

## 3D Dynamic Models of Line Changing Planes and Plane Revolution Methods in Descriptive Geometry

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**Abstract.** This paper presented 3D dynamic models representing two fundamental methods in descriptive geometry changing planes and revolution. The objective is to display the true size of the objects analyzed in 3D space and their adjacent views. Despite the relevance of the topics in descriptive geometry, these are contents that students struggle to consolidate. Thus, it has happened because the widely applied resources in the teaching-learning process do not support the procedures required to understand them. To fulfill this demand, we built 3D dynamic models regarding the line changing planes and the plane revolution in GeoGebra. After creating the models, a survey questionnaire was sent to specialists, for accessing and evaluating. As a result, the answers indicated that the 3D dynamic models conveyed the information regarding descriptive methods effectively. Additionally, above 60% of the participants intend to adopt the dynamic models in their classes.

**Keywords:** Descriptive Geometry, GeoGebra, 3D dynamic models, Descriptive methods, Adjacent view.

### 1 Introduction

True size representation is primordial to obtain the exact size of the lines and angle measurement of the objects in the 3D domain. The true size is obtained with the descriptive methods applications. However, descriptive methods are considered advanced topics which students in technical courses and college struggle most, due to their underdeveloped visual skills requirements at the beginning of the course (Contero et al., 2005). Additionally, some resources in the teaching-learning process are not enough to support this demand. Descriptive geometry is frequently taught by drawing utensils. Nevertheless, it does not fit with the current college students' profile which requires more and more interaction with digital resources (Di Paola et al., 2013). This demand is intensified, due to the necessity of imagining the represented object moving or rotating. In the changing planes method, it needs to imagine the represented object fixed and moving one of the projections. Conversely, the revolution process entails inserting the object inside a plane and rotating, to obtain its true size planes (Asensi, 2000). In order to master visual skills, it is fundamental choosing the appropriate techniques to effectively support

students in assimilating the true-size objects concept (Ferdianová, 2017; García et al., 2007; Quintilla Castán & Fernández Morales, 2021).

To provision this demand, the authors developed dynamic 3D models in GeoGebra to represent the concepts of changing planes and revolution. These methods belong to the descriptive methods class (Asensi, 2000; Hohenwarter, 2006, 2023; Pirklová & Bímová, 2019).

This paper aims to create interactive 3D models that allows students to associate the space and adjacent view representations of a line and a plane on each dynamic model in the mentioned descriptive methods applications. Likewise, they are didactic resources for teachers to support the teaching learning process of their descriptive geometry classes.

Therefore, this paper is sorted into the following sections. First, the introduction. Then, the literature review with the theoretical fundamentals concerns descriptive geometry and its association with GeoGebra. Next, the method is applied in the paper. Besides that, it presented the construction principles of the two dynamic models, the questionnaire elaboration, and the survey participants' profiles. The following section regards the analysis of the results collected in a survey questionnaire. Finally, the conclusions.

## **2 Literature Review**

The discussion about the descriptive geometry teaching-learning process is frequent. Even currently when plenty of graphic resources are available for deployment due to two main reasons. First, the modifications concern the educational environment. Beyond that, the offer of online teaching and hybrid structures are increasing after the COVID-19 pandemic and the students' demands (Almeida & Castro, 2021). Second, the aspects regarding the teaching-learning process in descriptive geometry. The following references pointed out some aspects, which are fundamental to consider in carrying out new tools for descriptive geometry teaching.

García et al. (2007) shed light on the teaching-learning process in descriptive geometry. In this sense, they researched their teaching techniques and the media applied. It objected to investigate the factors that influence visual skills learning, for elaborating a multimedia tool to teach descriptive geometry. Hence, they found the theoretical explanation combined with the dynamic computer images is the ideal resource to deploy in classes. Beyond that, they discovered that students require a logical sequence and image association among distinct representations of the same object, in the explanation, to assimilate it accurately.

To figure out the most attractive strategy for the students for learning visual skills, Contero et al. (2005) tested the visual abilities of Engineering undergrads, with a myriad of visual and spatial skills tests. Hence, they conclude the deployment of graphic design software is helpful to consolidate and acquire

visual rotation skills because it supports object movement comprehension. This reference brought the essential aspect involved in descriptive methods teaching-learning process, the visual rotation skills. Descriptive methods involve visual rotation to change the position of the projection planes or the object. They correspond to the changing planes and the revolution, respectively.

