# Applying Geogebra® in Descriptive Geometry Online Teaching to Model Fundamental Concepts

Juliane Silva de Almeida[1], Márcio Schneider de Castro[1]

[1] Federal University of Santa Catarina (UFSC), Brazil.
juliane.almeida@gmail.com
marcio.schneider@ufsc.br

**Abstract.** Gamification tools are becoming increasingly common, especially due to the COVID-19 pandemic, as it has been necessary to adapt subjects for distance learning. Gamification tools include software classified as digital interactive technologies for teaching. Hence, it propitiates the student to learn through software interaction. One renowned gamification tool available in calculus and geometry is Geogebra® with an attractive and interactive graphic interface. Owing to these features, this article will present the authors' didactic materials developed in Geogebra®. They consist of 3D dynamic models, which illustrate fundamental concepts of descriptive geometry. Thus, the models aim to support spatial vision acquisition in the teaching-learning process in descriptive geometry courses. This course is part of the undergrad engineering curricula at the Universidade Federal de Santa Catarina, Brazil. After testing the new materials, the students classified the 3D dynamic models of Geogebra® as useful to comprehend the content and exercises.

**Keywords:** Geogebra®; Descriptive Geometry; 3D Dynamic Models; Online Teaching Geometry; Adjacent View.

## 1 Introduction

The COVID-19 pandemic has brought challenges to online teaching, especially in practical subjects such as technical drawing and descriptive geometry. Hence, it has become crucial to find solutions and adapt the teaching-learning process of these subjects. In Brazil, descriptive geometry is typically taught only with drawing utensils. Besides, the students' motivation seems to diminish as they realize that drawing with utensils will not be applicable in their professional lives.

Additionally, the students usually struggle to acquire spatial vision by drawing with utensils. Along these lines, it is noticeable that representations with drawing utensils do not easily convey the relationship between the representation in perspective and its adjacent views. Consequently, the pandemic has enforced the incentives to modernize the descriptive geometry teaching-learning process (Álvaro-Tordesillas et al., 2020; Gorjanc & Jurkin, 2015).

To provide affordable resources to support the teaching-learning process of the engineering undergrads, the authors developed 3D dynamic models to implement in a descriptive geometry course, using the software Geogebra® (Hohenwarter & Preiner, 2007; Mackrell, 2011; Tim Brzezinski, 2021). The focus of these models consists of representing fundamental concepts of descriptive geometry, such as the representations of the basic 3D geometry elements to support the students' spatial vision. Moreover, the 3D models demonstrate the relationship with the elements' representations in perspective and adjacent views, by applying the dynamic resources. These 3D dynamic models were created as a complement to support the resolution of the drawing utensil activities. Additionally, they are a consequence of the project carried out by professors from the Graphic Communication and Arts at Universidade Federal de Santa Catarina (UFSC), who aim to develop 3D dynamic models as didactic material for descriptive geometry courses.

Therefore, this article will shed light on the first results regarding the application of the 3D dynamic models developed in Geogebra® in a descriptive geometry course for engineering undergrads. The students evaluated how the 3D models impact their teaching-learning process in acquiring spatial vision. The first evaluated 3D dynamic models displayed representations of points, lines, and planes in a 3D space and their adjacent views. They also assessed the benefits of 3D models in the comprehension and resolution of drawing utensil activities.

## 2 Literature Review

The research about the development of 3D dynamic models adopted in descriptive geometry courses began with the search for evidence of the resources utilized that aim to enhance the teaching-learning process.

Gorjanc, Hallas and Jurkin (2015) demonstrated their experience in using Computer-Aided Design (CAD) software to create examples of 3D representations in the undergrad descriptive geometry course of Civil Engineering at the University of Zagreb. As a result of their study, they presented the students' increased interest to enroll in the descriptive geometry course after the inclusion of the CAD resources.

The deployment of digital resources in geometry and drawing courses was studied by Di Paola, Pedone and Pizzurro (2013). These authors examined applications of Innovation and Communication Technologies (ICTs) in descriptive geometry at Palermo University. However, it did not present strong evidence that the incorporation of ICTs in descriptive geometry provides benefits to the teaching-learning process for students. This study affirms that the ICTs could attract the students' attention to the activities, because of their interest in digital resources. Nevertheless, they did not exhibit significant results related with student performance.

