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NUMERICAL EVALUATION OF A SANDWICH VISCOELASTIC MODEL

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Abstract. *Passive control has been used to reduce vibration amplitudes. This is the case of composite structures with elastic layers and viscoelastic core. These structures, which are called by sandwich, present a high damping ratio and simple application. In order to design sandwich structures, many aspects ranging from computer modeling to laboratory tests should be considered. In this paper, an approach that involves a theoretical/numerical Golla-Hughes-McTavish (GHM) based model is presented. The GHM method is applied to modeling viscoelastic materials and, consequently, to aid the design of sandwich structures. In that way, starting from dynamic properties of a viscoelastic material, numerical models are used to evaluate the behavior of sandwich structures, showing the advantages and disadvantages of the presented methodology. Comparisons with uncontrolled structures are also presented, showing the dissipative characteristics of this type of passive control.*

Keywords: *Viscoelastic Material, Sandwich Model, Vibration Control.*

1. INTRODUCTION

The development of new construction materials, the advancement of knowledge about materials behavior and the domain of sophisticated construction techniques, allowed the construction of lighter and high bearing capacity structures. This process extends to the present days and imposed the need to check, during the design phase, the dynamic behavior of structures, with few exceptions.

Structural vibrations are undesirable not only for the discomfort caused to users, but also for the fatigue process, which is accelerated by dynamic oscillations. These effects may be detected specifically in structures with low stiffness and low natural frequencies, leading to large displacement amplitudes.

Aiming the reduction of structural vibrations, several techniques were developed to increase structural damping. Among these techniques, the passive control with viscoelastic materials has shown reasonable efficiency. These materials have low bearing capacity with high dissipative capacity when subjected to cyclic deformations. That is the main reason why viscoelastic materials are applied in sandwich layers with stiff elastic materials [1]. In that way, in order to effectively reduce structural vibrations using viscoelastic materials, it is important to understand the dynamic behavior of the structure and the used viscoelastic material (VEM).

Within this context, this paper will discuss the computational modeling of viscoelastic materials and their use for reducing vibrations in structures, working as a passive control mechanism in sandwich layers. A computational viscoelastic sandwich model, based on GHM method, is analised and validated. Finally, comparisons with uncontrolled structures are also presented, showing the dissipative characteristics of this type of passive control.

2. THE GHM MEHOD FOR VISCOELASTIC MATERIALS MODELING

The stress-strain relation on Laplace's domain as mentioned by reference [2] may be written as:

$$\sigma(s) = [E_0 + h(s)]\varepsilon(s)\,, \tag{1}$$

where s is the Laplace operator, $\sigma(s)$ and $\varepsilon(s)$ are, respectively, the stress and strain on Laplace's domain, E_0 is the elastic fraction of complex modulus and $h(s)$ is the relaxation function.

Function $h(s)$ can be written using Biot's [3] series, using with two terms:

$$h(s) = \frac{\alpha\left(s^2 + \beta s\right)}{s^2 + \beta s + \delta}\,, \tag{2}$$

where α, β and δ are materials constants and $(\alpha, \beta, \delta) \geq 0$.

Starting from the equation of motion in the Laplace domain:

$$\left\{M^L s^2 + K^L\right\}q(s) = f^L(s)\,, \tag{3}$$

where, M^L, K^L and $f^L(s)$ are respectively the mass, stiffness and external loading in the Laplace domain, where:

$$K^L = [E_0 + h(s)]K_v\,, \tag{4}$$

where: K_v is the rigidity fraction associated with geometrical characteristics of the model. The GHM model defines the equation of motion in the time domain as [1]:

$$\begin{bmatrix} M & 0 \\ 0 & \frac{\alpha}{\delta}K_v \end{bmatrix}\begin{Bmatrix} \ddot{q} \\ \ddot{z} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{\alpha\beta}{\delta}K_v \end{bmatrix}\begin{Bmatrix} \dot{q} \\ \dot{z} \end{Bmatrix} + \begin{bmatrix} K* & -\alpha K_v \\ -\alpha K_v & K_v \end{bmatrix}\begin{Bmatrix} q \\ z \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix}, \tag{5}$$

where: z is the auxiliary variable introduced into the problem, called dissipation variable, and $K^* = K_v(E_0 + \alpha)$.

Generalizing equation (5) for n degrees of freedom, equation (6) may be written as:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \frac{\alpha}{\delta}\mathbf{I} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \ddot{\hat{\mathbf{z}}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\alpha\beta}{\delta}\mathbf{I} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{\hat{\mathbf{z}}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathbf{v}}(E_0 + \alpha) & -\alpha\mathbf{R} \\ -\alpha\mathbf{R}^T & \alpha\mathbf{I} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \hat{\mathbf{z}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f(t)} \\ \mathbf{0} \end{Bmatrix}, \tag{6}$$

where:

$$\mathbf{K}_{\mathbf{v}} = \mathbf{T}^T \mathbf{\Lambda} \mathbf{T}, \tag{7}$$

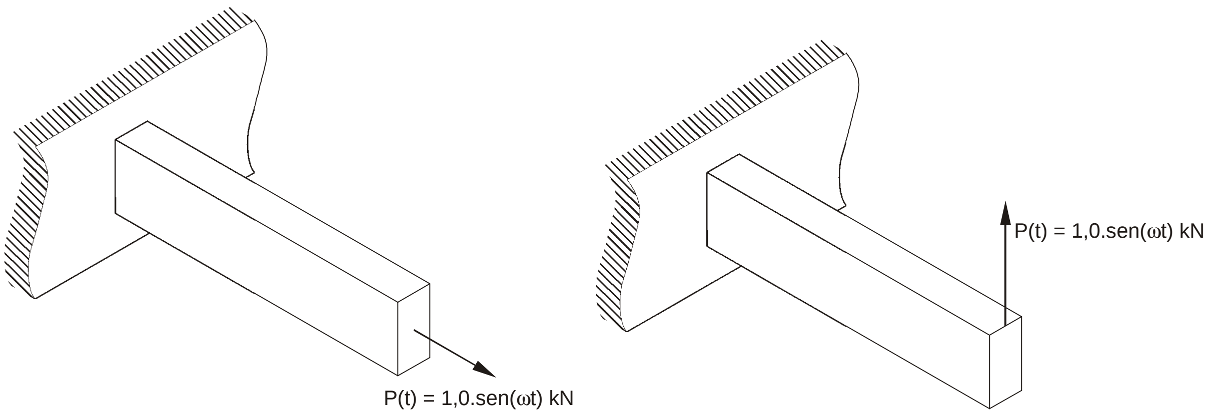
and $\mathbf{\Lambda}$ is a diagonal matrix consisting of the non-zero eigen-values of the stiffness matrix normalized with respect to the elastic modulus; $\mathbf{T}$ the matrix of vectors corresponding to the non-zero eigen-values of the matrix $1/E\,\mathbf{K}_{elástic}$; $\mathbf{R} = \mathbf{T}\mathbf{\Lambda}^{1/2}$ and $\hat{\mathbf{z}} = \mathbf{R}\mathbf{z}$.

As shown in equations (2) and (6) the number of dissipative degrees of freedom associated with viscoelastic elements depends on the number of terms used in relaxation function and the number of rigid body motions [4]. It should be noted that the greater the number of terms used to write function relaxation more accurate modeling [1].

Using equations (6) and (7), it is possible to determine stiffness, mass and damping matrices, for any kind of Finite Element.

3. VALIDATION OF GHM METHOD

In order to verify the accuracy of GHM Method, results obtained by means of classical modeling and those obtained by GHM will be analyzed. The following viscoelastic bars presented in figure 1 were used to develop the analysis. Dimensions for length, width and height are 1,00 m, 0,30 m and 0,15 m respectively.



(a) Bar with axial loading (b) Bar with transversal loading

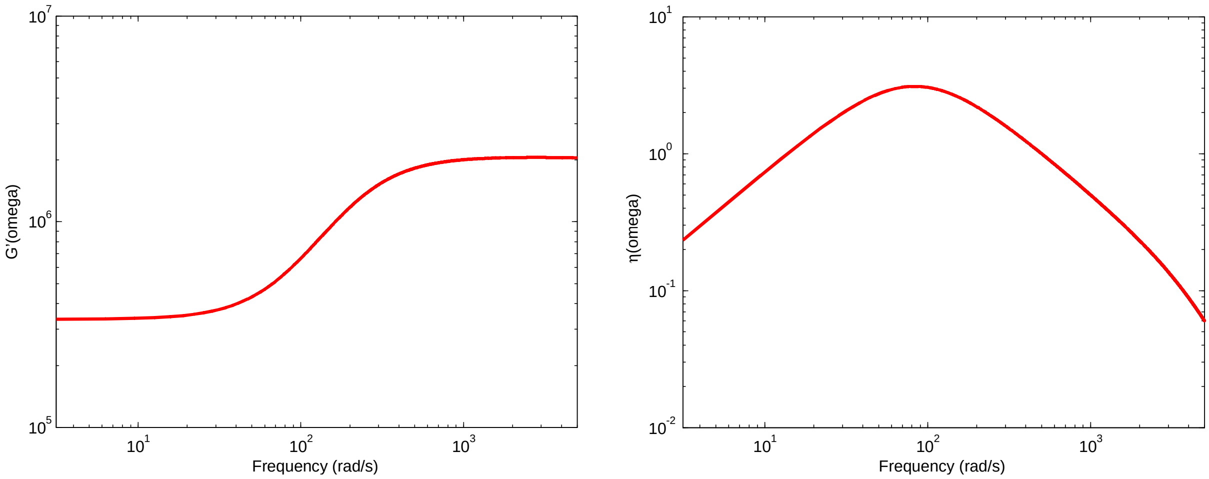
Figure 1. Bars used in GHM validation.

Using the dissipation function $h(s)$ as shown in equation (2), the storage modulus

($G'(\omega)$) and the damping factor ($\eta(\omega)$) are written as:

$$G'(\omega) = E_0 + \alpha \frac{\omega^2(\omega^2 - \delta + \beta^2)}{(\delta - \omega^2)^2 + \beta^2\omega^2}, \quad \eta(\omega) = \frac{\alpha\beta\delta\omega}{(\delta - \omega^2)^2 + \beta^2\omega^2} \frac{1}{G'(\omega)}. \tag{8}$$

In the present example, adopting E_0 =1,0MPa, α =5,0MPa, β =6,0.10³ s^{-1} and δ =1,2.10^{6}s^{-2}; the frequency dependent properties of the used viscoelastic material are ploted in figure 2. The other mechanical properties of these bars are described in Table 1.



(a) $G'(\omega)$ (b) $\eta(\omega)$

Figure 2. $G'(\omega)$ and $\eta(\omega)$ functions for viscoelastic material considered.

Table 1. Mechanical properties adopted for validation structure.

Mechanical properties	Value
Density	1120 kg/m³
Poisson's coefficient	0,25

Matrices of viscoelastic Finite Elements formulated by GHM Method are obtained with equations (6) and (7) and matrices for classical elements formulation are obtained by changing, in the correspondent elastic matrix, the elastic modulus by complex modulus.

