

## TOPOLOGY DESIGN OF STRUCTURE LOADED BY EARTHQUAKE

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**Abstract.** *The contribution deals with optimal topology design of civil structures in dynamics. Earthquake loading is considered. The earthquake excitation has multi-frequency content. The dynamic input is defined on the base of Eurocode 8. The push-over curve is applied in dynamic analysis.*

*The practice in civil engineering requires cost minimization and fulfillment the safety conditions. The optimal topology design of structure loaded by earthquake is focused. The topology problem is defined as a material distribution problem. Densities in discretized elements of the structure are the variables of the optimization. The relationship between mass and the stiffness is defined on the base of the SIMP method. The objective of the structural optimization is the minimum weight of the structure. The design space is constrained by conditions based on natural frequencies and on the maximum displacement defined on the push-over curve.*

**Keywords:** *Topology optimization of structures, Dynamics, Earthquake.*

### 1. INTRODUCTION

The civil engineering and mechanical engineering practice requires solution of topology problems of structures in dynamics. The contribution proposes optimal topology design of structures in dynamics loaded by excitation with multi-frequency content. The excitation with multi-frequency content appears in civil engineering during earthquakes, by wind loading, loading induced by human activities. In mechanical engineering the multi-frequency excitation is caused by machines. The paper focuses the earthquake excitation.

### 2. DYNAMICS

The dynamic analysis of the structure is repeated in each optimization step. The nonlinear analysis in time domain is very expensive and not acceptable in the optimization procedure. The frequency domain enables the efficient way of the solution. The recourses provided in Eurocode 8 [3] are applied. The consideration of risk is in this approach included.

The analysis in dynamics consists of following steps:

1. Solution of the eigenvalue problem

$$\left( \mathbf{K}(\rho) - \omega_i^2 \mathbf{M} \right) \Phi_i = 0, \quad i = 1, \dots, N. \quad (1)$$

with stiffness matrix  $\mathbf{K}(\rho)$ , mass matrix  $\mathbf{M}(\rho)$ , natural frequency  $\omega_i$  and modes  $\Phi_i$ .

2. The pseudo-acceleration spectrum is from Eurocode 8 [3] selected.

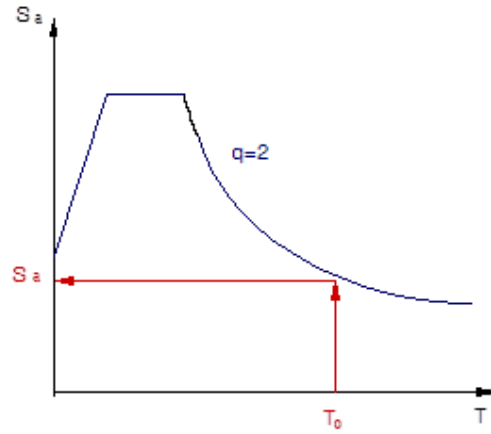


Figure 1. The pseudo-acceleration spectrum

3. To the structural period  $T_0$  the corresponding spectral acceleration  $S_a$  is found.

4. The equivalent static force vector  $\mathbf{f}_s$  is calculated as follows ([4], [5]):

$$\max \mathbf{f}_s = \mathbf{M} \Phi \Gamma^* S_a. \quad (2)$$

where

$$\Gamma^* = \frac{L^*}{m^*}. \quad (3)$$

$$L^* = \Phi^T \mathbf{M} \mathbf{e} = \sum_{i=1}^N m_i \Phi_i. \quad (4)$$

$$m^* = \Phi^T \mathbf{M} \Phi = \sum_{i=1}^N m_i \Phi_i^2. \quad (5)$$

with static force vector  $\mathbf{f}_s$ , mass  $m$ , modes  $\Phi$ , modal participation factor  $\Gamma^*$ , coefficient vector  $L^*$ , spectral acceleration  $S_a$ .

5. Applying equivalent static forces, displacements  $u$  can be calculated

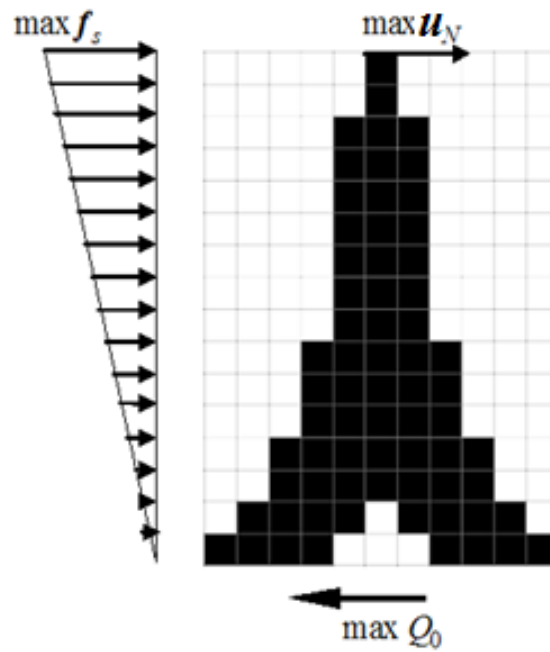


Figure 2. Equivalent forces and displacements.

