From Sacred Traditions to Digital Syntax: Navigating Formal Grammar's Role in Modelling Contemporary Islamic Geometric Patterns

Ghazal Refalian[1], Eloi Coloma[2], Joaquim Moya[3]
[1,2,3] Universitat Politecnica de Catalunya, Barcelona, Spain

Abstract. Islamic geometric patterns (IGP) have been an integral component of Islamic art for centuries. Developed using traditional techniques and tools, these patterns have been profoundly influenced by them. However, the emergence of digital tools has posed novel challenges to the creation and preservation of these intricate designs, leading to compatibility issues between original methodologies and new technologies. Consequently, the process of generating new patterns based on the same syntax has become notably more intricate. This paper delves into the application of Formal Grammar Methodology (FGM), specifically the string rewriting system (SRS), for digital modeling of star patterns that originated in historical Iran, substantiated by existing evidence. We assess the potential of adapting these patterns to the digital realm while respecting their cultural context for design and production purposes. Our study draws upon both historical and contemporary sources to gather, scrutinize, and analyze patterns utilizing both traditional and innovative approaches. The outcomes reveal an 85% success rate for the introduced method, as opposed to 55% for the traditional approach in production.

Keywords. Islamic geometric pattern, Formal grammar, Shape grammar, String rewriting system, L-system.

1 Introduction

In the realm of contemporary architecture, the interplay between tradition and innovation has become increasingly critical while our built environments evolve rapidly. As architectural practices are shaped by technological advancements, there is a growing interest in harnessing the potential of digital tools and computational techniques while preserving the rich cultural heritage embedded within architectural traditions. This paper explores the seamless integration of FGM as a transformative approach to digitally model and fabricate historical and contemporary Iranian Islamic geometric patterns.

This study is constrained to the star patterns created by disciples of artisans which have been documented in historical records, and as well, in contemporary references and possibly manifest in a few contemporary architectural structures. These references distinctly identify these Islamic geometric patterns (IGPs) as masonry "Girih," setting them apart from their carpentry counterparts. Figure1 shows a few pages of one of these references prepared by Mahmoud Maheronnaqsh.

The primary objective of this research is to investigate the application of formal grammar as a means to bridge the gap between tradition and contemporary architectural practices. By leveraging formal grammar, which encompasses a set of rules and principles governing the arrangement of design elements, we aim to develop a novel approach for the digital modeling of Iranian Islamic geometric patterns.

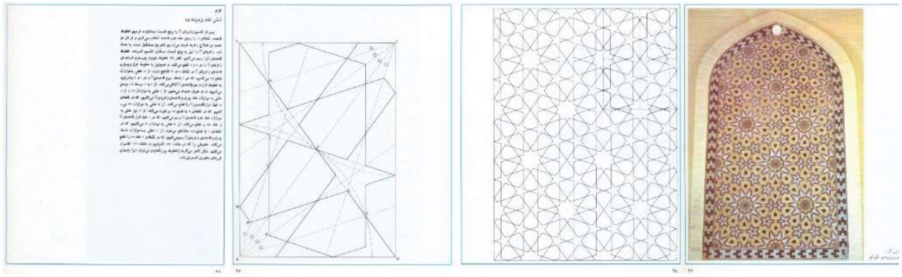


Figure 1. From the Book Five Volumes on Geometric Patterns (Persian), by Mahmoud Maheronnaqsh.

The integration of Iranian Islamic geometric patterns into contemporary architecture holds immense cultural, aesthetic, and functional significance. These patterns act as a bridge, linking modern architectural expressions with the profound artistic heritage of historical architecture. Through the revival and incorporation of these geometric designs into the built environment, architects can infuse spaces with a strong sense of cultural identity and evoke a timeless aesthetic appeal. This integration fosters a renewed appreciation for architectural heritage, underscoring the importance of cultural preservation in an era of rapid technological advancement. By embracing these design elements, architects can enrich the built environment with narratives that transcend time, creating spaces that feel both relevant and firmly rooted in cultural heritage.