Bozza (2015) agreed with Di Paola, Pedone and Pizzurro (2013) regarding the increased student motivation when ICTs are applied in the descriptive geometry courses. This author mentions that the incorporation of ICTs in the subject approach the students' reality.

Mackrell (2011) conducted a literature review about the main digital technologies directed to geometry instruction and their primary features. This author claimed that a user-friendly interface and the resources involved in the software chosen are fundamental for success in the students' teaching-learning process. She described a variety of software deployed, such as Geogebra® and Cabri 3D.

Álvaro-Tordesillas et al. (2020) developed an innovative project to employ gamification and collaboration techniques in the undergrad descriptive geometry course for architecture at the Universidad de Valladoid. After three years of evaluating the experience, the authors concluded that the students presented more motivation for the subject and improved their performance. The results demonstrated that more students completed the descriptive geometry course than in years that the innovative project was not applied. Besides, the authors assessed the relationship with the students' performance, the year they participated (2015, 2016, 2017 or 2018) and the when the project occurred. They concluded that the students exhibited better performance in the second semester of the assessed years. Nonetheless, they did not find a relationship between the year of participation and the students' performance. During the years that the project was running, the students presented similar performance.

Geogebra® is a software created by Hohenwarter & Preiner (2007) to develop graphic representation of mathematical models, making it possible to represent 3D graphics. The Geogebra® app is available for download on the Geogebra website, developed by Brzezinski (2021).

There were no Geogebra®-type applications in the descriptive geometry mentioned in the literature. Most are in calculus and statistics because the software was originally developed to represent mathematical models. However, there is the work of Ferdiánová (2017), who created an e-book in Geogebra® with descriptive geometry activities. She constructed 3D dynamic models in Geogebra® and integrated this material with the Learning Management System (LMS) platform, Moodle. She reported that this practice improved student performance in a descriptive geometry course, during the two semesters evaluated. The students who used the ICT resources scored 8.62% higher in their exams than those who did not use ICTs.

Pirklová (2019) and Pirklová and Bímová (2019) developed an e-book with an emphasis on Monge Projection teaching (Monge et al., 1851). The e-book contained Monge Projection exercises applying the dynamic features of their models in the 3D construction steps. In addition, the 3D models presented in the e-book enabled student interaction. Hence, they illustrated and explained each step of the descriptive geometry exercises.

These last three references demonstrated the potential of using Geogebra® as a gamification tool to create 3D dynamic models for descriptive geometry courses. However, examples of models that focus on the different forms of 3D object representation were not found in the literature. Therefore, it was not possible to obtain references that highlight the different representations, such as perspective and adjacent views. One of the most common challenges for students in the teaching-learning process of descriptive geometry is the association between the representation in perspective and adjacent views.

Due to the absence of these topics in the literature, this article aims to present a proposal for implementing Geogebra® in the teaching-learning process of descriptive geometry. Thus, it consists of the development of 3D dynamic models to support spatial vision acquisition. Furthermore, these 3D dynamic models associate a 3D perspective of the represented elements and their adjacent views. The methodology involved in the creative process of the 3D models and the students' evaluations will be presented in the next section.

## 3 Methodology

The methodology consists of three parts. First, the professors carried out literature research for the project beforehand to understand previous work. The literature review aimed to consolidate the main idea of the project presented in this article as well. Thus, the potential of implementing Geogebra® in the online teaching of descriptive geometry was observed because of the similarity of previous experiences reported in the literature.

Nevertheless, the authors identified that another strategy could be pursued, by focusing on the association between the descriptive geometry representations of the object in 3D and 2D. Many students struggle with understanding Monge Projections and the formation of the adjacent views, which are based on these fundamentals (Di Pietro, 1985; Ledneczki, n.d.; Monge et al., 1851; Stachel, n.d.). This difficulty is related to the absence of association involved in the 2D representation of objects which are in 3D. When this association is exposed, it frequently occurs from 2D representation to 3D, which is counterintuitive. Consequently, it is not productive for student comprehension.

