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DYNAMIC BEHAVIOR OF FRAMED STRUCTURES WITH AN ELASTIC INTERNAL HINGE

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Abstract. *The study of the dynamic properties of framed structures is extremely important in the field of structural engineering. In this paper the first natural frequencies of transverse vibration of frames are determined. The elastic structural system consists of a beam supported by a column. The presence of an internal hinge located in different positions of the beam is considered. The hinge is elastically restrained against rotation and translation. Attention is given to the way in which supports are modeled. It is known that ideal supports used in many structural models do not fit exactly with the real supports. Here the column is considered not rigidly connected to the foundation. The displacement of the component elements are assumed to be described by the theory of Euler-Bernoulli. The governing equations of the system, together with the boundary and compatibility conditions are obtained using the technique of variational calculus. Applying the method of separation of variables, the exact values of the natural frequencies of the model are obtained. Results are given for different cases, which arise from combining different magnitudes in the internal elastic hinge. These results are compared with those obtained using the finite element method, and in particular cases they are also compared with values available in the literature. Finally, an experimental device allows verifying the procedure.*

Keywords: *Vibration, Frame, Modal shapes, Elastically supported, Hinge.*

1. INTRODUCTION

As Laura et al. pointed in [1], many excellent books and technical papers deal with vibrating frames, such as Warburton [2], Blevins [3], Clough and Penzien [4], Timoshenko and Young [5], Karnosky and Lebed [6], among others. The title problem is of importance in practically all fields of engineering given that frame structures resist by virtue of its geometry, ranging from big scale like bridges and buildings placed in seismically active regions, to micro-frames used in modern electronic equipment subject to vibratory environments.

Many researchers have analyzed the vibration of frames. Lin and Ro [7] proposed a hybrid analytical/numerical method to do dynamic analysis of planar serial-frame structures. Wu [8] presented an elastic-and-rigid-combined beam element to determine the dynamic characteristics of a two-dimensional frame composed of any number of beam segments. In his paper Mei [9] considered the vibration in multi-story planar frame structures from the wave vibration standpoint. Laura et al. [1] determined the in-plane vibration of frames with concentrated masses attached and Filipich and Laura [10] analyzed in-plane vibrations of portal frames with elastically restrained ends. An approximate solution is obtained by means of a variational method.

In the particular case of L-frame structures, early studies have been done by Bang [11], Grügöze[12] and Oguamanam et al. [13]. In 2003 Heppler et al. [14] extend the previous paper [13] by relaxing the restrictions on the motion of the open frame. In 2005 Abarracín and Grossi [15] determined natural frequencies and mode shapes of elastically restrained L-frames. They applied the separation of variables method for the determination of the exact eigenfrequencies and mode shapes and calculated the eigenvalues numerically by applying the Newton method strategy to the corresponding frequency equation. Lee and Ng [16] used a formulation by the Rayleigh–Ritz method together with the introduction of artificial linear and torsional springs for computing the natural frequencies and modes for the in-plane vibrations of complex planar frame structures.

The presence of an internal hinge in beams has been treated in several papers by Wang and Wang [17], Lee et al. [18], Chang et al. [19], Grossi and Quintana [20], Quintana et al. [21]. Here, we deal with the vibration of L-frames assuming an internal hinge in different positions of the horizontal part of the frame. The two members of the L-shaped geometry are joined at right angle, with the end of one of them clamped and the end of the other elastically restrained. Figure 1 depicts the structure under study.

The presence of the hinge allows the simulation of a crack model as developed by Chondros et al. [22], and some numerical experiments are included. It is assumed that the beams are adequately modeled using Euler -Bernoulli theory and the method of separation of variables is used to obtain the exact values of the natural frequencies of the model.

Numerical results are obtained for different magnitudes of the internal elastic hinge and the boundary conditions by means of MATHEMATICA [23] code. These results are compared with results obtained with the finite element method in ALGOR [24]. Additionally, some particular cases are compared with values available in the literature and experimental results.

2. THEORY AND FORMULATIONS

The L-frame under study has elastic restrain and clamped ends as shown in Figure 1. The structure is composed of three members, a vertical beam (column) of length l_1 and two consecutive horizontal beams of lengths l_2 and l_3 respectively (see Figure 1). At the intermediate point, the horizontal beams have an internal hinge elastically restrained against rotation between them and the hinge is externally restrained by translational and rotational springs.