2 Journey from Tradition to Code

Understanding the embedded geometrical methods of IGPs has long challenged researchers. The patterns' origins and development are intricate, historically shrouded in secrecy between masters and apprentices (Maheronnaghsh, 1976). This scarcity of evidence, like treatises or sketches, hinders a full explanation. Scholars like Chorbachi (1989)and NecipOglu (1995)mention scrolls and manuscripts, yet these lack a comprehensive insight. In 1879, Bourgoin (1974) released an extensive compilation of IGPs and their sub-grids, positing them as the initial impetus for these designs. This pioneering work was followed by analogous analyses by Critchlow (1976) Wade (1976), and El Said and Parman (1988). Conversely, Salman and Abbas (1992) critique these scholars' methods as excessively intricate, lacking an explanation for pattern evolution. They advocate a logical framework: patterns begin with fundamental shapes, progressing to overlaid shapes, geometric constructions on familiar forms, and concealed grid patterns.

Another approach involves group theory, elucidating pattern structures mathematically. Symmetric patterns are categorized into seventeen types, defining minimal information for generating specific symmetries. Scholars, especially in digital IGP generation, use this method. Grunbaum and She- phard (1987), intricately examine periodic patterns in symmetric groups, albeit excluding IGPs. Abas and Salman (1992), employ group theory to devise an IGP algorithm. Ostromoukhov (1998), extends Abas and Salman's approach to provide systematic guidelines for Moorish art's two predominant symmetry families.

Hankin (1925), introduced an archaeological observation of geometric patterns in an Indian Turkish bath, featuring an underlying polygonal grid. He proposed the "polygon in contact" method, later expanded upon by Lee(2015) in 1987, demonstrating how patterns could extend from tiling edges. Bonner (2017) further refined this concept, offering an extensive collection of IGPs following this methodology. While useful, finding the polygonal grid in some cases is as intricate as the main pattern itself.

Various attempts to digitally encode IGPs and provide parametric models have emerged. Group theory and symmetry-based frameworks, such as Kaplan's algorithm, offer greater flexibility but may overlook core attributes of IGPs. Aljamali et al (2009) emphasize unit pattern attributes, motif components, and geometric properties, introducing the IGP Designer tool. Conversely, Khamjane and Benslimane (2018) employed symmetry groups theory to automate periodic star and rosette patterns, introducing novel variations. The template motifs are constructed based on continuous strands. Later in this research, we will utilize the capacities of continuous strands to code IGPs.

Nadyrshine et al. (Nadyrshine et al., 2021) extended this, introducing the N-angle star pattern for increased parametric ornamentation. Lahcen et al. (2021) introduced Hasba, a symmetric-based approach using software to generate numerous IGPs. Kaplan and Salesin (2004) explored a tiling-based system with parameterized stars and rosettes. They developed an algorithm and tool, called "Najm", to computationally generate IGPs, particularly star patterns, enabling designers to explore the design space. Izadi et al. (2010) devised a computerized algorithm for both traditional and novel geometric patterns through tiling techniques.

Although previously discussed digital tools may not be widely accessible or discontinued, Kaplan's algorithm remains functional within platforms like Rhinoceros-Grasshopper, enabling IGPs' parametric production.

An alternative significant approach for analyzing IGPs is through shape grammar, a mechanism for generating shapes based on an initial shape (axiom) and transformation rules. This concept was initially introduced by Stiny and Gips (1972), although their primary focus was not specifically on geometric patterns. They, beside Lionel March (1981) , extensively formalized the utilization of shape grammar as a system for shape production. Cenani (2006) leveraged shape grammar to explore novel design potentials in IGP creation, deriving new patterns from basic shapes through production rules. Ulu and Sener (2009) applied shape grammar to analyze Penrose patterns and model decagonal IGP patterns. Another utilization of shape grammar for IGPs, pursued by Sayed, Ugail et al (2016), largely pertains to 3D applications of IGPs. Lalvani (1989) introduced shape and morph codes, providing rules and information for generating diverse IGPs through symmetry groups.

The last approach which is topic of this research is implementation of FGM and specifically, string rewriting system (SRS), to generate IGPs. As it is mentioned above, the concept of developing grammars to generate IGP is not new. However, using SRS technique for this purpose is current. In the next section we will discuss this approach.

3 Syntax of Design: The Formal Grammar Framework

A string rewriting system, known as string grammar or production system, is a formal framework that generates symbol sequences using defined rules. It operates on character or word strings, where an initial string transforms into a new one through repetitive rule application. Each rule replaces a pattern with a corresponding string, iterating until no more rules apply or a desired string is achieved. In computer science, these systems are valuable for code generation.