Second, the 3D dynamic model representative of basic concepts of descriptive geometry were developed, using the Geogebra® Classic applet (Hohenwarter & Preiner, 2007; Tim Brzezinski, 2021). Geogebra® Classic was chosen to create the 3D models because it includes 3D and 2D windows simultaneously. Thus, it permits the application of dynamic features in 3D graphics created in the 3D environment of Geogebra®. The importance of aggregating dynamic features in the 3D dynamic models must be highlighted. It integrates changing the 3D object to a 2D representation by adjacent views. Hence, the variety of object representations in the space is addressed. The incorporation of the dynamic features in the 3D models are highlighted in the next section.

After testing the 3D dynamic models, one of the professors applied them in two of the groups of a descriptive geometry course for engineering undergrads. This action aimed to evaluate the students' perspective and the effectiveness of the new didactic materials. In order to report the evaluation, the students received a questionnaire about each model.

## 4 Results Discussion

Using a different method than the one presented in Ferdiánová (2017), Pirklová and Bímová (2019), Pirklová (2019), the authors of this article applied the dynamic features of the 3D models in the coordinates of the points. This gave more versatility in displaying the 3D element positions and the composition of the adjacent view. They prioritized these strategies because of the students' challenges in the construction process of the adjacent view.

Ferdiánová (2017), Pirklová and Bímová (2019) and Pirklová (2019) developed their models in Geogebra® using the 2D environment. Despite the facility in integrating dynamic properties in the 2D environment of Geogebra®, this strategy has the disadvantage of not demonstrating the relationship between the object representations. In this context, the cited authors opted to deploy the dynamic features of their models in the construction steps.

Consequently, they prioritized the students' memorization of the construction steps instead of the comprehension of visual thinking.

In order to focus on the teaching-learning process of spatial vision skills, the authors of this article prioritized the creation of 3D models with Geogebra®, exposing a closer representation of the reality than the ones created in 2D. Additionally, the 3D interface in Geogebra® is interactive for the user. The student can change the views of the perspective and rotate the 3D graphics, obtaining various points of view. The developed 3D dyamic models are available at Almeida (2021b) and Almeida (2021a).

Another feature of the 3D models created is the transition from 3D space representation (perspective) to the object adjacent views. To include this feature, it was necessary to use the slider control called *Epura* and integrate the 2D and 3D environments in Geogebra®. Examples of how this transition works are illustrated in Figures 1 and 2. Thus, the transition from 3D space representation to adjacent views of the objects are highlighted in items (b) and (c) of Figures 1 and 2.

Finally, it is important to mention that the slider controls were applied to the point coordinates in both constructed 3D dynamic models as well. The objective of this strategy is to enable the visualization of the perspective and the adjacent view of the line in different positions of the space in the same 3D model. Hence, the line position can be modified by the interaction between the student and the software because of the slider control.

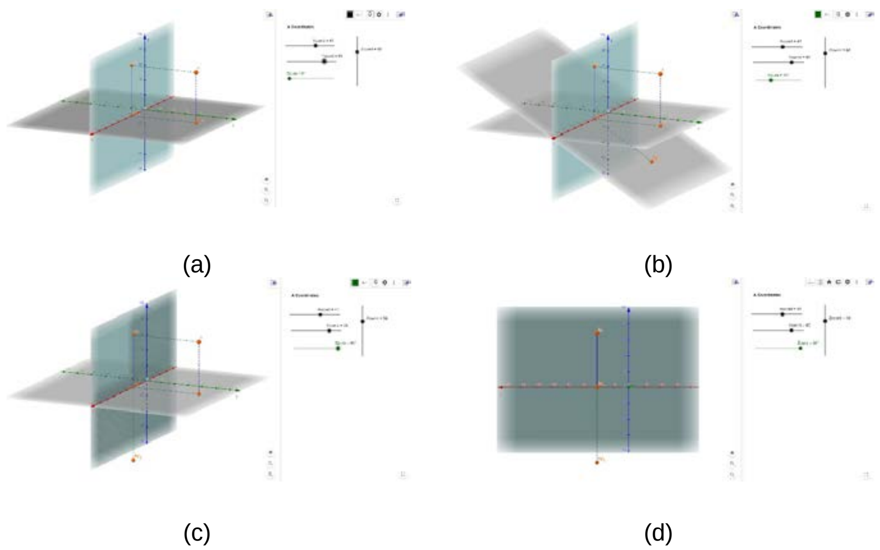


**Figure 1:** 3D dynamic model of the point representations. (a): 3D space point representation. (b): Transition from 3D space representation to the points' adjacent view.