Solutions in time domain are not trivial to classical formulation, since the complex modulus is frequency dependent. However, solutions in frequency domain, where, for example, the amplitude of horizontal displacement on free end of the bar is calculated to a harmonic loading as shown in Figure 1.a, the solution in the classical formulation becomes very simple. From the equation that express displacements for the model of figure 1 can be easily reach:

$$\mathbf{d} = \left[\mathbf{K}_{classical}(\omega) - \omega^2 \mathbf{M}_{classical}(\omega)\right]^{-1} \mathbf{f}\,, \tag{9}$$

where: $\mathbf{K}_{classical}(\omega)$ and $\mathbf{M}_{classical}(\omega)$ are, respectively, the global stiffness and mass matrix, $\mathbf{d}$ is the displacement vector and $\mathbf{f}$ is the external force vector. In the case of GHM formulation, the analogous equation is:

$$\begin{Bmatrix} \mathbf{d} \\ \mathbf{z} \end{Bmatrix} = \left[\mathbf{K}_{GHM} + i\omega \mathbf{D}_{GHM} - \omega^2 \mathbf{M}_{GHM} \right]^{-1} \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \end{Bmatrix}, \tag{10}$$

where: $\mathbf{K}_{GHM}$, $\mathbf{D}_{GHM}$ and $\mathbf{M}_{GHM}$ are, respectively, the global stiffness, damping and mass matrix.

As one can observe, the basic differences between the equations (9) (classical formulation) and (10) (GHM formulation) are:

- Matrices in equation (10) have additional rows and columns associated to dissipation dofs;
- The stiffness, damping and mass matrices in equation (10) are constant for any frequency value, unlike matrices of equation (9) are frequency dependent.

3.1. Spatial Frame Element

Discretizations with viscoelastic frame element of the studied bar are shown in figures 3 and 4. This figures show the bar modeled with five Spatial Frame Finite Elements.

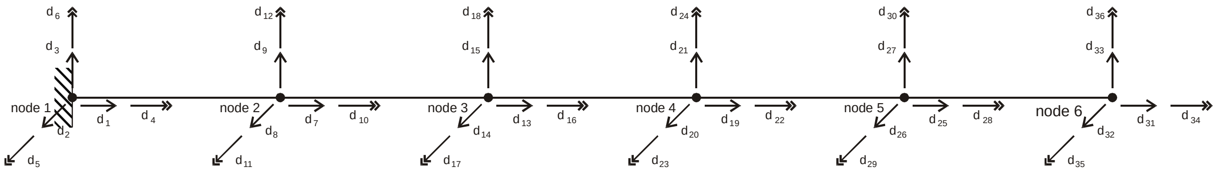


Figure 3. Bar discretized with five classical viscoelastic frame elements.

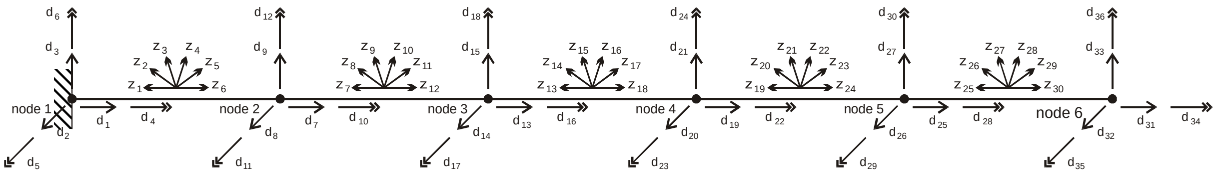
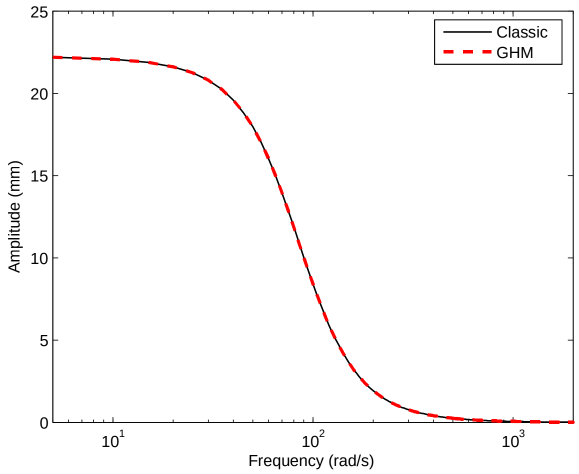
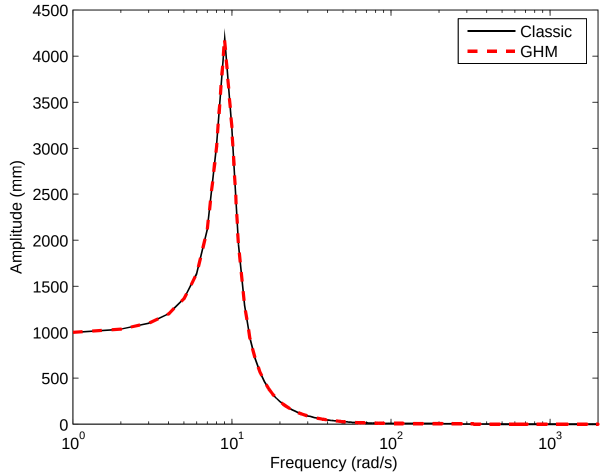


Figure 4. Bar discretized with five GHM viscoelastic frame elements.

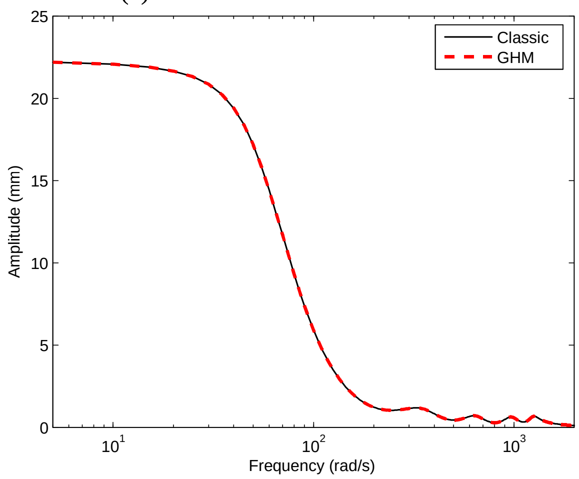
Figure 5 presents comparisons in terms of displacement amplitudes of the free extremity of the analysed bar for classical and GHM models, varing the excitation frequency from 0 to 2000 rad/s. It is clearly notice that classical and GHM solutions are practically identical.