Graphical counterparts arise in Lindenmayer systems (L-systems), introduced by Aristid Lindenmayer (1990) in 1968 to model plant growth. Prusinkiewicz and Hanan (1988) expanded L-systems to fractals and tiling. Paul Bourke (1991) extended this to recursive tiling. Refalian et al (2021) explored rewriting systems for modeling Islamic geometric patterns (IGPs), employing specific rules in a step-by-step process. Each step builds patterns using axioms and production rules. Unlike other methods, this resembles machine language, enabling efficient coding. It models patterns based on samples without drawing techniques or hidden sub-grids, while geometric details like angles and proportions define the grammar. Current study expands this methodology and develop it to be applied on a larger family of patterns and examine its capacities in this regard.

In a string rewriting system, each shape has a string equivalent and an alphabetic character as its name. Each symbol in the string represents a line with a certain length, or an angle to define the direction of the next line. For example, a pentagon in the fivefold family can be presented as "P→y+y+y+y+y", which 'y' is equal to a line with the unit length and "+" means a 72 degrees clockwise rotation to start the next line.

To understand the relation between strings and the graphics, one should imagine that

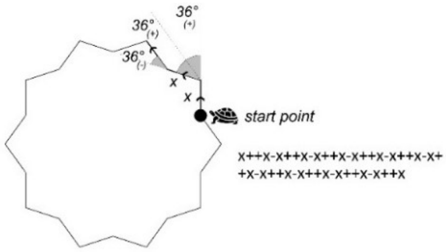


Figure 2. Making strings from a geometric shape in the pattern.

these strings guide a pen through a path to draw the pattern (This Considering the mentioned feature, defining a morphological grammar for IGPs means defining the connections between shapes in a manner that they can be described as axioms and rewriting rules. For example, in (Figure 3), the central star of patterns and the surrounding pentagon can be written as:

S→ "x++x-x++x-x++x-x++x-x++x-x++x-x++x-x++x-x++x-x++x"

P→ "y++y++y++y++y"

("x"= 1 unit, "y"= 1.618 unit, "+" =36° clockwise, "-"=36° counter clockwise)

To draw the pattern in (Figure 3), the drawing path is similar to what we can see in (**¡Error! No se encuentra el origen de la referencia.**, image 4). It means that while we are drawing S, we have to insert the P inside the strings of the star. We start from the start point to draw one '-x' and later continue by drawing the pentagon. In the end, we add another 'x' and repeat this action 5 times. The following procedure can be shown schematically as: "-xPx-xPx-xPx-xPx-xPx-xPx-xPx-xPx-xPx-xPx".

To proceed with the pattern in (Fig 3., image 5), this time we should take the strings for the pentagon and insert the new shape's sentence inside it. If the new shape is:

T→ "y++x-x++y++y++x-x++y"

Then we need to insert it through the pentagon as follows: "y++yTy++y++y".

To write the complete grammar, an axiom of "A+A+A+A+A" can be considered, in which the five 'A's respond to the 5-fold symmetry. Each 'A' will be replaced

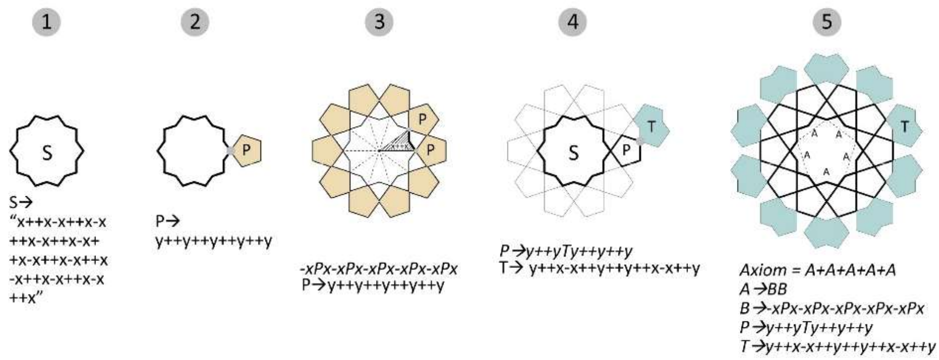


Figure 3. Steps of making the grammar for the pattern.

by "BB", to achieve to the ten branches of the star; then inside 'B' there is a parameter called 'P' and inside 'P' there is another parameter called 'T'.