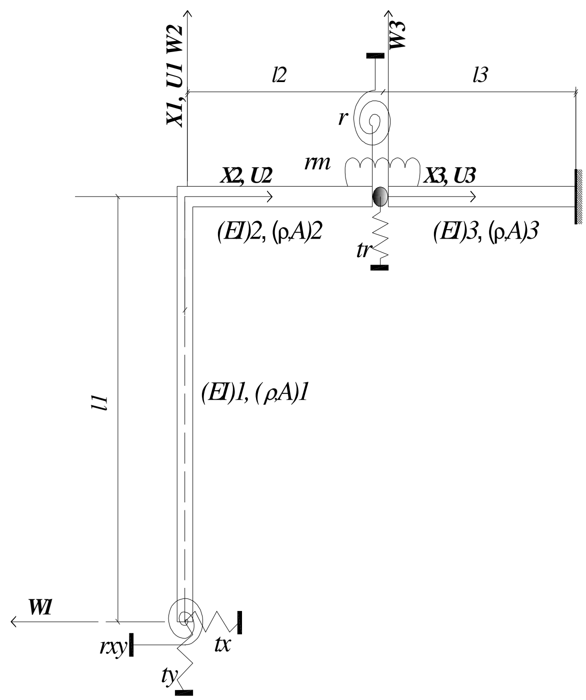

Figure 1. L-frame structure.

The rotational restraints are characterized by the spring constants *rxy, r* and *rm.* and the translational restraints by the spring constants *tx, ty* and *tr*. The behaviors of the individual members of the frames are assumed to be governed by Euler-Bernulli beam theory and the axial deformation effects are also included.

For this case let us consider a three-element frame, i=1, 2, 3. The transverse and axial displacements are described by the functions:

$$W_i = W_i(X_i, t);\ \ i = 1,2,3\,.$$

$$U_i = U_i(X_i, t),\ \ i = 1,2,3\,.$$

The kinetic energy of the mechanical system, at time t, is given by:

$$\begin{aligned} T = &\frac{1}{2}\int_0^{l_1}(\rho A)_1\left[\left(\frac{\partial W_1}{\partial t}(X_1,t)\right)^2 + \left(\frac{\partial U_1}{\partial t}(X_1,t)\right)^2\right]dX \\ &+\frac{1}{2}\int_0^{l_2}(\rho A)_2\left[\left(\frac{\partial W_2}{\partial t}(X_2,t)\right)^2 + \left(\frac{\partial U_2}{\partial t}(X_2,t)\right)^2\right]dX \\ &+\frac{1}{2}\int_0^{l_3}(\rho A)_3\left[\left(\frac{\partial W_3}{\partial t}(X_3,t)\right)^2 + \left(\frac{\partial U_3}{\partial t}(X_3,t)\right)^2\right]dX; \end{aligned} \tag{1}$$

where $(\rho A)_i$ denotes the mass per unit length of the members of the frame.

On the other hand, the potential energy of the mechanical system is given by:

$$
\begin{aligned}
U =& \frac{1}{2}\int_0^{l_1}\left[(EI)_1\left(\frac{\partial^2 W_1}{\partial X_1^2}(X_1,t)\right)^2 + (EA)_1\left(\frac{\partial U_1}{\partial X_1}(X_1,t)\right)^2 \right] dX_1 \\
&+\frac{1}{2}\int_0^{l_2}\left[(EI)_2\left(\frac{\partial^2 W_2}{\partial X_2^2}(X_2,t)\right)^2 + (EA)_2\left(\frac{\partial U_2}{\partial X_2}(X_2,t)\right)^2 \right] dX_2 \\
&+\frac{1}{2}\int_0^{l_3}\left[(EI)_3\left(\frac{\partial^2 W_3}{\partial X_3^2}(X_3,t)\right)^2 + (EA)_3\left(\frac{\partial U_3}{\partial X_3}(X_3,t)\right)^2 \right] dX_3 \\
&+\frac{r}{2}\left(\frac{\partial W_2}{\partial X_2}(l_2,t)\right)^2 + \frac{tr}{2}\left(W_2(l_2,t)\right)^2 + rm\left(\frac{\partial W_2(l_2,t)}{\partial X_2} - \frac{\partial W_3(0,t)}{\partial X_3}\right)^2 + \\
&\frac{rxy}{2}\left(\frac{\partial W_1}{\partial X_1}(0,t)\right)^2 + \frac{tx}{2}\left(U_1(0,t)\right)^2 + \frac{ty}{2}\left(W_1(0,t)\right)^2;
\end{aligned}
\tag{2}
$$

where l_i, $(\rho A)_i$ and $(EI)_i$ denote the length, the flexural rigidity and the axial rigidity that correspond to each member *i* of the frame.

