f(R,T) gravity from null energy condition

Alvarenga, F. G.[1]; Houndjo, M.J.S.[2]; Monwanou, A. V.[2]; Orou, J.B.C.[2]

[1] Departamento de Ciências Naturais, Universidade Federal do Espírito Santo, Vitória, ES, Brasil
[2] Institut de Mathématiques et de Sciences Physiques, Porto-Novo, Benin

Abstract

We consider f(R,T) theory of gravity, where R is the curvature scalar and T is the trace of the energy momentum tensor. Attention is attached to the special case, f(R,T) = R + 2f(T), and we assume for the function f(T), $a_1T^n+b_1/a_2T^n+b_2$, where a_1, a_2, b_1, b_2 and n are input parameters. We observe that by adjusting suitably these input parameters, the null energy condition can be satisfied.
Keywords: (modified gravity, energy conditions, singularity).

1. Introdução

In this present paper, attention is attached to so-called f(R,T) theory of gravity, where T denotes the trace of the energy momentum tensor. This generalization of f(R) gravity has been made first by Harko et al [1]. The dependence on T can take source from the introduction of exotic imperfec fluid or from quantum effect. In this theory, the equations of motion show the presence of an extra-force acting on the test particles, and the motion are generally non-geodesic. This theory also relates the cosmic acceleration, not only due to the contribution of geometrical terms, but also to the matter contents. Our task in this paper is to check the viability of a particular model f(R,T) according to the null energy condition. We assume a special form, f(R,T)=R + 2f(T), the usual Einstein-Hilbert term plus a T dependent f(T)= $a_1T^n+b_1/a_2T^n+b_2$.

2. General formalism

Let us assume the modified gravity replacing the Ricci scalar R in Einstein gravity by an arbitrary function f(R, T), and writing the total action as

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[f(R,T)+L\right], \qquad (1)$$

where $\kappa^2=8\pi G$, G being the gravitational constant and $T=g^{\mu\nu}T_{\mu\nu}$ the trace of the matter energy momentum tensor $T_{\mu\nu}$. The equations of motion, using the metric formalism, have been explicitly obtained as

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f_R(R,T)g_{\mu\nu}+(g_{\mu\nu}\Delta-\nabla_\mu\nabla_\nu)f_R(R,T)= \kappa^2T_{\mu\nu}-f_T(R,T)T_{\mu\nu}-f_T(R,T)\Theta_{\mu\nu}, \qquad (2)$$

where $f_R(R,T)$ and $f_T(R,T)$ denote the derivative de $f_R(R,T)$ with respect to R e T, respectively, and $\Theta_{\mu\nu}$ is defined by $\Theta_{\mu\nu}=-2T_{\mu\nu}-pg_{\mu\nu}$ for the stress-energy tensor of matter $T_{\mu\nu}=(\rho+p)u_\mu u_\nu-pg_{\mu\nu}$.

From now, we will treat the models of type f(R,T) = $f_1(R)+f_2(T)$ and set $\kappa^2=1$. Then, the equation (2) becomes

$$f_{1R}(R)R_{\mu\nu}-\frac{1}{2}f_1(R,T)g_{\mu\nu}+(g_{\mu\nu}\Delta-\nabla_\mu\nabla_\nu)f_{1R}(R)= T_{\mu\nu}+f_{2T}(T)T_{\mu\nu}+\left[f_{2T}(T)p+\frac{1}{2}f_2(T)\right]g_{\mu\nu} \qquad (3)$$

3. Energy conditions

The energy conditions are essentially based on the Raychaudhuri equation that describes the behaviour of a congruence of timelike, spacelike or lightlike curves. It is commonly used to study and establish singularities of the spacetime [2].

For equivalence to GR, by just dividing by $f_1(R)$, one can cast Eq. (4) in the following form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\iota}R=T^{eff}_{\mu\nu}, \qquad (4)$$

where the effective energy momentum tensor is defined

$$T^{eff}_{\mu\nu}=\frac{1}{f_1(R)}\left\{\begin{array}{l}T_{\mu\nu}+f_{2T}(T)T_{\mu\nu}+\frac{1}{2}\left[2f_{2T}(T)p+f_2(T)+f_1(R)-Rf_{1R}(R)\right]g_{\mu\nu}\\ -(g_{\mu\nu}\Delta-\nabla_\mu\nabla_\nu)f_{1R}(R)\end{array}\right\} \qquad (5)$$

By assuming that the total content of the universe behaves as perfect fluid, we can replace ρ and p, by ρ_{eff} and p_{eff} respectively (the effective energy density and effective pressure). Thus, the energy conditions for the effective perfect fluid reduce to:

