resonance (DER), the Residual Influence Line (RIL) or the Proportional Dynamic Signature (DER) [1]. These methods are formulated for isostatic structures and do not consider time integration; however they provide a speed envelope from which it is possible to forecast resonance in structures. On the other hand, when the combined response of bridge and vehicle is required, models that account for the dynamic interaction should be used. In this case, bridge and vehicles can be simulated as two different structures interacting with each other by means of forces and displacements prescribed at contact points, i.e. the wheels and rails. This type of interaction is nonlinear and time dependent since the contact points are mobile along time as a result of the relative motion between systems. Some interaction approaches use to analytically develop a system of dynamic equations for a specific vehicle-structure configuration. Both system interact by means of contact forces which are interpolated to bridge nodes [4][6][11]. Those methods show long analytical deductions in order to obtain the dynamic governing equations. Following such approach, new vehicle-structure configurations need additional analytical work. Considering this fact, this work proposes a simple method to treat vehicle-structure interaction using a single system that comprises vehicle and structure connected by means of an interface element based on [7]. This interface element aims to transmit forces and displacements at contact points between vehicle and structure and is constituted by a series of springs and dampers arranged over the bridge. The springs and dampers properties are variable in such a way that there is force transmission only through bridge nodes that are located near a wheel. Finally, the vehicle, the bridge and the interface element are combined systematically based on its degrees of freedom to assemble the mass, stiffness and dumping global matrices to be used to integrate the equation of motion:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{F}\,, \tag{1}$$

where $\mathbf{M}$ is the system mass matrix, $\mathbf{C}$ is the dumping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{y}$ is the displacements vector and $\mathbf{F}$ is the vector with external forces.

2. VEHICLE MODELING

Rail vehicles are mechanical systems with several degrees of freedom whose suspensions systems comprises springs with linear and nonlinear behavior and dumpers which can be hydraulic or pneumatic. During the passage of the vehicle over a bridge structure, their own weight in combination with the inertia of its mass can cause vibrations which may affect the structural integrity of the bridge. Some dynamic analysis of vehicle-structure interaction considers the vehicle as a concentrated moving load [4][5][8][10][13]. This approach leads to greater responses for bridge displacements and can be considered conservative. Therefore, it is convenient to represent the vehicle through a simplified mass-spring-dumper system for both two and three dimensions [1][6][11]. In this work, the vehicle is represented by a set of interconnected basic elements (mass, spring and dumper). All basic elements have their own stiffness, mass and dumping matrices which are then combined to obtain the corresponding matrices for the vehicle. Considering structures as a set of basic elements has the advantage of allowing the use of superposition to mount the stiffness, dumping and mass matrices for vehicles with different settings and levels of complexity.

2.1 Simplified model 1 (SM1)

Considering a train as a set of concentrated loads over each independent axle, this model represents the load related to one axle. An axle and its suspension system are represented by a spring-mass-dumper system (Figure 1) which properties are obtained from the vehicle features. In the figure, y_s is the vertical displacement of the suspended mass m_v (part of carriage box and bogie), y_r is the vertical displacement of the non-suspended mass m_r

(wheel), k_s and c_s are respectively the equivalent stiffness and damping coefficients from the axle suspension system.

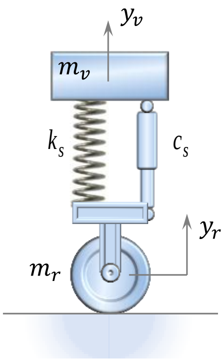

Figure 1. Simplified model 1.