It is convenient to introduce dimensionless variables:

$$x_i = X_i / l_i\ ;\quad i = 1,2,3;\ \text{with}\ x_i \in [0,1]\ \forall i = 1,2,3.$$

W_i and U_i may be expressed in terms of the dimensionless coordinates:

$$w_i = \frac{W_i(x,t)}{l_i}, i = 1,2,3,\quad x \in [0,1],\quad u_i = \frac{U_i(x,t)}{l_i}, i = 1,2,3,\quad x \in [0,1];$$

and it is useful to define the following dimensionless parameters:

$$v_{li} = \frac{l_i}{l},\ v_{EIi} = \frac{(EI)_i}{EI},\ v_{\rho Ai} = \frac{(\rho A)_i}{\rho A},\quad i = 1,2,3;$$

$$R = \frac{r \times l}{EI};\ Rm = \frac{rm \times l}{EI};\ Tr = \frac{tr \times l^3}{EI};$$

$$Rxy = \frac{rxy \times l}{EI}; Tx = \frac{tx \times l^3}{EI}; Ty = \frac{ty \times l^3}{EI};$$

with $l = l_1;\ E = E_1;\ \rho = \rho_1;\ A = A_1;\ I = I_1;$

where *l*, *(ρA)* and *(EI)* correspond to the characteristics of the member 1 of the frame.

2. 1 Expression of the functional

Hamilton's principle requires that between times t_a and t_b, at which the positions of the mechanical system are known, the system should execute a motion which makes the functional stationary on the space of admissible functions

$$J\left(\boldsymbol{w}\right)=\int_{t_a}^{t_b}\left(T-U\right)\,dt\,,$$

where $\boldsymbol{w}=(w_i,\ u_i)$.

The stationary condition required by Hamilton's principle is given by:

$$\delta J\left(w,v\right)=0,\forall v\in D_0\,;$$

where D_0 is a space of admissible direction at $\boldsymbol{w}$ for the domain D of the functional.

Then, the expression of the functional is:

$$\begin{aligned}
J(w)=\frac{1}{2}\Bigg\{\int_{t_1}^{t_2} B_1\int_0^1&\left[\left(\frac{\partial w_1}{\partial t}(x_1,t)\right)^2+\left(\frac{\partial u_1}{\partial t}(x_1,t)\right)^2\right]dx_2\\
&+B_2\int_0^1\left[\left(\frac{\partial w_2}{\partial t}(x_2,t)\right)^2+\left(\frac{\partial u_2}{\partial t}(x_2,t\right)^2\right]dx_2\\
&+B_3\int_0^1\left[\left(\frac{\partial w_3}{\partial t}(x_3,t)\right)^2+\left(\frac{\partial u_3}{\partial t}(x_3,t)\right)^2\right]dx_3\Bigg\}dt-\\
-\frac{1}{2}\Bigg\{\int_{t_1}^{t_2} C_1\int_0^1&\left[\left(\frac{\partial^2 w_1}{\partial {x_1}^2}(x_1,t)\right)^2+\left(\frac{\partial w_1}{\partial x_1}(x_1,t)\right)^2\right]dx_1\\
&+C_2\int_0^1\left[\left(\frac{\partial^2 w_2}{\partial {x_2}^2}(x_2,t)\right)^2+\left(\frac{\partial u_2}{\partial x_2}(x_2,t)\right)^2\right]dx_2\qquad;\\
&+C_3\int_0^1\left[\left(\frac{\partial^2 w_3}{\partial {x_3}^2}(x_3,t)\right)^2+\left(\frac{\partial u_3}{\partial x_3}(x_3,t)\right)^2\right]dx_3\Bigg\}dt-\\
-\frac{1}{2}\Bigg[C_2\,R\int_{t_0}^{t_1}&\left(\frac{\partial w_2}{\partial x_2}(1,t)\right)^2 dt+D_2\,Tr\int_{t_0}^{t_1}w_2^2(1,t)\,dt\\
&+C_2\,R_m\int_{t_0}^{t_1}\left(\frac{\partial w_2}{\partial x_2}(1,t)-\frac{\partial w_3}{\partial x_3}(0,t)\right)^2 dt+\\
&C_1\,R_{xy}\int_{t_0}^{t_1}\left(\frac{\partial w_1}{\partial x_1}(0,t)\right)^2 dt+D_1\,Tx\int_{t_0}^{t_1}u_1\left(0,t\right)dt+\\
&D_1\,T_y\int_{t_0}^{t_1}w_1\left(0,t\right)dt\ \Bigg]
\end{aligned}\qquad(3)$$

where

$$B_i = \rho Al \times v_{\rho Ai} \times v_{li},\ C_i = \frac{EI}{l^3} \times \frac{v_{EIi}}{(v_{li})^3},\ D_i = \frac{EA}{l} \times \frac{v_{EAi}}{v_{li}};\ i = 1,2,3\,.$$