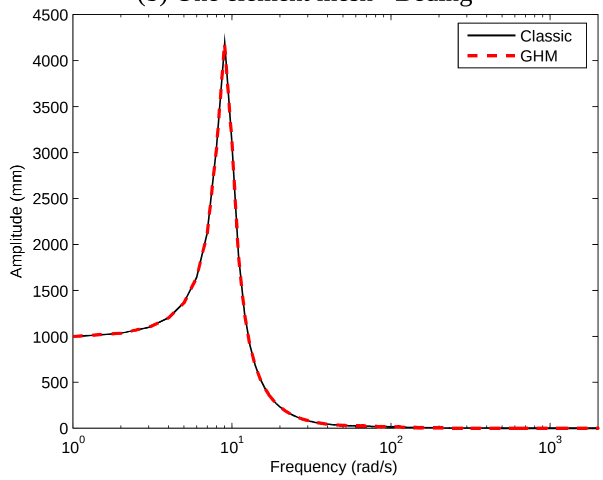
(a) One element mesh - Tension



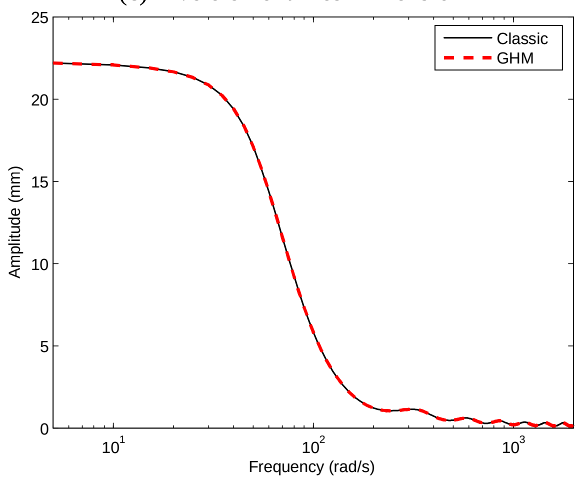
(b) One element mesh - Beding



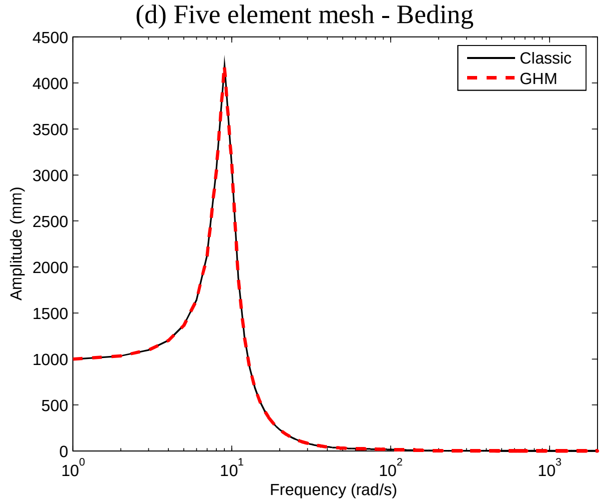
(c) Five element mesh - Tension



(d) Five element mesh - Beding



(e) Ten element mesh - Tension

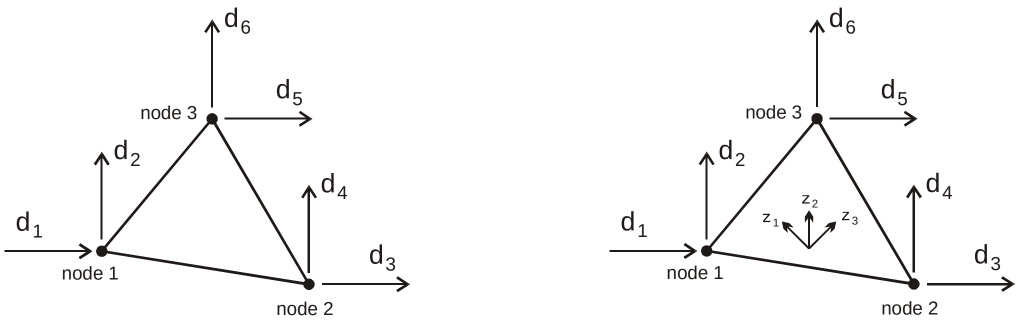


(f) Ten element mesh - Beding

Figure 5. Analysis of convergence for viscoelastic frame elements.

3.3. Constant Strain Triangular element (CST)

The classical and GHM viscoelastic CST elements used to model the bars are shown in figure 6. Obviously this kind of Finite Element is not the more appropriated, but the main goal of the present analysis is to validate GHM model.



(a) Classical viscoelastic CST element (b) GHM viscoelastic CST element

Figure 6. Viscoelastic CST elements.

An example of the bar domain discretization with GHM viscoelastic CST Finite Elements is shown in figure 7, where the domain is discretized with 16 Finite Elements.