The simplified model has two degrees of freedom, one related to the suspended mass displacement and another related to the wheel displacement. The displacement vector for this model is just given by:

$$\mathbf{y} = \begin{pmatrix} y_v \\ y_r \end{pmatrix} . \tag{2}$$

The mass, dumping and stiffness matrices for the spring element are respectively:

$$\mathbf{M}_k = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{C}_k = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{K}_k = \begin{pmatrix} k_s & -k_s \\ -k_s & k_s \end{pmatrix} . \tag{3}$$

For the dumper, the matrices are:

$$\mathbf{M}_c = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{C}_c = \begin{pmatrix} c_s & -c_s \\ -c_s & c_s \end{pmatrix} \quad \mathbf{K}_c = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} . \tag{4}$$

Finally, for both the suspended and non-suspended elements the matrices are:

$$\mathbf{M}_v = \begin{pmatrix} m_v & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{C}_v = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{K}_v = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \tag{5}$$

$$\mathbf{M}_r = \begin{pmatrix} 0 & 0 \\ 0 & m_r \end{pmatrix} \quad \mathbf{C}_r = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \mathbf{K}_r = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} . \tag{6}$$

All matrices described above are combined by superposition in order to mount the final ones for this simplified model, which are:

$$\mathbf{M} = \begin{pmatrix} m_v & 0 \\ 0 & m_r \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} c_s & -c_s \\ -c_s & c_s \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} k_s & -k_s \\ -k_s & k_s \end{pmatrix} . \tag{7}$$

2.2 Simplified model 2 (SM2)

Aiming to improve the modeling of the behavior of suspended masses a new model is considered. In such model two axles are considered and they interact simultaneously by means of a rigid bar element with mass m_v and moment of inertia J_v. Figure 2 shows schematically the model configuration.

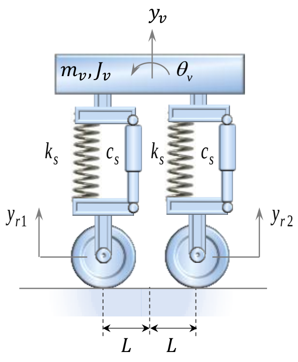

Figure 2. Simplified model 2.

According to Figure 2, y_v e θ_v are respectively the vertical displacement and rotation of the suspended mass m_v, y_{r1} and y_{r2} are the vertical displacements from wheels. Each axle is located at a distance L from the model center of mass and the suspension system comprises springs with stiffness k_s and dumpers with damping coefficient c_s. The model has four degrees of freedom, two displacements related to the wheels and one related to the suspended mass and a rotation related only to the suspended mass. The vector of degrees of freedom is given by:

$$\mathbf{y} = \begin{pmatrix} y_{r1} \\ y_{r2} \\ y_v \\ \theta_v \end{pmatrix}. \tag{8}$$

Using the superposition principle, the mass, dumping and stiffness matrices from constituent elements are combined to find the corresponding matrices for the simplified model 2. Those matrices are:

$$\mathbf{M} = \begin{pmatrix} m_{r1} & 0 & 0 & 0 \\ 0 & m_{r2} & 0 & 0 \\ 0 & 0 & m_v & 0 \\ 0 & 0 & 0 & J_s \end{pmatrix}, \tag{9}$$

$$\mathbf{C} = \begin{pmatrix} c_s & 0 & -c_s & c_sL \\ 0 & c_s & -c_s & -c_sL \\ -c_s & -c_s & 2c_s & 0 \\ c_sL & -c_sL & 0 & 2c_sL^2 \end{pmatrix} \text{ and} \tag{10}$$

$$\mathbf{K} = \begin{pmatrix} k_s & 0 & -k_s & k_sL \\ 0 & k_s & -k_s & -k_sL \\ -k_s & -k_s & 2k_s & 0 \\ k_sL & -k_sL & 0 & 2k_sL^2 \end{pmatrix}. \quad (11)$$

Continuing to use the principle of superposition it is possible to combine the composed models as SM1 and SM2 to obtain new, different and more complex vehicle models.

2.3 Complete model (CM)

This model represents a complete carriage. For this purpose the carriage box and four axles are considered together with the suspension system. Considering composed models as previously viewed and the superposition principle, the complete model can be formed by three SM2 elements as it is shown in Figure 3.