Taking into account the boundary conditions at the ends, the compatibility and equilibrium conditions at the joints between column and beam, and the compatibility and equilibrium conditions at the two horizontal beams, and applying the procedure of calculus of variations in Eq. 3, the following boundary and eigenvalue problem is obtained:

$$\frac{\partial^4 w_1}{\partial x_1{}^4}(x_1,t) + k_1^4 \frac{\partial^2 w_1}{\partial t^2}(x_1,t) = 0, \quad k_1^4 = a^4 \frac{(\rho A)_1}{(\rho Al)} \frac{EI}{(EI)_1} \left(\frac{l_1}{l}\right)^4; \tag{4}$$

$$\frac{\partial^2 u_1}{\partial x_1{}^2}(x_1,t) - p_1^2 \frac{\partial^2 u_1}{\partial t^2}(x_1,t) = 0, \quad p_1^2 = a^4 \frac{(\rho A)_1}{(\rho Al)} \frac{EI}{(EI)_1} \frac{I}{Al^2} \left(\frac{l_1}{l}\right)^2; \tag{5}$$

$$\frac{\partial^4 w_2}{\partial x_2{}^4}(x_2,t) + k_2^4 \frac{\partial^2 w_2}{\partial t^2}(x_2,t) = 0, \quad k_2^4 = a^4 \frac{(\rho A)_2}{(\rho Al)} \frac{EI}{(EI)_2} \left(\frac{l_2}{l}\right)^4; \tag{6}$$

$$\frac{\partial^2 u_2}{\partial x_2{}^2}(x_2,t) - p_2^2 \frac{\partial^2 u_2}{\partial t^2}(x_2,t) = 0, \quad p_2^2 = a^4 \frac{(\rho A)_2}{(\rho Al)} \frac{EI}{(EI)_2} \frac{I}{Al^2} \left(\frac{l_2}{l}\right)^2; \tag{7}$$

$$\frac{\partial^4 w_3}{\partial x_3{}^4}(x_3,t) + k_3^4 \frac{\partial^2 w_3}{\partial t^2}(x_3,t) = 0, \quad k_3^4 = a^4 \frac{(\rho A)_3}{(\rho Al)} \frac{EI}{(EI)_3} \left(\frac{l_3}{l}\right)^4; \tag{8}$$

$$\frac{\partial^2 u_3}{\partial x_3{}^2}(x_3,t) - p_3^2 \frac{\partial^2 u_2}{\partial t^2}(x_2,t) = 0, \quad p_3^2 = a^4 \frac{(\rho A)_3}{(\rho Al)} \frac{EI}{(EI)_3} \frac{I}{Al^2} \left(\frac{l_3}{l}\right)^2; \tag{9}$$

where $a^4 = (\rho A / EI) l^4$.

$$EA_1 \frac{\partial u_1}{\partial x_1}(0,t) = -tx \times u_1(0,t); \tag{10}$$

$$Rxy \frac{\partial w_1}{\partial x_1}(0,t) = \frac{\partial^2 w_1}{\partial x^2}(0,t); \tag{11}$$

$$\frac{\partial^3 w_1}{\partial x^3}(0,t) = ty \times w_1; \tag{12}$$

$$w_1(1,t) = -u_2(0,t), \tag{13}$$

$$u_1(1,t) = w_2(0,t) \tag{14}$$

$$\frac{\partial w_1}{\partial x}(1,t) = \left(\frac{l_1}{l_2}\right)\frac{\partial w_2}{\partial x}(0,t) \tag{15}$$

$$\frac{EI_1}{l_1^2}\frac{\partial^2 w_1}{\partial x}(1,t) = \frac{EI_2}{l_2^2}\frac{\partial^2 w_2}{\partial x}(0,t) \tag{16}$$

$$EI_1\frac{\partial^3 w_1}{\partial x}(1,t) = -EA_2\frac{\partial u_2}{\partial x}(0,t)\,; \tag{17}$$

$$EI_2\frac{\partial^3 w_2}{\partial x}(0,t) = EA_1\frac{\partial u_1}{\partial x}(1,t)\,; \tag{18}$$

$$u_2(1,t) = u_3(0,t)\,; \tag{19}$$

$$w_2(1,t) = w_3(0,t)\,; \tag{20}$$

$$Rm\left(\frac{\partial w_2}{\partial x_2}(1,t) - \frac{\partial w_3}{\partial x_3}(0,t)\right) - R\frac{\partial w_2}{\partial x_2}(1,t) = \frac{\partial^2 w_2}{\partial {x_2}^2}(1,t)\,; \tag{21}$$

$$Rm\left(\frac{\partial w_2}{\partial x_2}(1,t) - \frac{\partial w_3}{\partial x_3}(0,t)\right) = \frac{\partial^2 w_3}{\partial {x_3}^2}(0,t)\,; \tag{22}$$

$$R\frac{\partial w_2}{\partial x_2}(1,t) = \frac{\partial^2 w_2}{\partial {x_2}^2}(1,t) - \frac{\partial^2 w_3}{\partial {x_3}^2}(0,t)\,; \tag{23}$$

$$\left(\frac{\partial^3 w_2}{\partial {x_2}^3}(1,t) - \frac{\partial^3 w_3}{\partial {x_3}^3}(0,t)\right) = Tr\ w_2(1,t)\,; \tag{24}$$