6. The displacement constraint is on the push-over curve checked.

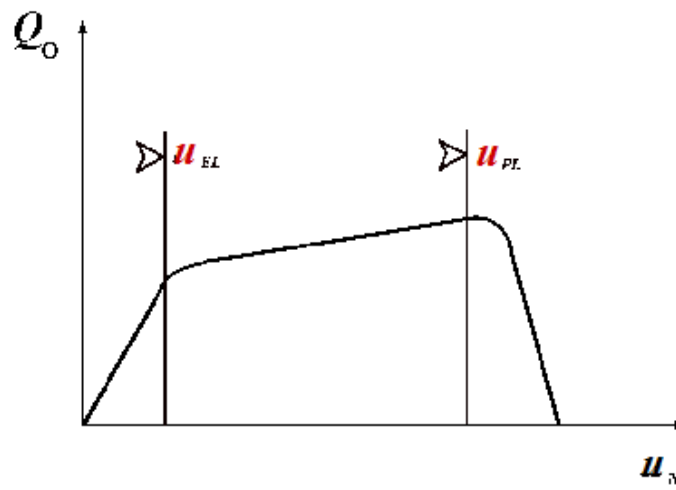


Figure 3. Push-over curve.

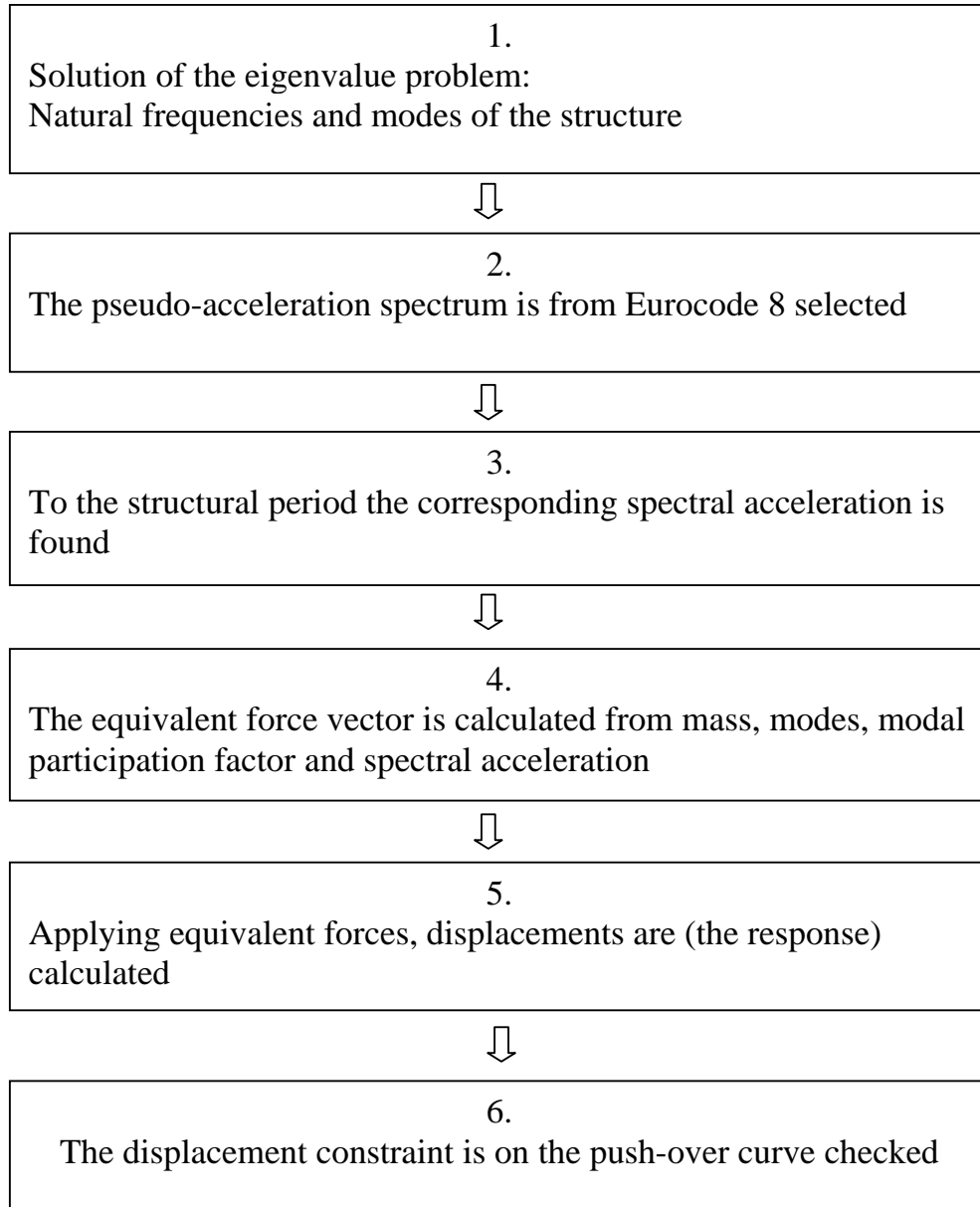


Figure 4. Flowchart of the dynamic analysis.

### 3. TOPOLOGY OPTIMIZATION

#### 3.1. Topology problem definition

The topology problem is defined as a material distribution problem. Densities  $\rho_e$  in discretized elements ( $e$ ) of the structure are the variables of the optimization.

$$0 < \rho_{\min} \leq \rho_e \leq 1, \quad e = 1, \dots, N. \quad (6)$$

The relationship between mass and the stiffness is defined on the base of the SIMP method [1], [2].

$$\mathbf{M}(\boldsymbol{\rho}) = \sum_{\substack{matr \\ e=1}}^N \rho_e V_e \quad \mathbf{K}(\boldsymbol{\rho}) = \sum_{\substack{matr \\ e=1}}^N \rho_e^p \mathbf{K}_e. \quad (7)$$

The objective is the minimum weight of the structure.