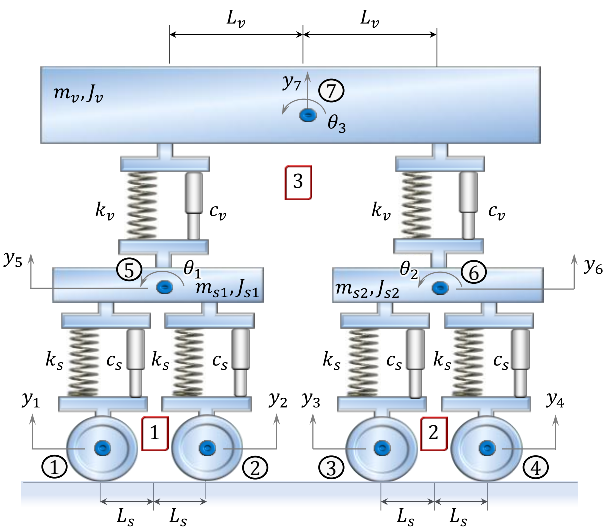

Figure 3. Modelo de interação completa.

Still according to Figure 3, model nodes are numbered from 1 to 7, y_1 to y_4 represent the wheels displacements, y_5 and y_6 represent the bogies displacements, y_7 represents the box displacement, θ_1 and θ_2 represents the bogies rotations and θ_3 represents the box rotation. The distance between wheels in each bogie is equal to $2L_s$ and the distance between bogies is equal to $2L_v$. Suspension systems in bogies have springs and dumpers with stiffness and dumping coefficient equal to k_s and c_s, respectively. The box suspension system has springs and dumpers with stiffness and dumpers coefficient equal to k_v and c_v, respectively. This model has a total of ten degrees of freedom among displacements and rotations according to:

$$\mathbf{y} = (y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7 \quad \theta_1 \quad \theta_2 \quad \theta_3)^T. \quad (12)$$

The mass, stiffness and dumping matrices for this model are obtained by mounting the corresponding matrices from SM2 constituent elements.

3. BRIDGE MODELING

A bridge is represented by means of a set of conventional Euler-Bernoulli beam elements. Each beam element has six degrees of freedom which are two displacements and two rotations as illustrated in Figure 4. Eq. 13 shows the vector containing all degrees of freedom.

$$\mathbf{y} = (y_1 \quad y_2 \quad \theta_1 \quad y_3 \quad y_4 \quad \theta_2)^T \,. \tag{13}$$

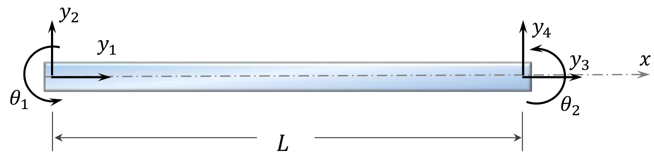

Figure 4. Beam elemento with all degrees of freedom.

Stiffness and mass matrices are given by Eqs. 14 e 15, where E is the Young modulus, I is the section inertia, A is the section area, L is the element length and ρ the density per unit length.

$$\mathbf{K} = \begin{pmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{pmatrix} \tag{14}$$

$$\mathbf{M} = \frac{\rho L}{420} \begin{pmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22L & 0 & 54 & -13L \\ 0 & 22L & 4L^2 & 0 & 13L & -3L^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13L & 0 & 156 & -22L \\ 0 & -13L & -3L^2 & 0 & -22L & 4L^2 \end{pmatrix} \tag{15}$$

The dumping matrix **C** for the bridge is obtained using the Rayleigh-Ritz method. This method assumes that the damping matrix could be estimated as a linear combination of stiffness and mass matrices as:

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \,. \tag{16}$$

Taking a dumping ratio ζ for the first and second vibration modes, scalars α and β are given by:

$$\alpha = \frac{2\zeta\omega_1\omega_2}{\omega_1+\omega_2} \text{ and} \tag{17}$$

$$\beta = \frac{2\zeta}{\omega_1+\omega_2} \,, \tag{18}$$

where ω_1 e ω_2 are the first and second natural frequencies, respectively, of a simply beam.