$$\frac{\partial w_3}{\partial x}(1,t) = 0\,; \tag{25}$$

$$w_3(1,t) = 0\,; \tag{26}$$

$$u_3(1,t) = 0\,; \tag{27}$$

2.2. Determination of the exact solution

Using the well-known separation of variables method, solution of Eqs. (4) to (9) are assumed to be of the form:

$$w_1(x_1,t)=\sum_{n=1}^{\infty} w_{1n}(x_1)T(t); \qquad u_1(x_1,t)=\sum_{n=1}^{\infty} u_{1n}(x_1)T(t); \tag{28}$$

$$w_2(x_2,t)=\sum_{n=1}^{\infty} w_{2n}(x_2)T(t); \qquad u_2(x_2,t)=\sum_{n=1}^{\infty} u_{2n}(x_2)T(t); \tag{29}$$

$$w_3(x_3,t)=\sum_{n=1}^{\infty} w_{3n}(x_3)T(t); \qquad u_3(x_3,t)=\sum_{n=1}^{\infty} u_{3n}(x_3)T(t). \tag{30}$$

The functions w_1, w_2, w_3, u_1, u_2 and u_3 represent the corresponding transverse and longitudinal modes of natural vibration of each member and are given by:

$$w_{1n}(x_1)=c_1\cosh(\lambda\alpha_1 x_1)+c_2 senh(\lambda\alpha_1 x_1)+c_3\cos(\lambda\alpha_1 x_1)+c_4 sen(\lambda\alpha_1 x_1); \tag{31}$$

$$u_{1n}(x_1)=c_5\cos\left(\lambda^2\beta_1 x_1\right)+c_6 sen\left(\lambda^2\beta_1 x_1\right); \tag{32}$$

$$w_{2n}(x_2)=c_7\cosh(\lambda\alpha_2 x_2)+c_8 senh(\lambda\alpha_2 x_2)+c_9\cos(\lambda\alpha_2 x_2)+c_{10} sen(\lambda\alpha_2 x_2); \tag{33}$$

$$u_{2n}(x_2)=c_{11}\cos\left(\lambda^2\beta_2 x_2\right)+c_{12} sen\left(\lambda^2\beta_2 x_2\right) \tag{34}$$

$$w_{3n}(x_2)=c_{13}\cosh(\lambda\alpha_3 x_2)+c_{14} senh(\lambda\alpha_3 x_2)+c_{15}\cos(\lambda\alpha_3 x_2)+c_{16} sen(\lambda\alpha_3 x_2) \tag{35}$$

$$u_{3n}(x_3)=c_{17}\cos\left(\lambda^2\beta_3 x_3\right)+c_{18} sen\left(\lambda^2\beta_3 x_3\right) \tag{36}$$

where:

$$\alpha_i=\sqrt[4]{\frac{v_{\rho Ai}}{v_{EIi}}}\;v_{li};\quad \beta_i=\sqrt[4]{\frac{v_{\rho Ai}}{v_{EAi}}\frac{I}{Al^2}}\;v_{li};\quad i=1,2,3.$$

Finally the natural frequency coefficients of the vibrating system in the adimentional form is expressed:

$$\lambda = \lambda_n = a\sqrt[4]{\omega_n^2}\ ;\ a^4 = (\rho A/EI)l^4 .$$

2.3. Finite element method

Numerical examples are solved by means of the finite element method, using the software ALGOR 23.1 [24]. The column and the beam are divided into 100 beams elements respectively, each beam element with three degrees of freedom.

The internal hinge elastically restrained was modeled by a very small beam element, 300 times smaller than the length of the beam. The moment of inertia of the section was varied in order to obtain stiffness values that are equivalent to the stiffness constants of the spring connecting the two sections of the lintel.

2.4. Experimental model

An experimental device was built to verify the analytical and numerical models developed. A frame of two uniform members of equal length l (Figure 2) was tested under different boundary conditions (clamped and free) at the lower end of its vertical member ($x_1 = 0$). The other end ($x_3 = 1$) is clamped. The presence of internal hinge is not considered.
The magnitudes of the frame are: l=0.50 m, A= 4.064×10^{-5} m^2, I= 3.468×10^{-11}m^4 and E=2.1×10^{6}kg/cm^2.

Figure 2. Experimental set-up for clamped-free model.

In order to measure the natural frequencies, an optical proximity sensor was used.

Figure 3 shows the spectrum of the first natural frequency of the frame clamped-clamped and clamped-free respectively.