Axiom = A+A+A+A+A
A→BB
B→-xPx-xPx-xPx-xPx-xPx
P→y++yTy++y++y
T→y++x-x++y++y++x-x++y

As we can see, to model an IGP by SRS technique, three steps are considered:

1. *Shape Definition: converting the shapes of the pattern into their corresponding string representations.*
2. *Branch-making: Creating a linked sequence of shapes originating from the central star form.*
3. *Re-writing formula: a convertor to generate polylines from strings*

The fact is that defining the formal grammar for such a complicated object like IGP is not easy. Moreover, there are hundreds of patterns, and defining such grammar manually is a hard and time-consuming work, which increases the chance of getting lost in the path and it is possible to confuse all the letters and symbols. This is the point that a computational aided tool is required to assist the process.

In constructing a CAD toolkit, completing steps one and three suffices for applying this method. The user will handle the second step, aided by CAD tools. It results in the development of two primary components: The first is the String Body (StB), which generates a string representation of a shape or pattern (Figure 4). This facilitates creating a collection of shapes and patterns that can be easily shared using the string formula. The second component, the Shape Body (ShB), converts a string back into its original shape or pattern form.

Figure 5, in step 1 shows the shape component produced by FGM and use the strings as input, and step 2, uses SRS and turtle graphics basics to convert the strings to the geometry. The order or connecting the shapes follows the branch making procedure.

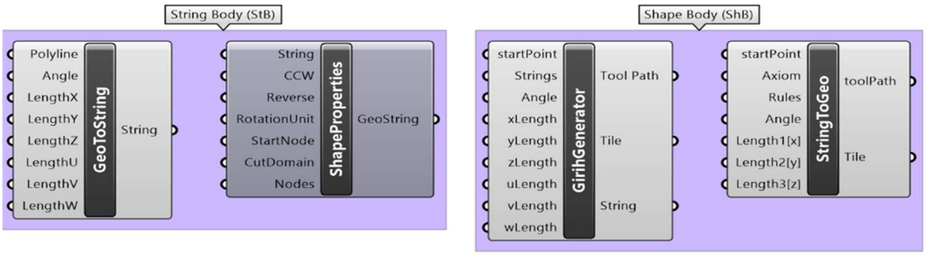


Figure 4. Main components of the tool kit. From Left to right, 1.GeoToString: convert a polyline shape to its string equivalent. 2.ShapeProperties: Gives access to the markers of a shape to make the connections between shapes and to manage them. 3.GirihGenerator: Converts the final produced string to a polyline by turtle graphics rules. 4.StringToGeo: Uses SRS and only strings to produce the patterns without any graphical mediation.

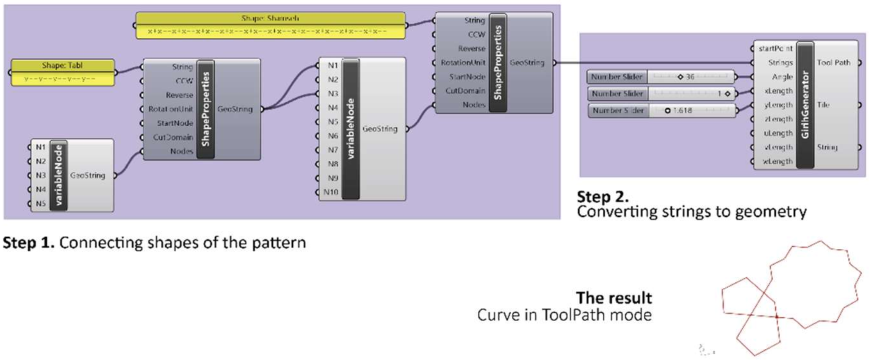


Figure 5. A close up of the pattern coding with the toolkit.