4. VEHICLE-STRUCTURE INTERACTION MODELING

The currently widely used numerical approaches, e.g. [4][9][11], consider vehicle-structure interaction analysis by means of interaction beam elements with variable properties and geometry according to the position of the passing vehicle. However, the approach presented here considers vehicle and structure linked together by an interface element that allows automatic transmission of forces and displacements at contact points between both systems.

This interface element is based on the element proposed by [7] to simulate interaction of reinforcements (bar elements) arranged arbitrarily in finite element meshes. Figure 5 shows spring interface elements used to link a bar element with a solid element at points where nodal locations do not match. At these non-matching points, displacements are made compatible using an interpolation procedure. One advantage of the interface element proposed by [7] is that the bar element can slip inside the solid element when contact strength is reached.

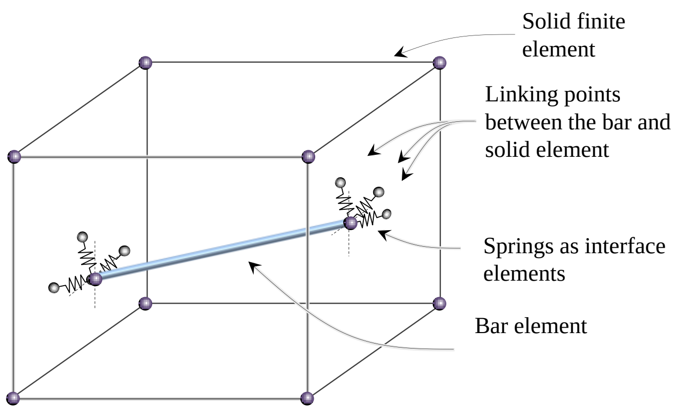

Figure 5. Interface element proposed by [7] for the analysis of reinforcements (modified).

Similar to [7], the interface element formulated for this work aims to transmit forces and to make displacements compatible at contact points between a moving carriage and a bridge. This element made of set of linking springs that connect wheels nodes to the bridge beam nodes without increasing degrees of freedom to the global system. In addition to the linking springs, dumpers are added in order to minimize impact effects at contact points. For instance, Figure 6a shows schematically a wheel connected to a three beam elements bridge by means of an interface element. Figure 6b illustrates the interface element separately constituted by four springs and four dumpers arranged over the bridge nodes. For this particular case, the interface element has five nodes and five displacement degrees of freedom. It can be noted that node 5 is drawn at different locations but all of these represent a unique node that matches the wheel node position.

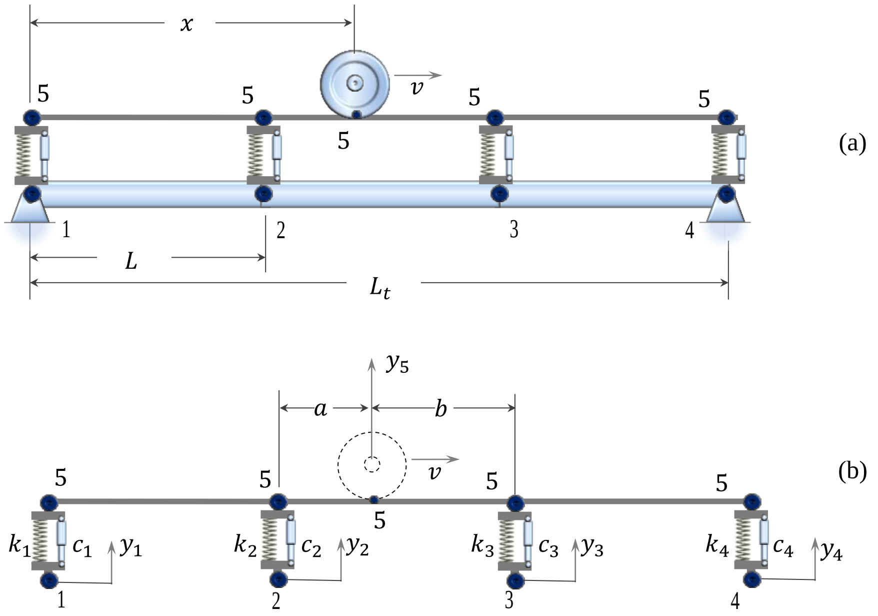

Figure 6. a) Interface element linking a vehicle wheel with a bridge; b) Interface element represented separately.