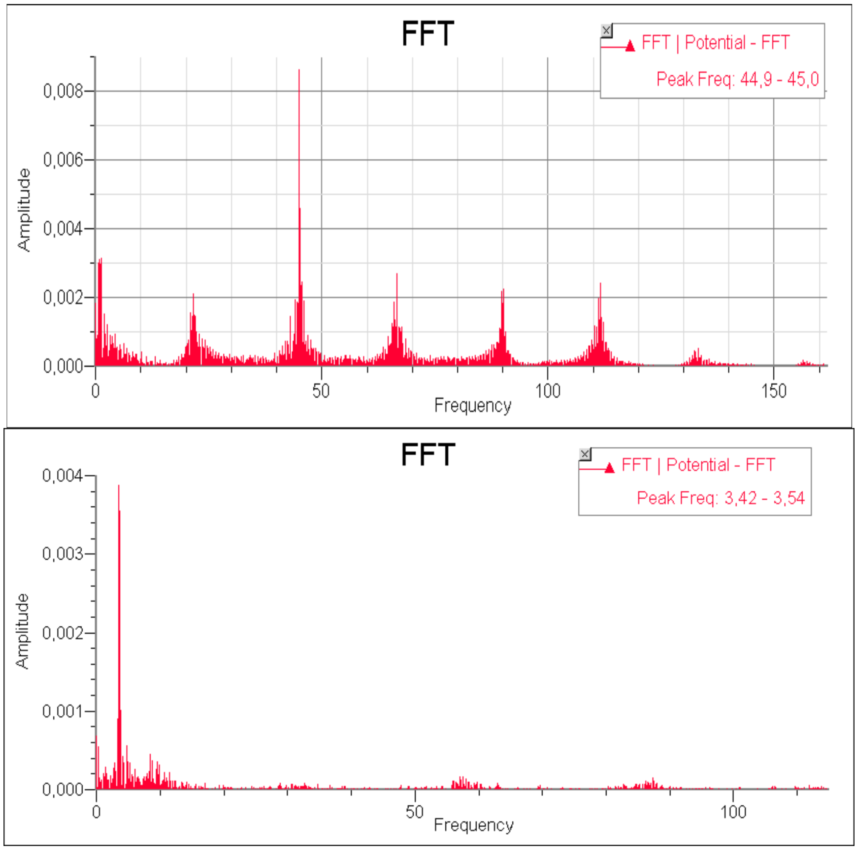

Figure 3. Spectrum of the first natural frequency.

3. NUMERICAL RESULTS

3.1. Validation of the model

The proposed approach allows solving many special cases. Some numerical examples are developed. In all of them it is supposed that the whole frame is of the same material and have equal stiffness:

$$v_{EIi} = \frac{(EI)_i}{EI} = 1; \; v_{\rho Ai} = \frac{(\rho A)_i}{\rho A} = 1; \; i = 1,2,3 .$$

$$v_{EIi} = \frac{(EI)_i}{EI} = 1, \; v_{\rho Ai} = \frac{(\rho A)_i}{\rho A} = 1, \quad i = 1,2,3$$

The relation between lengths is constant, $l_1 = l_2 + l_3$, while the relative lengths of the lintel, l_2 and l_3 may change.

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Table 1 presents the first three coefficients of natural frequency of vibration of a frame clamped-clamped without internal hinge. Both members of the frame have the same length *l*. The values without internal hinge were obtained by the finite element method (FEM) [24] and were compared with those calculated by Albarracín and Grossi [15].

Table 1. Frequency coefficients: $\lambda_i = a\sqrt[4]{\omega_n^2}$, $a^4 = (\rho A/EI)\, l^4$, clamped-clamped frame without internal hinge

	λ_1	λ_2	λ_3
Analytical	3.9234	4.6605	7.0430
FEM	3.9248	4.7239	7.0574
[15]	3.9225	4.7142	7.0376

Figure 4 shows the first three mode shapes for an L-frame clamped-clamped without internal hinge. They correspond to the frequency coefficients presented in Table 1.

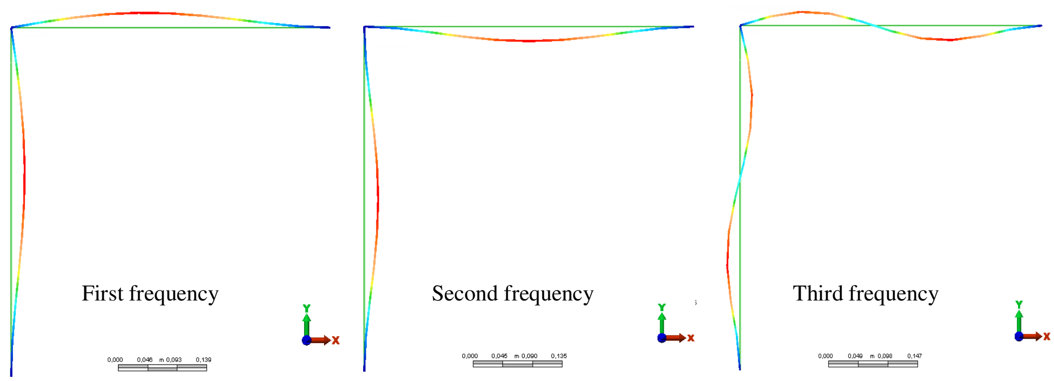

Figure 4. Mode shapes of clamped-clamped frame without internal hinge.