4 Assessing the Formal Grammar Methodology

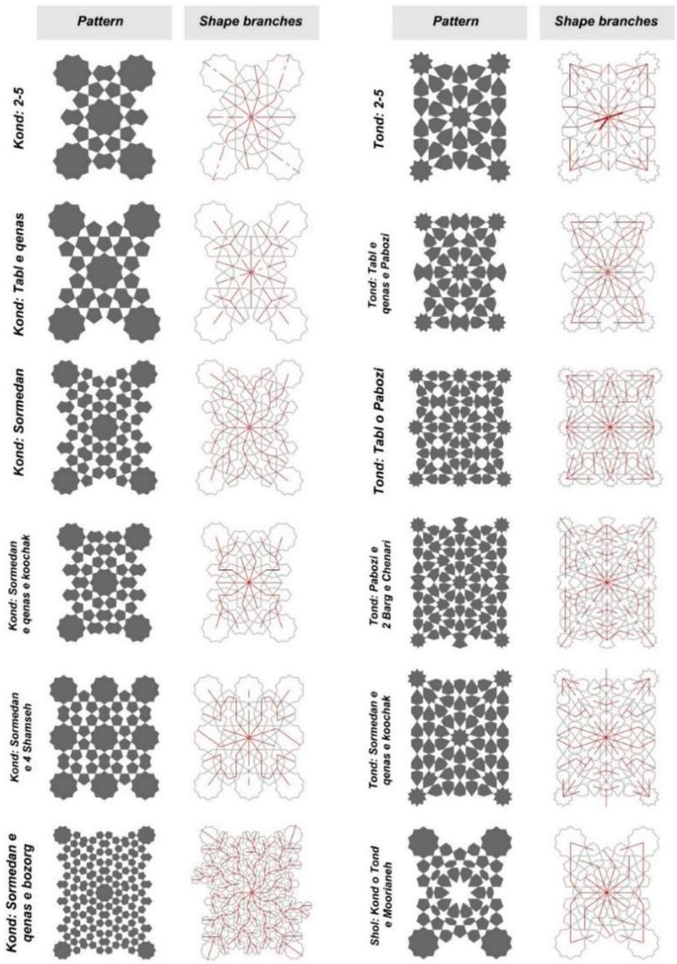
The Formal Grammar Methodology has emerged as a potent approach in computational design, particularly for generating complex geometric patterns. To evaluate the efficacy and applicability of this methodology, we embarked upon an investigation aimed at developing an add-on for Grasshopper plug-in in Rhino3D software, serving as a toolkit for designers. This section navigates the systematic assessment of the FGM and string rewriting system through its utilization in this context and its subsequent evaluation in an experimental workshop involving architecture students. The development of a computational toolkit utilizing the SRS technique aimed to provide designers with a versatile means for generating intricate geometric patterns. The process involved encoding the generative rules and instructions into the toolkit, enabling designers to effortlessly produce a wide array of patterns. To gauge the practical effectiveness of the methodology in a real-world scenario, an experimental workshop was conducted involving two groups of 12 volunteers of architecture students without or with low previous experiment on IGP. Two introduction sessions were provided in order to make a preparation with the topic. Later, 42 patterns by four prominent Persian references in

Islamic art were selected to implement the application of the developed toolkit for modelling the case studies:

1. Shaarbaaf (from the book Girih & Karbandi (2006))
2. Helli (from the book Girih and arc (1987))
3. Lorzadeh (from the book Revival of the Forgotten Arts: Principles of the Traditional Architecture in Iran According to Hossein Lorzadeh (1995))
4. Maheronnaqsh (from the book Five Volumes on Geometric Patterns (1976))

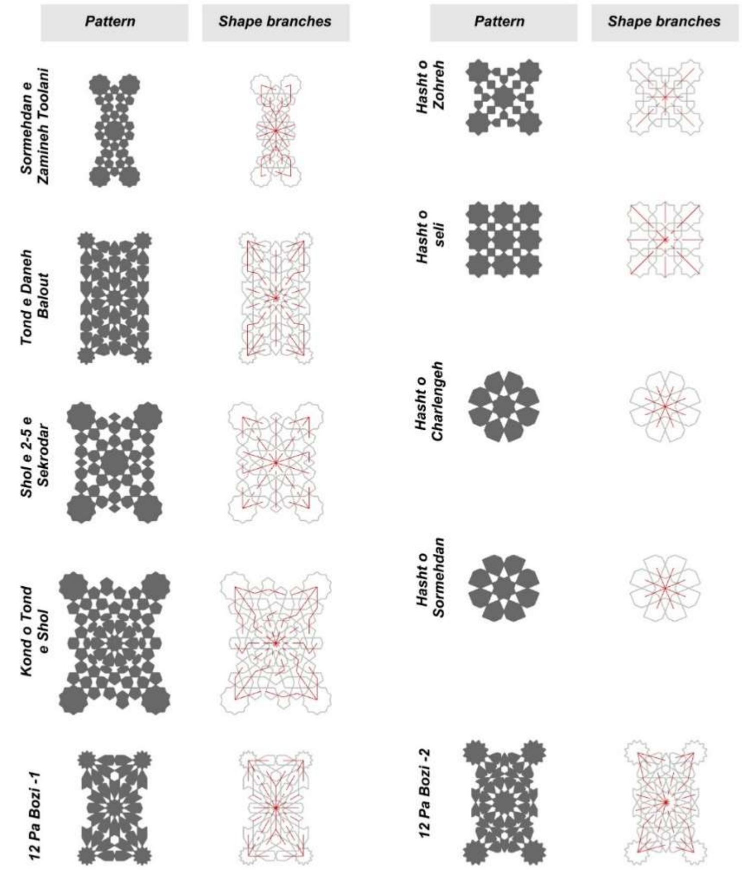
Table 1 shows the 22 pattern that successfully modeled for this study. 11 patterns had geometric mistakes that the original designer tried to hide the tolerances during the drawing. 7 patterns did not have sufficient drawing instructions and due to time limitation were eliminated from the process, and 2 patterns were not adaptable with this methodology (it was not possible to find a unit length or angle). The evaluation of the Methodology was carried out based on an analysis of the workshop outcomes in three categories:

Table 1 library of the patterns which are produced with FGM. Second column shows the branching step, done by users.



1. **Pattern Accuracy:** Results indicated that approximately 25% of the original patterns exhibited mistakes, showing the importance of improving the accuracy of the decoding process.
2. **Instruction Sufficiency:** Around 15% of the patterns were associated with insufficient instructions, impeding participants' ability to accurately reproduce the desired patterns. This highlighted the need for comprehensive and precise rule formulation.
3. **Adaptability Challenges:** A subset of patterns, constituting 5%, were found to be incompatible with the FGM. This matter raised awareness regarding the limitations of the methodology in handling certain intricate design patterns.

Table 1. Continued.



pattern generation in comparison to conventional design techniques. For this survey the following parameters were questioned (Fig 6):

1. Accuracy of pattern generation (SRS:100% - TT:60%)
2. Flexibility and customization (SRS:70% - TT:40%)
3. Time efficiency (SRS:60% - TT:40%)
4. Innovative design exploration (SRS:80% - TT:60%)
5. Overall efficiency (SRS:85% - TT:55%)

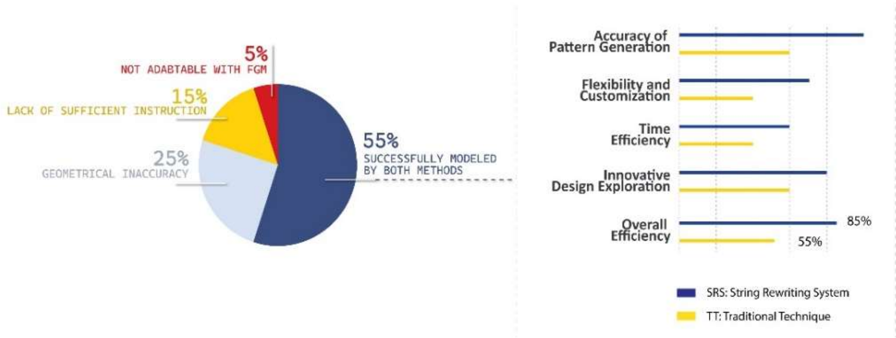


Figure 6. The evaluation the FGM and the toolkit.

Considering the innovation and design exploration, in one case reported a generation of over 3200 variations of a single pattern by replacing specific strings of the pattern with new alternations:

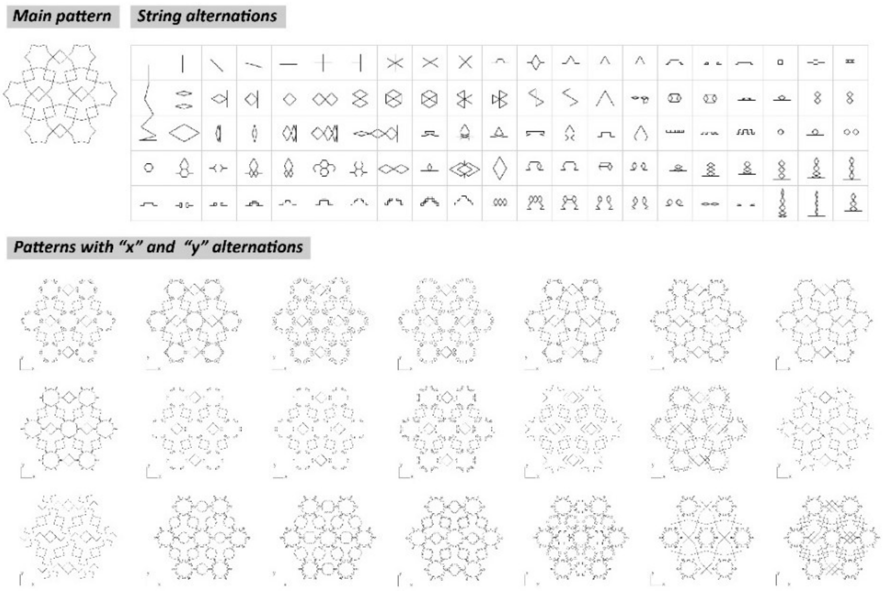


Figure 7 Design space exploration for generating new alternatives by replacing "x" and "y" characters with new strings. 50 strings for “x”, and 55 strings for "y" reached to 2750 alternatives. 21 cases were selected for printing purposes. (Developed by Mahdi Akbari Ahangar)

5 Discussion and conclusion

This research explored the relationship between conventional approaches and digital advancements in the field of Islamic geometric patterns (IGP). The study focused on the Formal Grammar Methodology (FGM), particularly the string rewriting system (SRS), as a conduit for digitally modeling star patterns within Iran's historical context. The study found that IGPs are compatible with SRS. Through a branched morphological structure for an IGP and by placing marker symbols within the string equivalent of each shape, it is possible to model the shapes of a class of star patterns with SRS.

A toolkit developed in Rhino3D Grasshopper, and two experimental workshops with two groups of 12 volunteers were held to work on 42 patterns from five reference books. The comprehensive assessment of FGM unveiled its strengths, weaknesses, and the potential applications in computational design. The research compared traditional and introduced methods, resulting in a significant 85% success rate in pattern production using FGM, compared to the 55% success rate of conventional approaches. The favorable perception of participants regarding heightened efficiency in design workflows reaffirms the practicality of FGM in contemporary design contexts.

The applied technique made a significant contribution to the analysis of traditional instructions, digital modeling, and restoring a wide range of IGPs in Iran's cultural heritage. However, some observations were made:

1. A primary study and analysis are essential for utilizing the SRS method in star pattern modeling. SRS is primarily a modeling technique rather than a design tool, not recommended for starting star pattern development from scratch. However, once the model is prepared, adjusting parameters enables a wide range of new pattern iterations.
2. Traditional pattern classifications often differed from parameter sets within a class. Various original design techniques led to parameter variations. For instance, in the 4-fold class, some patterns utilized lines and grids, while others employed polygons in contact techniques. Patterns with the same technique shared similar parameters, distinguishing them from others.
3. Creating acceptable patterns while adhering to traditional principles presented challenges due to the strict domain of valid patterns in traditional practitioners' minds and their limited production techniques. However, leveraging computational tools allowed students to generate numerous new variations merely by replacing existing strings with new ones.
4.

Future directions involve refining FGM precision with algorithmic enhancements and integrating machine learning for greater automation. Additionally, expanding the pattern library to encompass diverse Islamic artistic styles, digitally preserving and restoring historical IGP, and developing educational tools for practitioner learning and production

Acknowledgements. We are grateful to all volunteers who actively participated in the experimental implementation of the methodology presented in this paper, including: Fatemeh Dehqan Shadkami, Rasha Kiani, Mohammaamind Matinfar, Fatemeh Mirmousavi, Rozita Partovi, Fatemeh Nazarian, Mahdi Akbari Ahangar, Ana Azhdar, Ayda Aliasgarian, Katayoun Moslemi, Kimia Salimzadeh, Mohammad Pourfuladi, Shadi Akbarian, Soroush Amidifard, Atefeh Shahrashoob, Elnaz Asemi, Elnaz Kakoui, Elham Ghaderi.

References

Abas, S. J., & Salman, A. (1992). Geometric and Group-theoretic Methods for Computer Graphic Studies of Islamic Symmetric Patterns. Computer Graphics Forum, 11(1), 43–53. https://doi.org/10.1111/1467-8659.1110043

A.J.LEE. (2015). Islamic Star Patterns Author(s): A. J. Lee Source: Brill, 4(1987), 182–197.

Aljamali, A. M. (2009). Classification and design of islamic geometric patterns using computer graphics. Proceedings - 2009 2nd International Conference in Visualisation, VIZ 2009, 3, 253–258. https://doi.org/10.1109/VIZ.2009.46

Bonner, J. (2017). Islamic Geometric Patterns: Their Historical Development and Traditional Methods of Construction. Springer.