Still for the interface element shown in Figure 6b, the vector containing the displacement is given by Eq. 19 and the stiffness matrix is given by Eq. 20.

$$\mathbf{y} = (y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5)^T \tag{19}$$

$$\mathbf{K} = \begin{pmatrix} k_1 & 0 & 0 & 0 & -k_1 \\ 0 & k_2 & 0 & 0 & -k_2 \\ 0 & 0 & k_3 & 0 & -k_3 \\ 0 & 0 & 0 & k_4 & -k_4 \\ -k_1 & -k_2 & -k_3 & -k_4 & k_1 + k_2 + k_3 + k_4 \end{pmatrix}. \tag{20}$$

In this work, the dumping matrix is estimated as being proportional to the stiffness matrix according to:

$$\mathbf{C} = \frac{\eta}{\omega_1}\,\mathbf{K}\,, \tag{21}$$

where η is an scalar to be adjusted and ω_1 is the bridge first natural frequency.

Figure 6, in particular, shows the instant when a wheel is in contact with the central beam element of the bridge. At this point, the load that may come over the wheel should be transmitted to the bridge by the springs located at nodes 2 and 3. For this reason, the stiffness coefficients k_1 and k_4 should be equal to zero and the coefficients k_2 and k_3 should get high values to avoid excessive relative displacements at nodes 2 and 3. Still, stiffness coefficients k_2 e k_3 should be proportional to a and b in order to transmit loadings to nodes 2 and 3 in an equivalent way as a simple beam subject to a concentrated load. In such sense, k_2 and k_3 are given by:

$$k_2 = \frac{b}{L} k_0 \quad \text{and} \quad k_3 = \frac{a}{L} k_0\,, \tag{22}$$

where k_0 represents a high value and is related with the wheel load P and the bridge maximum deflection δ_{max} by means of an scalar λ according to:

$$k_0 = \frac{1}{\lambda} \cdot \frac{P}{\delta_{max}} \, . \tag{23}$$

For a simple supported bridge of length L_t, Young Modulus E and section inertia I, Eq. 23 can be rewritten as:

$$k_0 = \frac{1}{\lambda} \cdot \frac{48EI}{L_t^3} \, . \tag{24}$$

Since the bridge can be discretized in any number of beam elements, the interface element should be properly constructed for each case in order to be connected to all bridge nodes. When the vehicle is represented by a model with several wheels, one interface element can be used for each wheel. To avoid the use of many interface elements, a single one could be used if it is assumed as the summation of the interface elements for each wheel. In this case, the stiffness and dumping matrices can be mounted using the superposition principle.

5. APLICATIONS

The proposed modeling was implemented in a computer program called PyDyn using Python language. The next subsections present two examples that show the method applicability. The first one analyses the deflection at the middle of the span in a bridge subject to a moving concentrated load. Numerical results are compared to analytical ones obtaining excellent accuracy. The second analysis studies the passage of a vehicle for which the complete model described in section 2.3 is used.

5.1 Bridge subject to a moving concentrated load

In this analysis a concentrated load moves with velocity v over a bridge of length L_t, as shown in Figure 7. The deflection in the middle of the span along time is compared with the analytical solution introduced by [2]. This solution is represented by two equations; the first one is used when the load is over the bridge (Eq. 25), and the second one is used when the load leaves the bridge which is then subject to free vibrations only (Eq. 26).