- null energy condition : $\rho_{eff}+p_{eff}\geq 0;$ (6)
- strong energy condition: $\rho_{eff}+p_{eff}\geq 0,$
  $\rho_{eff}+3p_{eff}\geq 0;$ (7)
- weak energy condition: $\rho_{eff}\geq 0,\ \rho_{eff}+p_{eff}\geq 0;$ (8)
- dominant energy condition (NEC): $\rho_{eff}\geq 0;$
  $\rho_{eff}+p_{eff}\geq 0, \qquad \rho_{eff}-p_{eff}\geq 0;$ (9)

Therefore, the energy conditions, as known in GR, can also be applied in this modified theory of gravity by substituting the ordinary energy density and pressure p in GR by the effective ones, ρ_{eff} and p_{eff}.

In what follows, we will analyze the null energy condition for models of type f(R,T) = R+2f(T), i.e., the usual Einstein-Hilbert term plus trace depending term 2f(T) and . This amounts to consider $f_1(R)$ = R and $f_2(T)$ = 2f(T). The factor 2 is used just for letting the field equations more easier to be treated. We will also assume that the ordinary content of the universe is pressureless and satisfies the energy conditions $\rho\geq 0$.

4. Testing model f(R,T) = R + 2f(T)

In this section we will present the conditions required on ρ and the algebraic function f(T) for realizing the null energy condition. For this end, we first need to establish the respective expression of the effective energy density ρ_{eff} and effective pressure p_{eff}. According to the assumptions made at the end of the previous section, Eq. (5) becomes

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\iota} R = T_{\mu\nu} + 2 f_T(T) T_{\mu\nu} + f(T) g\mu\nu. \quad (10)$$

Considering the flat FRW space-time described by the metric

$$ds^2 = dt^2 - a^2(t) dx^2 , \quad (11)$$

where a(t) is the scale factor. The 00 and ii components of (10) can be written as

$$3H^2 = \rho_{eff} = \rho + 2\rho f_T(T) + f(T), \quad (12)$$

$$-2\dot{H} - 3H^2 = p_{eff} = -f(T). \quad (13)$$

By using the above expressions of the effective energy density and pressure, we get the null energy condition:

$$\rho[1 + 2 f_T(T)] \geq 0 \quad (14)$$

We propose to test one model of f(T) in the way to make them satisfying the energy conditions. We consider a function f(T) such that for large and small values of the trace, it converges. We start by assuming first this function as $f(T) = a_1T^n+b_1/a_2T^n+b_2$, where a_1, a_2, b_1, b_2 and n are parameters to be adjusted in order to obtain models that satisfy some or all the energy conditions. A motivation in using this model is that when n > 0, it prevents divergence for large and small values of the trace. Remark that for $T \to \infty$, $f(T) = a_1/a_2$ is finite, and for $T \to 0$, $f(T) = b_1/b_2$ which is also finite. However, when n < 0, the situation inverts, and for large and small values of the trace, one obtains $f(T) = b_1/b_2$ and $f(T) = a_1/a_2$, respectively, and the divergence is still prevented. Let us now explore the cosmological feature of this model. Here we can discuss the occurrence of cosmic acceleration, which may impose some constraints to the parameters, reducing the degree of freedom of the model [3]. Searching for the power-like solutions, one has: $a = a_0t^x$, $H= xt^{-1}$, $H=-xt^{-2}$ and $f(T) \to g(t) = (a_1^* t^{-3xn} + b_1)/(a_2^* t^{-3xn} + b_2)$, with $a_i^* = a_i \rho_0 a_0^{-3}$ (i = 1, 2), where we assume that the ordinary content of the universe is dust, $T=\rho=\rho_0 a^{-3}$. With the scale factor being used here, the early universe corresponds to $t \to 0$ and the late time correspond to $t \to \infty$, where x > 0. At early time, the curvature scalar $R \to \infty$ (where the f(T) contribution may be neglected) and at late-time, the universe may be characterized by the ΛCDM model, i.e., our model must behave like R + 2Λ. For obtaining this feature, for n < 0, the cosmological constant reads $\Lambda = a_1^*/a_2^* = a1/a2$, while for n > 0, one gets $\Lambda = b_1/b_2$. By calling ω_{eff} the effective parameter of the equation of state, one has $\omega_{eff} = p_{eff} / \rho_{eff} = -f(T)/(\rho+2\rho f_\rho+f(T))$. The requirement of guaranteeing the acceleration of the universe, without falling into phantom model, is $-1 < \omega_{eff} < -1/3$. Observe that at time, $(t \to \infty)$, !eff = −1 (this holds only when n > 0). Thus, we see that some values of the input parameters can make the model providing the late time acceleration, i.e., for n > 0, $a_1/a_2 > 0$ and $b_1/b_2 = \Lambda$. Our task here is to put out the constraints on the input parameters in order to get a R + 2f(T) type model that satisfies the null energy condition. According to the sign of the parameter n, and assuming that a_2 and b_2 cannot be identically null, the model can be cast into two different forms. In fact, for the late time stage of the universe, by dividing the parameters of the model by a_2 (n > 0) and b_2 (n < 0), one gets respectively the models $f(\rho) = (\Lambda\rho^n + B_1)/(\rho^n + B_2)$ and $f(\rho) = (A_1\rho^n + \Lambda)/(A_2\rho^n + 1)$, where the cosmological constant is characterized by a_1/a_2 (for n > 0) and b_1/b_2 (for n < 0), and $A_1 = a_1/b_2$, $A_2 = a_2/b_2$, $B_1 = b_1/a_2$ and $B_2 = b_2/a_2$. In this case, the model which initially was four parameters dependent, under the cosmological constraints, becomes three parameters dependent, Λ, B_1 and B_2 for n > 0, and Λ, A_1 and A_2 for n < 0. Since the cosmological constant is known, the model turns into two parameters dependent. The first derivative of f(T) with respect to T (or the derivative of f(ρ) with respect to ρ) reads for n>0 and n<0, respectively