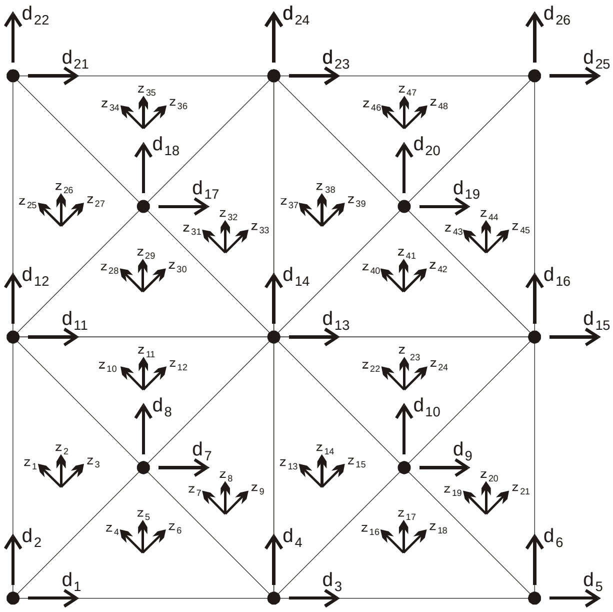
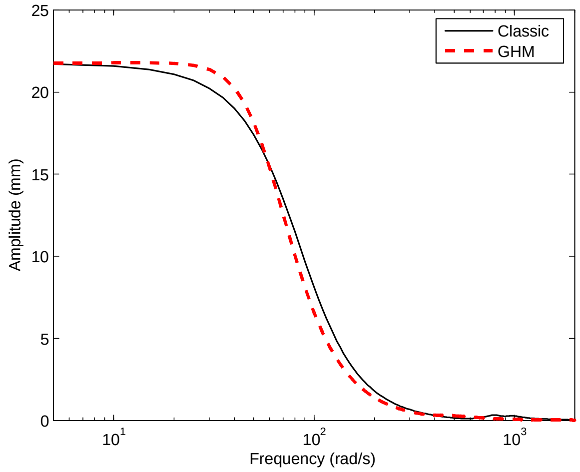
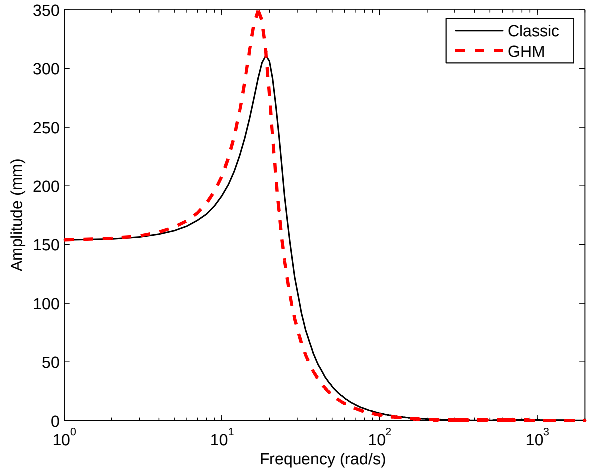


Figure 7. Bar discretized with 16 GHM viscoelastic CST elements (unscaled).

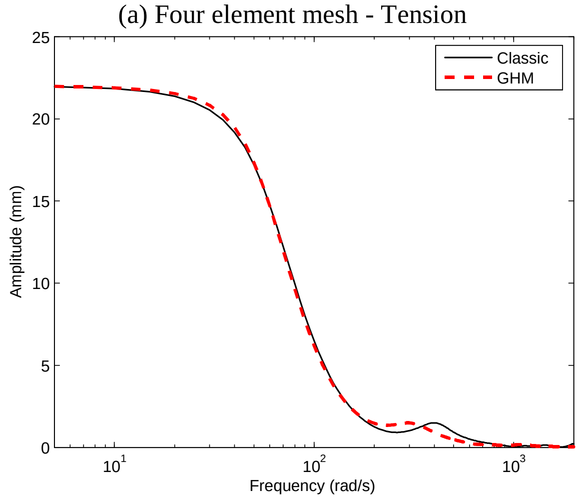
With meshes like the one showed in figure 7, one can draw the graphics in figure 8. As it was done previously, the excitation frequency range is from 0 to 2000 rad/s. In this figure, it is easy to see that classical and GHM solutions converge as the meshes are refined.