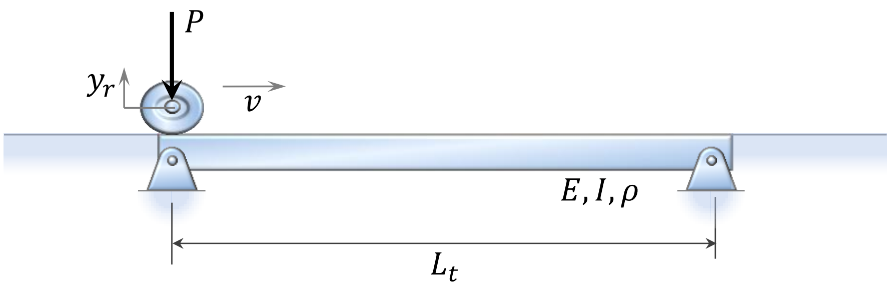

Figure 7. Initial configuration for the analysis of a concentrated load moving over a bridge.

$$y(t) = \frac{y_s}{1-r^2}\left[\text{sen}(r\omega_1 t) - r\exp(-\zeta\omega_1 t)\,\text{sen}(\omega_1 t)\right] \quad (25)$$

$$y(t) = \frac{y_s r}{1-r^2}\left[\text{sen}(\omega_1 t)\exp(-\zeta\omega_1 t) + \text{sen}\big(\omega_1(t-t_1)\big)\exp\big(-\zeta\omega_1(t-t_1)\big)\right] \quad (26)$$

In the equations above, t is the elapsed time the load spends moving from the beginning of the bridge, t_1 is the needed time for the load to cross the entire bridge, ω_1 is the first natural frequency, ζ is the dumping ratio, y_s is the static deflection at the middle of the span and r is given by:

$$r = \frac{\pi v}{\omega_1 L_t}. \quad (27)$$

To obtain the analytical solution, the following data is used: moving load $P = 203$ kN, $L_t = 15$ m, $v = 83.33$ m/s, $EI = 8.323 \times 10^6$ kNm^2 which is used to calculate ω_1 and y_s, $\rho = 2303$ kg/m, $y_s = 1.715$ mm, $\omega_1 = 83.389$ rad/s and $\zeta = 0.01$. The time the load needs to cross the bridge is $t_1 = 0.18$ s.

During the numerical analysis, the bridge is discretized in 40 beam elements. Regarding the interface element, the following coefficients are used $\lambda = 0.021$ e $\eta = 0.028$. The analysis is performed using the Newmark integration method for a total analysis time of 0.7 s with a time step equal to 0.001 s. Figure 8 shows the numerical results together with analytical values obtained by Eqs. 25 and 26. Excellent accuracy can be seen between numerical and analytical results.

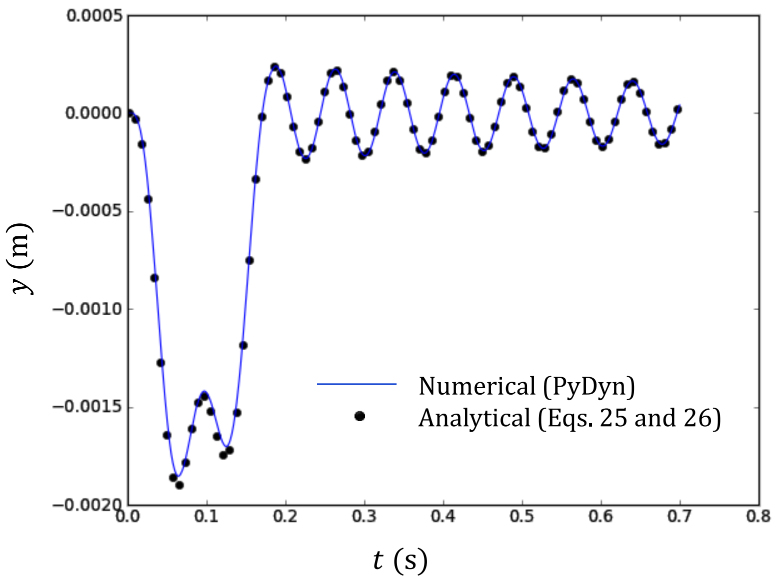

Figure 8. Comparison between numerical and analytical results for the deflection at the middle of the span in a bridge subjected to a concentrated moving load.