(c): 90° clockwise rotation to form the points' adjacent view. (d): 2D representation of the points' adjacent view. Source: Authors.

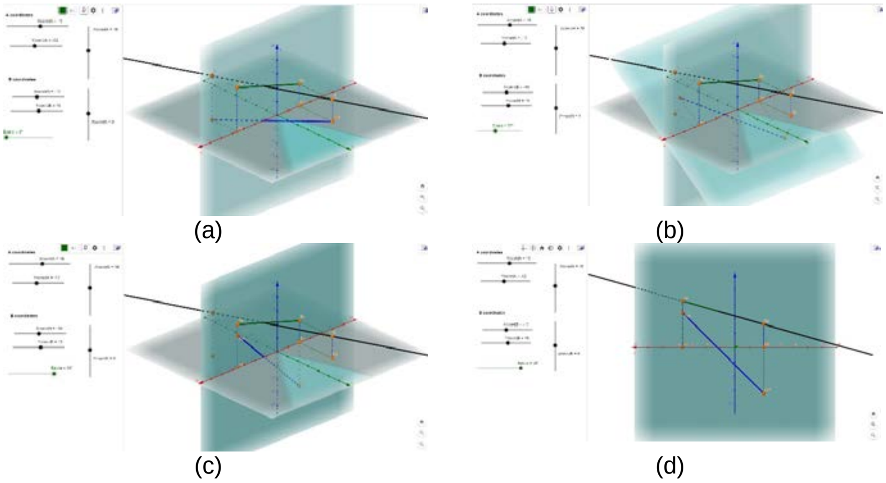


**Figure 2:** 3D dynamic model of the line representations. (a): 3D space point representation. (b): Transition from 3D space representation to the lines' adjacent view. (c): 90° clockwise rotation to form the lines' adjacent view. (d): 2D representation of the lines' adjacent view. Source: Authors.

As aforementioned in the Methodology section, the 3D dynamic models were presented to two groups of students in descriptive geometry courses. Subsequently, they received the questionnaire. Tables 1 and 2 exhibit the questions applied to the students in each questionnaire. There were yes/no questions and scoring from 1 to 5. The students in the 1st college semester group received the questionnaire to fill out during the course. Regarding the answers received, the amount was significant. Of 43 students, 40 answered the questionnaire from Table 1, and 38 answered the questionnaire from Table 2.

**Table 1:** Questionnaire for the 3D dynamic model of the point representations with the average answers.

| Questions | Type | Most Recurrent/Average Answers |
|---|---|---|
| Did you enjoy the 3D dynamic models of the point representations in Geogebra? | Yes/No | Yes |
| Did you see the representation of the three-point coordinates in the 3D space? | Yes/No | Yes |

| | | |
|---|---|---|
| Did you see the representation of the three-point coordinates in the adjacent view? | Yes/No | Not answered |
| How clear was the point representation in the 3D space? | Scoring from 1 to 5 | 4.3 |
| How clear was the point representation in the adjacent view? | Scoring from 1 to 5 | 4.08 |

Source: Authors, 2021.

**Table 2:** Questionnaire for the 3D dynamic model of the line representations with the average answers.

| Questions | Type | Most Recurrent/Average Answers |
|---|---|---|
| Did you enjoy the 3D dynamic models of the line representations in Geogebra? | Yes/No | Yes |
| Did you see the representation of the three-line coordinates in the 3D space? | Yes/No | Yes |
| Did you see the representation of the three-lines coordinates in the adjacent view? | Yes/No | Not answered |
| How clear was the line representations in the 3D space? | Scoring from 1 to 5 | 3.92 |
| How clear was the line representations in the adjacent view? | Scoring from 1 to 5 | 3.79 |

Source: Authors, 2021.