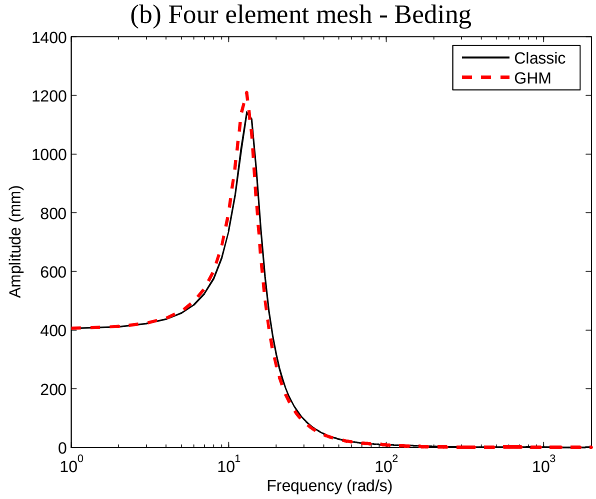
(a) Four element mesh - Tension



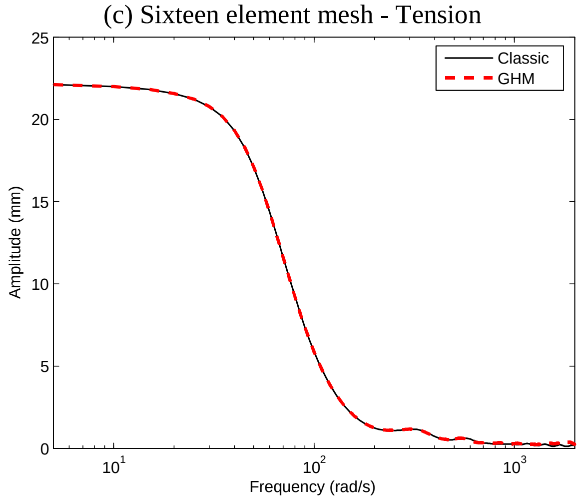
(b) Four element mesh - Beding



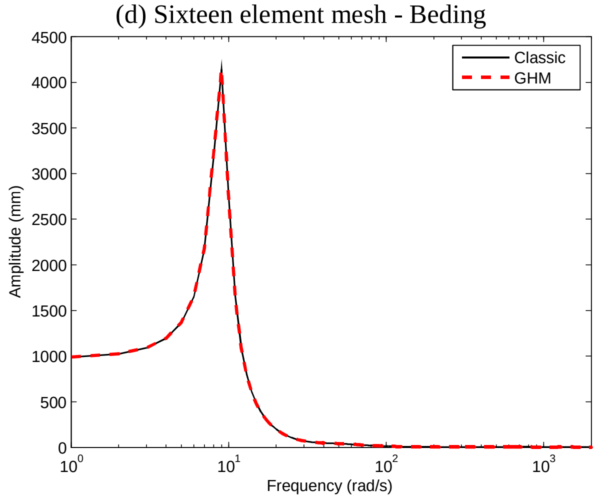
(c) Sixteen element mesh - Tension



(d) Sixteen element mesh - Beding



(e) 120 element mesh - Tension

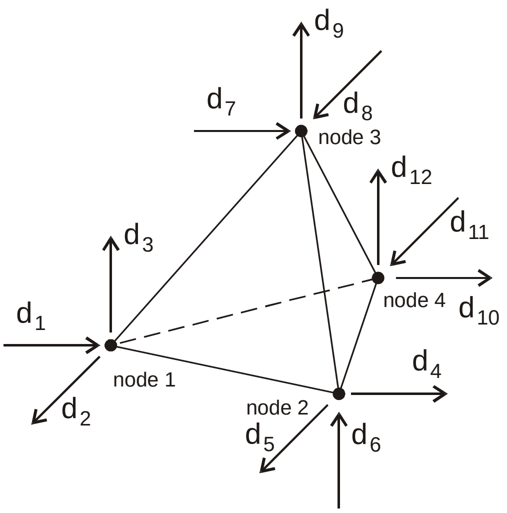


(f) 120 element mesh - Beding

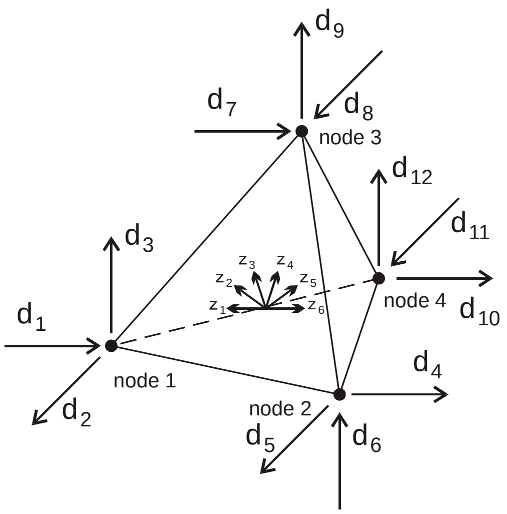
Figure 8. Analysis of convergence for viscoelastic CST elements.

3.4. Linear Tetrahedron element (T4)

The classical and GHM viscoelastic T4 elements used to model the bars are shown in figure 9.



(a) Classical viscoelastic T4 element



(b) GHM viscoelastic T4 element

Figure 9. Viscoelastic T4 elements.

In figure 10 can be seen an example of the bar domain discretization with 480 Tetrahedral Finite Elements.

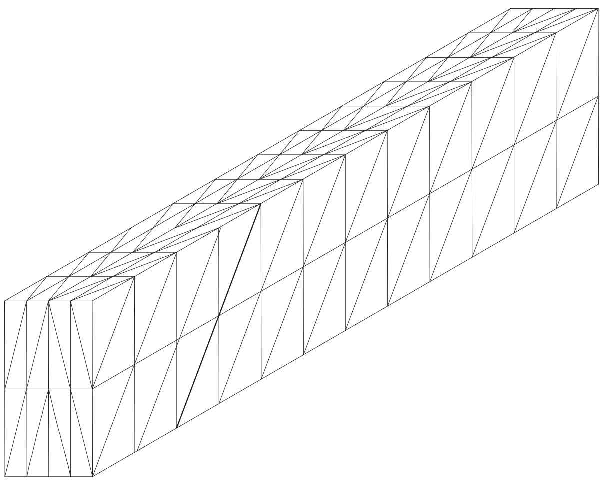
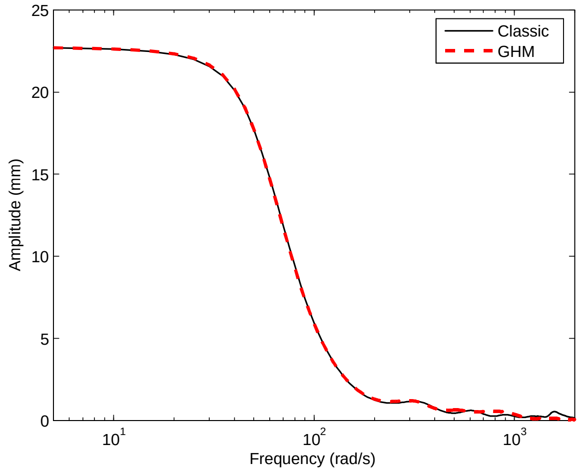
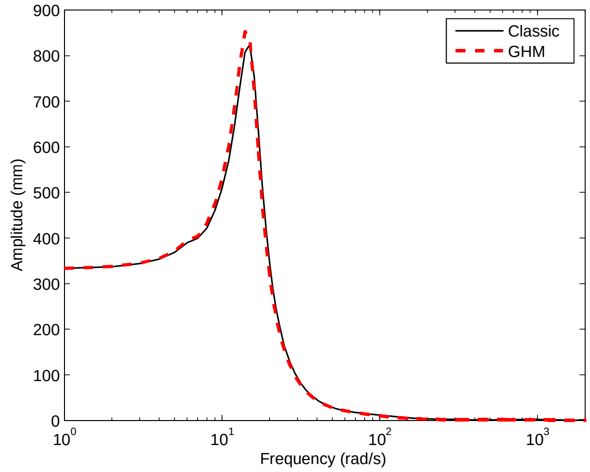


Figure 10. Bar discretized with 480 GHM viscoelastic T4 elements.