5.2 Bridge with a passing vehicle

In this analysis, the vehicle is represented by a CM (complete model) element as introduced in section 2.3 and the bridge is represented by a set of beam elements. The initial configuration for this analysis is illustrated in Figure 9. The vehicle properties, according to Figure 3, are: $L_v = 4$ m, $L_s = 1$ m, $m_v = 35300$ kg, $m_s = 2800$ kg, $k_v = 2600$ kN/m $k_s = 90$ kN/m, $c_v = 12$ kNs/m, $c_s = 20$ kNs/m. The 20 m length bridge is discretized with 30 beam elements with the following properties: $EI = 8{,}6 \times 10^6$ kNm^2, $\rho = 2300$ kg/m, $\zeta = 0{,}01$. The interface element is arranged linking all bridge nodes with the wheels nodes. The properties used for the interface, according to section 4, are $\lambda = 0{,}012$ and $\eta = 0{,}015$. A load equal to $P = 400$ kN related to the vehicle mass is applied to the node located at the box's center of gravity.

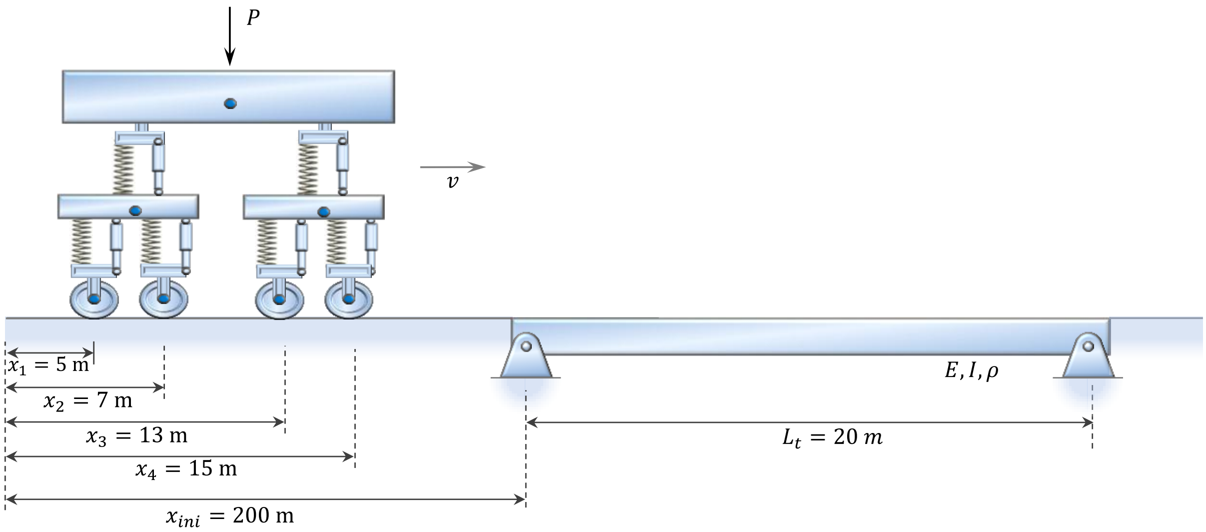

Figure 9. Configuration for the analysis of a moving carriage over a bridge.

The analysis is performed using the Newmark integration method for a total time of 13 s with a time step equal to 0.005 s. The time step is chosen small enough in order to minimize time integration errors. During the analysis, the vehicle starts its journey around 200 m before the bridge with constant velocity of 20 m/s. The time the first wheel needs to enter the bridge is 9.25 s and the time the last wheel needs to leave the bridge is 10.75 s. Figure 10 shows the deflection obtained at the middle of the span. It should be noted that the maximum deflection occurs about halfway through the route of the carriage on the bridge (10 s). After the vehicle leaves the bridge the last adopts a free vibration behavior as it is expected. Figure 11 shows the vertical displacement of the first vehicle's wheel which is consistent with the bridge deflection. Figure 12 shows the vertical acceleration of the box whose values are within the range of physical comfort. The slight oscillation in vertical acceleration obtained after the vehicle leaves the bridge is due to the influence of the suspension system of the carriage.