$$f_\rho(\rho) = \frac{n(\Lambda B_2 - B_1)\rho^{n-1}}{(\rho^n + B_2)^2} \; ; \; f_\rho(\rho) = \frac{n(A_1 - \Lambda A_2)\rho^{n-1}}{(A_2\rho^n + 1)^2} \, . \quad (15)$$

For ρ≥0, the condition (29) reduces to $1 + 2f_T(T) \geq 0$, (or $1/2 + f_\rho(\rho) \geq 0$). One can calculate $1/2 + f_\rho(\rho)$ as

$$f_\rho(\rho) + \frac{1}{2} = \frac{2n\rho^{n-1}(\Lambda B_2 - B_1) + (\rho^n + B_2)^2}{2(\rho^n + B_2)^2} \text{, for n>0,} \quad (16)$$

$$f_\rho(\rho) + \frac{1}{2} = \frac{2n\rho^{n-1}(A_1 - A_2\Lambda) + (A_2\rho^n + 1)^2}{2(A_2\rho^n + 1)^2} \text{, for n<0,} \quad (17)$$

whose the sign can just be characterised by that of the numerator, since the denominator is always positive. If we take the numerator as a function of the ordinary energy density ρ and the input parameters, we just need to analyse the sign of this latter. The evident conditions for which the numerator is positive are presented as follows:

- B1 > 0, B2 > 0, B1/B2 < Λ for n > 0 ,
- A1 > 0, A2 > 0, B1/B2 > Λ for n < 0 ,
- B1 < 0, B2 > 0, for n > 0 ,
- A1 < 0, A2 > 0, for n < 0 .

Indeed, the above conditions lead to the positivity $2n\rho^{n-1}(\Lambda B_2 - B_1) > 0$ for n > 0 and $2n\rho^{n-1}(A_1 - A_2\Lambda) > 0$ for n < 0. Observe that there are still situations in which the above quantities are negative but the numerators in (16) and (17) continuing positive, i.e.,

- $A_1 > A_2\Lambda$ and $|2n\rho^{n-1} (A1 - A_2\Lambda)| < |(A_2\rho^n + 1)^2|$ for n < 0 ,
- $\Lambda B_2 < B1$ and $|2n\rho^{n-1} (\Lambda B_2 - B_1)| < |(\rho^n + B_2)^2|$ for n > 0 .

In these cases, one can plot the function in terms of two of the parameters, fixing the other [4].

Finally, in this work we analyze the viability of the model $f(R,T)=R+(a_1T^n+b_1/a_2T^n+b_2)$ by investigating for which range of parameters, the models satisfy the null energy conditions.

Acknowledgement: M. J. S. Houdjo thanks CNPq/FAPES for financial suport.

5. References

[1] T. Harko, F. S. N. Lobo, S. Nojiri and S. D. Odintsov, “f(R, T) gravity,” Phys. Rev. D84 (2011).

[2] S. W. Hawking, G. F. R. Ellis., The Large Scale Structure of Space-Time, (Cambridge University Press, 1999).

[3] M. J. S. Houndjo, “ Reconstruction of f(R, T) gravity describing matter dominated and accelerated phases”, Int. J. Mod. Phys. D. 21, 1250003 (2012).

[4] F. G. Alvarenga, M. J. S. Houndjo, A. V. Monwanou, Jean B. Chabi Orou, Journal of Modern Physics, 4, 130-139(2013).