Bourgoin, J. (n.d.). Arabic Geometrical Pattern and Design (New editio). Dover Publications Inc.;

Bourke, P. (1991). Paul Bourke. http://paulbourke.net/fractals/lsys/

Cenani, S., & Cagdas, G. (2006). Shape Grammar of Geometric Islamic Ornaments. 290–297.

Chorbachi, W. K. (1989). In the Tower of Babel: Beyond symmetry in islamic design. Computers and Mathematics with Applications, 17(4–6), 751–789. https://doi.org/10.1016/0898-1221(89)90260-5

Critchlow, K. (1976). Islamic patterns: An analytical and cosmological approach. Thames and Hudson Ltd.

Grunbaum, B., & Shephard, G. C. (1987). Tilings and patterns. Dover Publications.

Hankin, E. H. (1925). The drawings of geometric patterns in Saracenic art. Memoirs of the Archaeological Society of India (Vol. 15). Government of India.

Helli, A. (1987). Girih and arches in islamic architecture. Mehr.

I. El-Said, A. P. (1988). Geometrical concepts in Islamic art. World of Islam Festival Publ. Co.

Izadi, A., Rezaei, M., & Bastanfard, A. (2010). A Computerized Method to Generate Complex Symmetric and Geometric Tiling Patterns. In SCI (Vol. 321).

Kaplan, C. S., & Salesin, D. H. (2004). Islamic Star Patterns in Absolute Geometry.

Khamjane, A., & Benslimane, R. (2018). A computerized method for generating Islamic star patterns. CAD Computer Aided Design, 97, 15–26. https://doi.org/10.1016/j.cad.2017.11.002

Lahcen, Y. (2021). New approach to construct a new Islamic geometric patterns using the Hasba method. Journal of Theoretical and Applied Information Technology. 3663-3672, 99(14)

Lalvani, H. (1989). Coding and generating complex periodic patterns. Visual Computer.Springer. Pp.180-202

Lindenmayer, A., & Prusinkiewicz, P. (1990). The Algorithmic Beauty of Plants. The virtrual labratoary. Springer.

Maheronnaghsh, M. (1976). Five Volumes on Geometric Patterns (Persian). Reza Abbasi Museum.

Mofid, H., & Raieszadeh, M. (1995). Revival of the Forgotten Arts: Principles of the Traditional Architecture in Iran According to Hossein Lorzadeh. Mola.

Nadyrshine, N., Nadyrshine, L., Khafizov, R., Ibragimova, N., & Mkhitarian, K. (2021). Parametric methods for constructing the Islamic ornament. E3S Web of Conferences, 274, 09009. https://doi.org/10.1051/e3sconf/202127409009

Necipoglu, G. (1995). The Topkapi Scroll: Geometry and Ornament in Islamic Architecture. GETTY CENTER FOR THE HISTORY.

Ostromoukhov, V. (1998). Mathematical tools for computer-generated ornamental patterns. Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), 1375, 193–223. https://doi.org/10.1007/BFb0053272

Prusinkiewicz, P., & Hanan, J. (1988). Lecture notes in Biomathematics. Springer-Verlag.

Refalian, G., Coloma, E., & Moya, J. (2021). FORMAL LANGUAGE METHODOLOGY FOR VISUALIZATION OF ISLAMIC GEOMETRIC. International Journal of Architectural Computing, 29–31.

Sayed, Z., Ugail, H., Palmer, I., Purdy, J., & Reeve, C. (2016). Parameterized Shape Grammar for n-fold Generating Islamic Geometric Motifs. Proceedings - 2015 International Conference on Cyberworlds, CW 2015, 79–85.

Shaarbaaf, A. (2006). Girih & Karbandi (3rd ed.). Miras Farhangi Institure.

Stiny, G., Gips, J., Freiman, C. V, & Processing, I. (1972). Shape Grammars and the Generative Specification of Painting and Sculpture by George Stiny and James Gips. 71, 125–135.

Stiny, G., & March, L. (1981). Design machines. Environment and Planning B, 8, 245–255.

Ulu, E., & Sener, S. M. (2009). A Shape Grammar Model To Generate Islamic Geometric Pattern. Proceedings of the 12h Generative Art Conference, 290–297. http://celestinosoddu.com/on/cic/GA2009Papers/p25.pdf

Wade, D. (1976). Pattern in Islamic Art. The Overlook Press,.