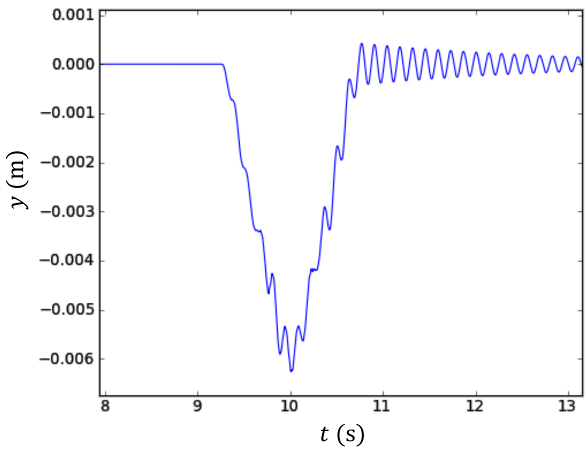

Figure 10. Deflection at the middle of the span.

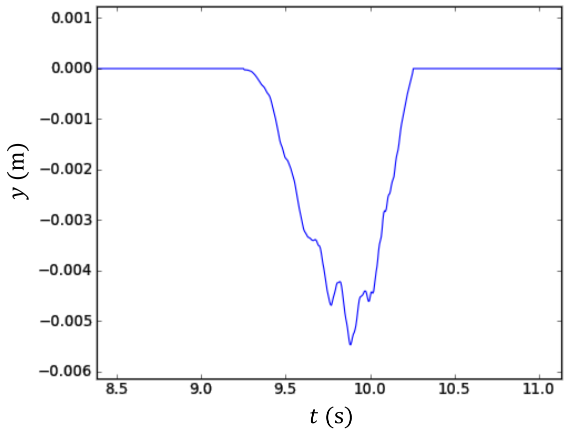

Figure 11. Vertical displacement at first vehicle wheel.

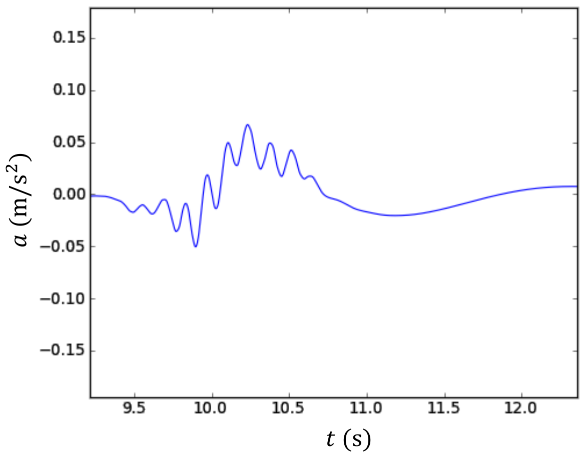

Figure 12. Vertical acceleration in the box.

6. CONCLUSIONS

A new approach for the numerical simulation of vehicle-structure interaction was presented. This approach considers the addition of an interface element to connect vehicle wheels to the bridge. The model was validated comparing numerical and analytical results of the deflection at the middle of the span of a bridge subject to a moving concentrated load. Excellent accuracy was obtained. An additional analysis was made to simulate a carriage crossing a bridge.

The key features of the proposed method are its simplicity and convenience to represent vehicles and their interaction with a bridge. Simple and complex vehicles models and even train compositions can be represented by grouping elements from basic models. Moreover, the interaction problem is reduced to the analysis of one single system. This allows reducing computational implementations besides simplifying the configuration of the analysis.

ACKNOWLEDGEMENTS

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