$$\min_{\boldsymbol{\rho}} V = \sum_{e=1}^N v_e \rho_e, \quad e = 1, \dots, N. \quad (8)$$

The constraints are defined according to topology densities (6) and maximum displacements

$$u \leq u_{pL}. \quad (9)$$

### 3.2. Optimization procedure

The genetic algorithm as optimization procedure controls the topology density variables in the optimization process.

### 3.3. Example

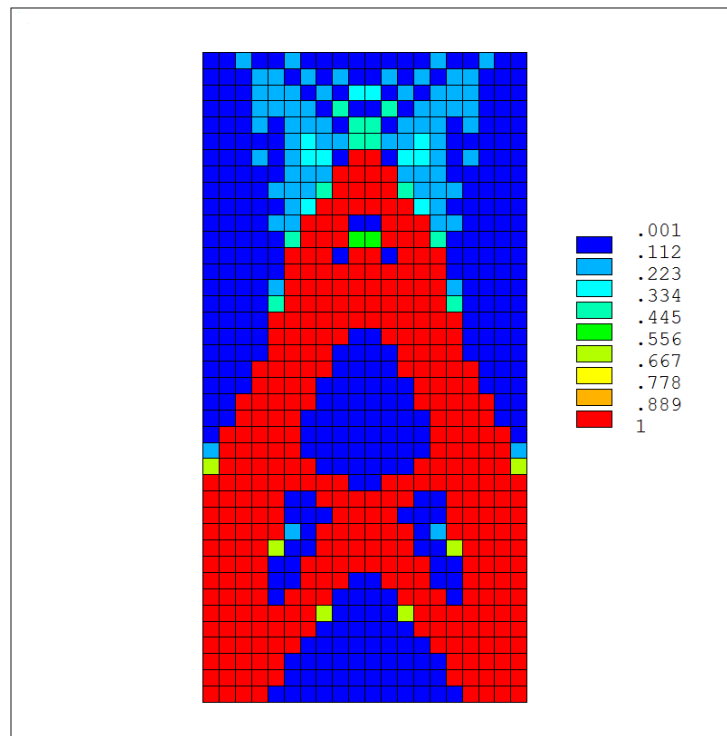


Figure 5. Optimal topology.

The 2-D example illustrates the presented theory.

#### **4. CONCLUSION**

The topology design of civil structure with earthquake loads was presented. The dynamic loading according Eurocode 8 was considered.

Presented procedure can be used for dynamic loading with multifrequency content: in civil engineering by wind loading, loading induced by human activities and in mechanical engineering by excitation caused by machines.

#### **Acknowledgements**

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