Regarding the students' perspective, they expressed that the 3D models supported their spatial vision acquisition, as presented in Tables 1 and 2. Beyond that, the students considered that the incorporation of Geogebra® as a gamification technology in the descriptive geometry course was beneficial. Most of them affirmatively answered that the 3D dynamic models of Geogebra® supported the students in locating the objects' coordinates and their 3D space representation. Likewise, the average scores attributed to the clarity of information (above 2.5 and close to 4) demonstrated that the 3D dynamic models helped the students to understand the spatial vision of the objects represented.

Nevertheless, the students did not reply to the 3rd question in both questionnaires, which asked about the adjacent view representations. Despite that, they responded to the 5th question. Hence, there was a slight contradiction in these answers because the 3rd and 5th questions are related, as demonstrated in Tables 1 and 2. The average scores attributed to the representation of the objects' adjacent view in the 5th question that evaluated both 3D dynamic models were above 2.5.

Furthermore, of the 43 students in the descriptive geometry course, 46% attributed a score of 5 for clarity, representing the points' adjacent view. Besides, 34% attributed a score of 4 for the adjacent view of the lines' 3D dynamic model. Each model received scores above 4 from 67% and 60% of the students, for the points' and lines' adjacent views, respectively. Thus, it requires more studies to verify if the adjacent view represented in the 3D dynamic models, in fact, support the students' teaching-learning process. Figures 3 and 4 illustrate the distribution of results obtained in the questionnaires in the graphics.

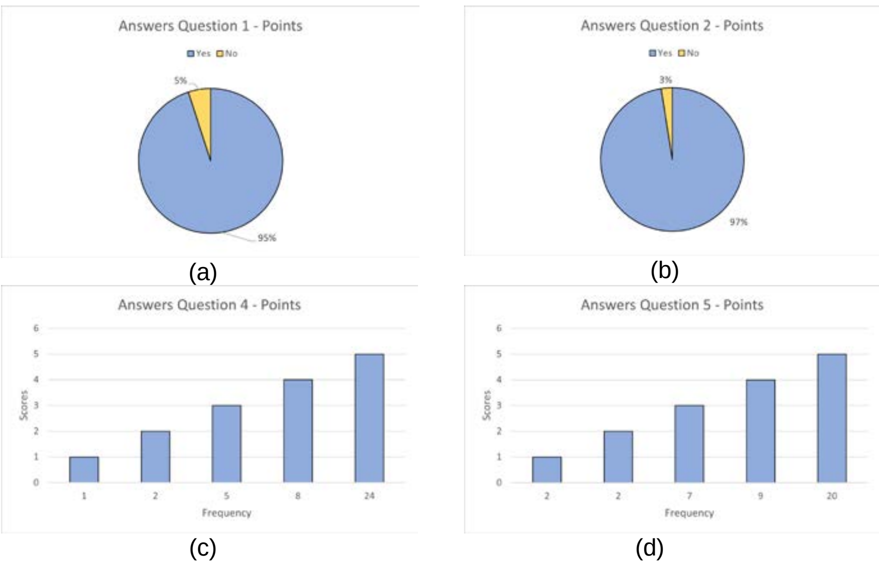


**Figure 3:** Answers from the questionnaire in Table 1. (a): answers to the 1st question. (b): answers to the 2nd question. (c): scores attributed to the 4th question. (d): scores attributed to the 5th question. Source: the authors.


**Figure 4:** Answers from the questionnaire in Table 2. (a): answers to the 1st question. (b): answers to the 2nd question. (c): scores attributed to the 4th question. (d): scores attributed to the 5th question. Source: the authors.

## 5 Conclusion

Despite the different approaches adopted in the literature and presented in this article, the results presented in Figures 3 and 4 validate the potential of using Geogebra® as a gamification tool in descriptive geometry courses as described in the literature. It is fundamental to mention that the literature and this article were based on experiments carried out in descriptive geometry courses for engineering undergrads. Consequently, this article demonstrates the potential of Geogebra® used as a graphic design tool, due to its graphic interface properties.

Additionally, the 3D dynamic models created in Geogebra were effective for student comprehension of the 3D space representations of points and lines, as exhibited in Figures 3 and 4. However, it requires more studies to properly verify the effectiveness of the 3D dynamic models when explaining the composition of the adjacent views to the student, due to the contradiction presented in the answers of the 3rd and 5th questions in Tables 1 and 2.