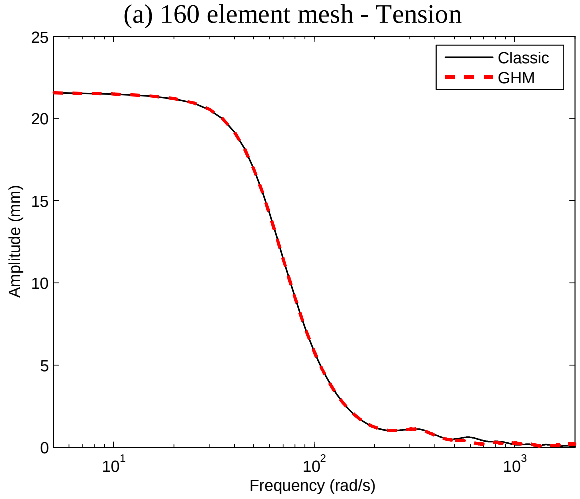
With meshes like the one plotted in figure 10, one can draw the graphics of figure 11. Once more, as the meshes are refined the classical and GHM models responses converge.



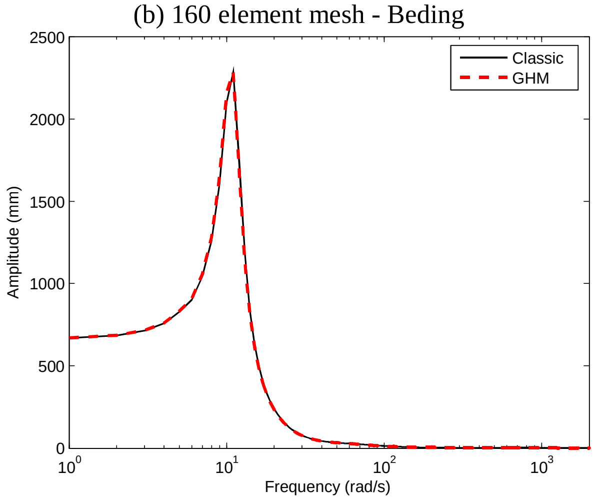
(a) 160 element mesh - Tension



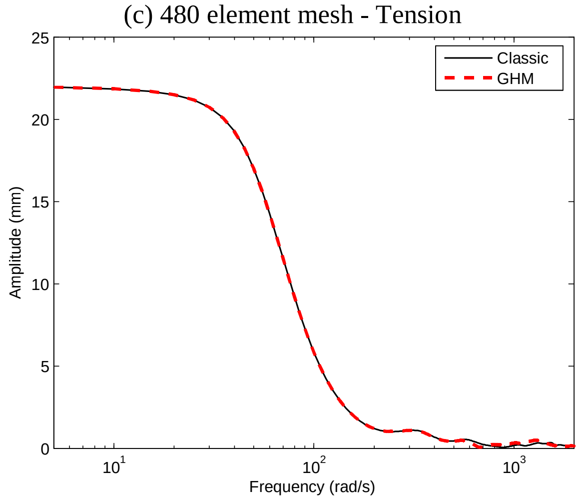
(b) 160 element mesh - Beding



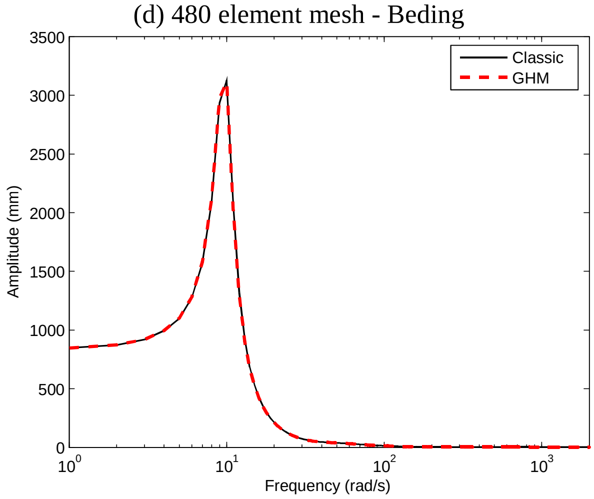
(c) 480 element mesh - Tension



(d) 480 element mesh - Beding



(e) 2880 element mesh - Tension



(f) 2880 element mesh - Beding

Figure 11. Analysis of convergence for viscoelastic T4 elements.

4. SANDWICH VISCOELASTIC MODEL

Once seen that the GHM model is able to produce results compatible with those obtained by classical formulation, it will be evaluated the damping capacity of a beam with annular section through computer simulation by GHM method.

The bar consists in a cantilever elastic beam with annular cross section, working as base structure and was applied in this structure two viscoelastic sandwich damping treatments: one with treatment on sectors 1 and 2 and another with treatment on four sectors, as shown in figure 12. Dimensions are presented in table 2. A similar example was analyzed by Borges [5].

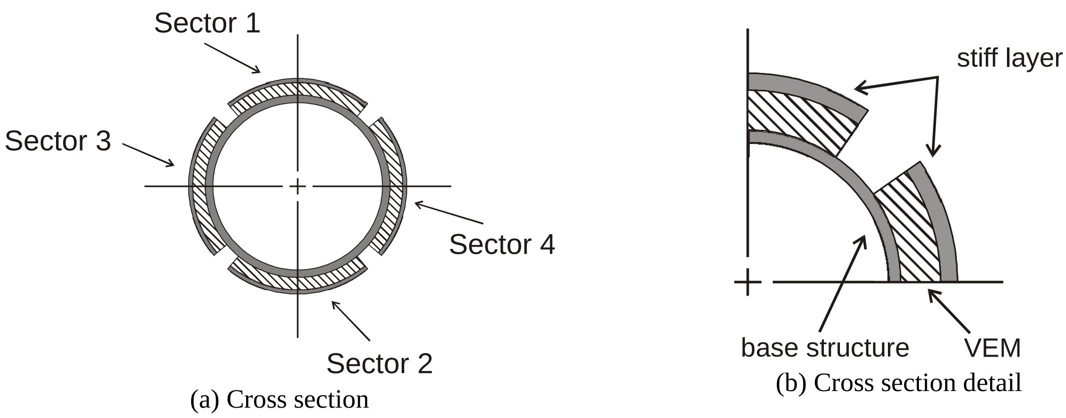


Figure 12. Cross sections of the analyzed beams.

Table 2. Mechanical properties adopted for the analyzed structure.