Table 2 shows the frequency parameters of an L-frame without internal hinge, when the spring constants at the outer end T_x, T_y and R_{xy} take three different values: 0, 10000 and $\rightarrow\infty$.

Table 2. Frequency coefficients for L-frame without hinge ($T_x = T_y = R_{xy}$)

$T_x = T_y = R_{xy}$	(f_1)	λ_1			λ_2		λ_3	
	Experimental $\lambda_1 = a^4\sqrt{\omega_1^2}$		Analytical	MEF	Analytical	MEF	Analytical	MEF
0	(3.51 Hz)	1.0960	1.0919	1.0890	1.8609	1.8612	3.9057	3.9059
10000	--	--	3.3980	3.4089	3.7011	3.7120	4.4729	4.4838
$\rightarrow\infty$	(45 Hz)	3.9123	3.9234	3.9248	4.6603	4.7239	7.0430	7.0574

As it can be seen in the Tables, all the results are consistent.

3.2. Analysis of the model in presence of a crack

For further analysis, the presence of a crack in the lintel is introduced. The crack is modeled as a hinge with a rotational spring using the formula proposed by Chondros et al. [22] for the crack flexibility. These authors proposed to model a crack as a continuous flexibility using the displacement field in the vicinity of the crack, found with fracture mechanics methods. The crack flexibility is assumed as:

$$\alpha_C = \frac{6\pi(1-\nu^2)h}{EI} I_C(h_c/h);$$

where h is the height of the cross-section, v is the Poisson ratio and h_c is the crack depth. I_c is defined by the expression:

$$I_c(z) = 0.6272\ z^2 - 1.04533 z^3 + 4.5948 z^4 - 9.973 z^5 + 20.2948 z^6 - \\ - 33.0351 z^7 + 47.1063 z^8 - 40.7556 z^9 + 19.6 z^{10};$$

with $z = h_c/h$.

Assuming like [22] that the effect of the supposed crack affects only in its neighborhood, the lintel can be treated as two uniform beams of length l_2 and l_3, connected by a rotational spring of local rigidity $R_m = 1/\alpha_C$ at the crack position.

Table 3 depicts values of coefficients λ_i, obtained by the analytical procedure, when the same type of the crack is considered in different positions of the lintel beam for a clamped-clamped frame. The assumed value for h_c is 2/3h.

Table 3. Frequency coefficients λ_i of the frame for different positions of the crack. $v_{li} = 1$, $Tx = Ty = Rxy \to \infty$, $v_{EI(2)} = v_{EI(3)} = 1$, $v_{\rho A(2)} = v_{\rho A(3)} = 1$ y $T = 0; R = 0$ $Rm = 700$.

l_2/l_1	λ_1	λ_2	λ_3
$1/3$	3.5154	4.4646	5.9115
$1/2$	3.9179	4.6653	6.1562
$2/3$	3.8811	4.5467	6.7124

As it can be observed, the position of the crack in the middle of the lintel influences very little the first two frequencies of vibration. That is expectable, since the solid undamaged model has an inflexion point near this position in its two first modes of vibration (See Figure 4).

Figures 5, 6 and 7 show the modal shapes for the first three frequency of vibration for different positions of the crack (the arrow indicates the position of the crack).

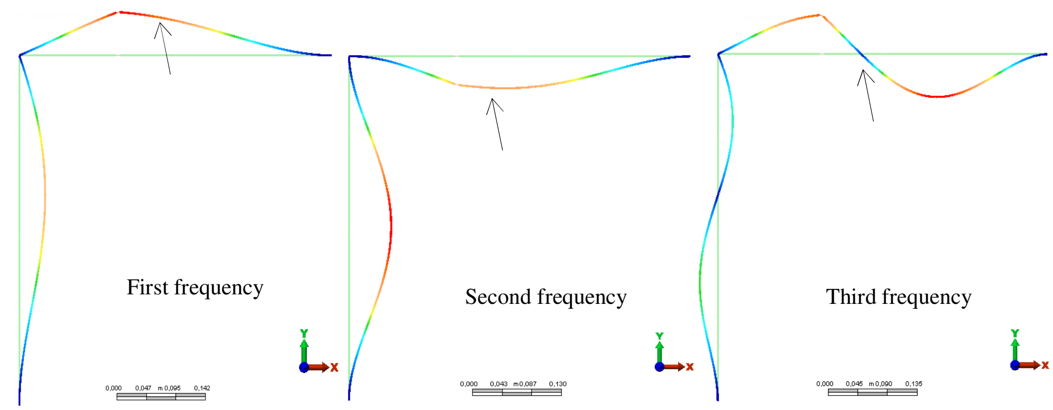

Figure 5. Mode shape of the frame with varying elastically hinge on the lintel, for $v_{li} = 1$, $l_2 = {}^{1}\!/_{3}\, l_1$; $v_{li} = 1$, $Tx = Ty = Rxy \to \infty$, $v_{EI(2)} = v_{EI(3)} = 1$; $v_{\rho A(2)} = v_{\rho A(3)} = 1$; $T = R = 0$; $Rm = 700$.