Therefore, the previous results obtained from the questionnaire applied in the descriptive geometry course reported the students' interest in using Geogebra® to support their studies as well as the potential of this tool. Moreover, with the construction of the first models, it was possible to compare with the experiences found in literature and analyze how each model could contribute to the teaching-learning process of the students' acquisition of spatial vision. Finally, it was possible to propitiate subsidies for future research in descriptive geometry studies.

**Acknowledgements.** The authors are glad to the participation of the students in descriptive geometry courses from Mechanical and Industrial Engineering at UFSC, at the 2nd semester of 2020 college year in this research. Additionally, we acknowledge the Prorectorate for Outreach and Extension to permit the execution of the extension project that originated this research.

## References

Almeida, J. S. De. (2021a). *3dLineSpace*. Geogebra. https://www.geogebra.org/m/cz7y5c6y

Almeida, J. S. De. (2021b). *3dPointSpace*. Geogebra. https://www.geogebra.org/m/tcdnz9hr

Álvaro-Tordesillas, A., Alonso-Rodríguez, M., Poza-Casado, I., & Galván-Desvaux, N. (2020). Gamification experience in the subject of descriptive geometry for architecture. Educacion XX1, 23(1), 373–408. https://doi.org/10.5944/educxx1.23591

Bozza, M. (2015). Pensando o Ensino de Geometria Espacial: Estratégias Didáticas que utilizam o Software GeoGebra e Materiais Concretos. Scientia Cum Industria, 3(3), 134–138. https://doi.org/10.18226/23185279.v3iss3p134

Di Paola, F., Pedone, P., & Pizzurro, M. R. (2013). Digital and Interactive Learning and Teaching Methods in Descriptive Geometry. Procedia - Social and Behavioral Sciences, 106, 873–885. https://doi.org/10.1016/j.sbspro.2013.12.100

Di Pietro, D. (1985). Geometria descriptiva. Alsina. https://books.google.com.br/books?id=8uZjAAAACAAJ

Ferdiánová, V. (2017). GeoGebra Materials for LMS Moodle Focused Monge on Projection. The Electronic Journal of E-Learning, 15, 259.

Gorjanc, S., & Jurkin, E. (2015). Introducing 3D Modeling into Geometry Education at Technical Colleges. In The Visual Language of Technique (pp. 57–67). Springer International Publishing. https://doi.org/10.1007/978-3-319-05326-4_6

Hohenwarter, M., & Preiner, J. (2007). Dynamic Mathematics with GeoGebra. Journal of Mathematical Association, 7(1448). https://www.maa.org/external_archive/joma/Volume7/Hohenwarter/

Ledneczki, P. (n.d.). Descriptive Geometry 1. Retrieved March 31, 2020, from http://en.wikipedia.org/wiki/Gaspard_Monge

Mackrell, K. (2011). Design decisions in interactive geometry software. ZDM - International Journal on Mathematics Education, 43(3), 373–387. https://doi.org/10.1007/s11858-011-0327-4

Monge, G., Heather, J. F., & Brisson, B. (1851). An Elementary Treatise on Descriptive Geometry: With a Theory of Shadows and of Perspective: Extracted from the French of G. Monge. To which is Added, a Description of the Principles and Practice of Isometrical Projection. The Whole Being Intended as an I. J. Weale. https://books.google.com.br/books?id=IHZGAQAAMAAJ

Pirklová, P. (2019). Monge Projection. Geogebra. https://www.geogebra.org/m/mjjxrgrt

Pirklová, P., & Bímová, D. (2019). GeoGebra book containing worksheets for Monge projection. In American Institute of Physics (Ed.), AIP Conference Proceedings 2172 (Vol. 2172, p. 040002). American Institute of Physics. https://doi.org/10.1063/1.5133512

Stachel, H. (n.d.). What is Descriptive Geometry for? Retrieved April 15, 2021, from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.10.5562&rep=rep1&type=pdf

Tim Brzezinski. (2021). GeoGebra. International Geogebra Institute. https://www.geogebra.org/