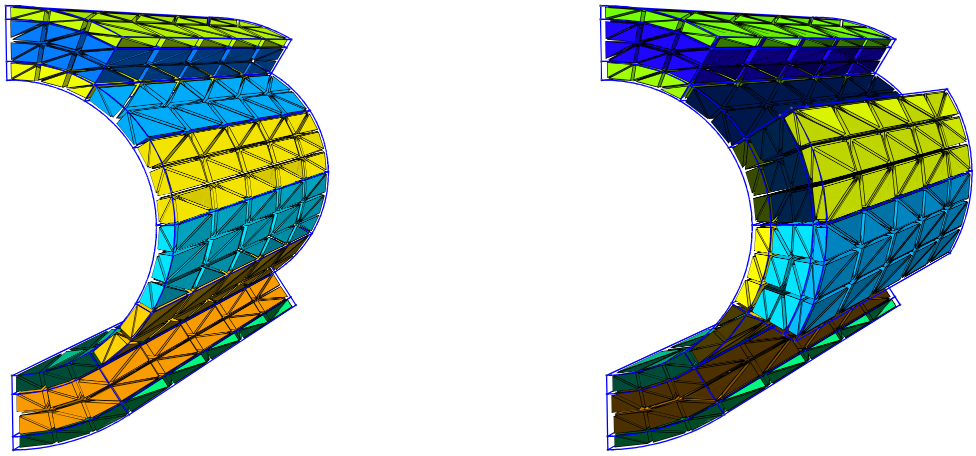
Layer	External radius (mm)	Thickness (mm)
Base	10	1
VEM	12	2
Restriction	13	1

The mechanical properties of elastic layers considered and viscoelastic layer are listed in table 3.

Table 3. Mechanical properties adopted for sandwich structure.

Mechanical properties	Elastic layer	Viscoelastic layer
Density	8794 kg/m³	795 kg/m³
Poisson's coefficient	0,33	0,49
Elastic Modulus	109,6 GPa	-
E_0	-	1,17 MPa
α	-	2,21 MPa
β	-	143000 s^{-1}
δ	-	$8{,}57.10^6$ s^{-2}

The structure was simulated under the action of a hammer impact at 20 cm from cantilever and, at same point, was observed the transversal displacement along the time. The domain of structure was dicretized with linear tetrahedral element meshes, as shown in figure 13. The observed displacements are plotted in figure 13. In this figure the efficiency of the viscoelastic sandwich treatment may be easily observed. Damping obtained with treatment on four sectors is practically identical to the one reached with the two sectors treatment.



(a) Beam with damping treatment on sectors 1 and 2 (b) Beam with damping treatment on four sectors.

Figure 13. Structural Finite Element discretization.

With these mesh ware simulated the base beam with the two damping treatment configurations and its time response are shown at figure 14.

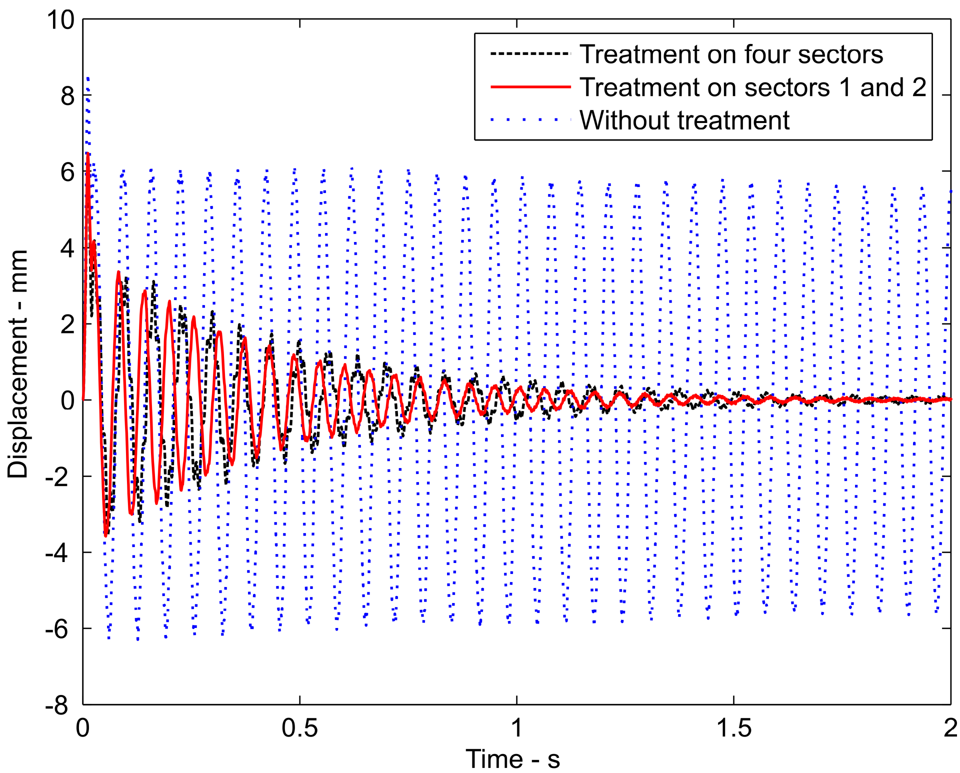


Figure 14. Time response for the beam with damping treatment.

5. CONCLUSIONS

This study evaluated the GHM method on computational modeling of viscoelastic materials acting as structural vibration dampers. The GHM was implemented in a finite element code and was observed that GHM produces results nearly to those with classical formulation when using a convenient mesh refinement. This fact allows the validation the GHM method, since this behavior occurred for all types of developed finite elements.

Analyzing the obtained responses for the cantilever beam, one can be observe that the adopted damping treatment, in both cases, considerably increased the damping ratio of the structure when compared with the elastic structure without damping treatment. This structural behavior allows the conclusion that viscoelastic materials may be used to reduce vibration oscillations.

Acknowledgements

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6. REFERENCES

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