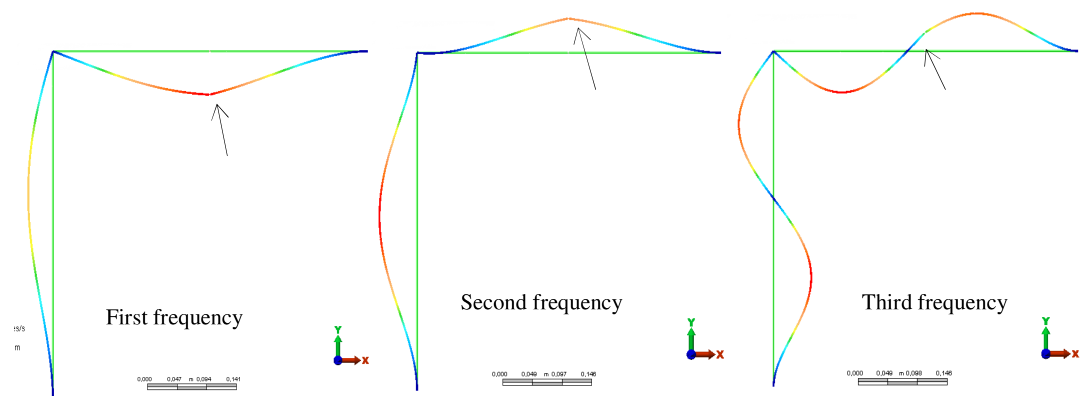

Figure 6. Mode shape of the frame with elastically hinge on the lintel, for for $v_{li} = 1$, $l_2 = {}^{1}\!/_{2}\, l_1$; $v_{li} = 1$, $Tx = Ty = Rxy \to \infty$, $v_{EI(2)} = v_{EI(3)} = 1$; $v_{\rho A(2)} = v_{\rho A(3)} = 1$; $T = R = 0$; $Rm = 700$.

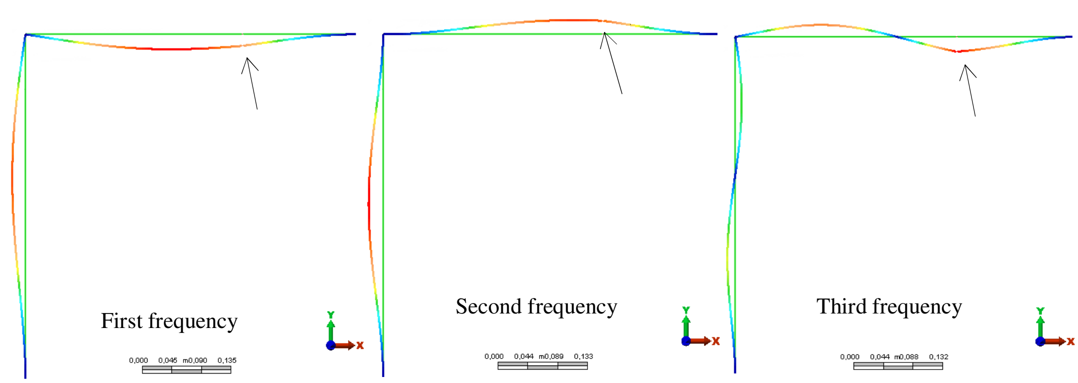

Figure 7. Mode shape of the frame with elastically hinge on the lintel, for for $v_{li} = 1$, $l_2 = {}^{2}\!/_{3}\, l_1$; $v_{li} = 1$, $Tx = Ty = Rxy \to \infty$, $v_{EI(2)} = v_{EI(3)} = 1$, $v_{\rho A(2)} = v_{\rho A(3)} = 1$; $T = R = 0$; $Rm = 700$.

4. CONCLUSIONS

In this paper the method of separation of variables combined with the variational calculation technique is used to deal with a difficult elastodynamics problem: the vibration of a plane frame with additional complexities as elastic external and internal elastic supports. The Euler-Bernoulli beam theory including the axial deformation is considered for each member of the structure. The values obtained with the proposed analytical approach are satisfactorily compared with particular cases available in the literature and with those acquired by means of a MEF code. In some cases, an experimental verification is performed. Values of the first natural frequencies of vibration and the corresponding modal shapes are presented. The model, also allows analyzing the influence of a crack in the dynamical behavior of the frame.

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