Considering the necessity of consolidating visual skills, as well, Ferdiánová (2017) applied GeoGebra resources in online teaching managed by an LMS platform. Likewise, it was one of the prior applications of GeoGebra in descriptive geometry online teaching. This author aimed to convey information about Monge Projection using interactive representations in GeoGebra. Hence, her work was fundament on the adjacent view representations. This is an important aspect of the teaching-learning process of descriptive methods because the association between the 3D representation of an object and its adjacent view is fundamental to understanding how descriptive methods work. Despite that, this paper did not cite the association aspect.

GeoGebra is an appealing tool for building descriptive geometry models, because of two main reasons (Hohenwarter, 2006). Primarily, it is possible to divide the software into different windows, as demonstrated in Almeida & Castro (2021). Besides that, it is possible to suit the interface. Finally, the accessibility is appealing. It is necessary to have an internet browser. Furthermore, it is feasible to download the GeoGebra app and the files \*.ggb on computers, cellphones, or tablets to avoid internet connection issues.

In the literature, after the pandemic, some papers regarding descriptive geometry GeoGebra applications were published (Khrishnamurti, 2020; Quintilla Castán & Fernández Morales, 2021). They contain the association between the 3D space and the adjacent view representations Almeida & Castro (2021) and Silva de Almeida & Schneider de Castro (2023). Hence, they applied descriptive geometry concepts, aiming at the visual aspect, also, and evaluated the models by questionnaires. Nevertheless, the last ones deployed a descriptive method application in a point.

In order to contribute to the literature this paper aims to propose a tool directed to descriptive methods to support the teaching-learning process of these advanced concepts. Hence, the method implemented to achieve this goal is described in the next section.

### **3 Method**

The method applied in this paper is based on Silva de Almeida & Schneider de Castro (2023) and Pirklová & Bímová (2019). Furthermore, we surveyed to acquire information about the interaction of the dynamic models.

First, we developed the 3D dynamic models in the GeoGebra app to represent the changing planes applied to a line and the revolution of a plane.

Then, we offered the 3D dynamic models to professors and lecturers, who lecture in Brazilian universities and technical schools for testing. Beyond that, there was a survey that follows them. We divided it into two sections. First, the section about the 3D dynamic model of Line Changing Planes. In the sequence, the section concerns the Plane Rotation 3D dynamic model. At the end of each section, there was a video tutorial explaining how each model precisely works. The participants must answer all the questions available in the two sections.

The questionnaire presented yes/no questions and questions based on the Likert scale of 5 points (Joshi et al., 2015). In this sense, the first ones intended to confirm if the user comprehended how the dynamic models work. Additionally, they checked if the participants knew the concepts involved in them. On the other hand, the questions based on the Likert scale aimed to evaluate the users' comprehension level of the models and their first impressions. They ranked the 3D dynamic models by giving a grade from 1 to 5.

Next, the analysis of the answers. To do the analysis properly, we calculated the average grades resulting from the Likert scale questions and their standard deviation. Moreover, we evaluated the percentage of affirmative answers in the yes/no questions.

Finally, we organized the analysis results and pointed out the contributions of this paper in the conclusion.

## 4 Results and Discussion

The questionnaire was applied for 1 month and 1 week, from July to August 2023. Moreover, it was sent to 100 Brazilian professors of descriptive geometry and technical drawing, who work in technical schools, public and private universities. 19 answered the questionnaire. Among them, 15% received training courses in other 3D dynamic models, with similar features. To clarify the information, the projection planes of each system were identified as demonstrated in Table 1.

Table 1. Identification of the projection planes in the dynamic models.

Figures		Words
The horizontal plane	original projection	Pi1
The vertical plane	original projection	Pi2

The horizontal plane	The modified projection	Pi3
The vertical plane	The modified projection	Pi4

In this sense, it was fundamental to establish a convention in the projection planes' identification. Hence, we attributed the characters Pi1, Pi2, Pi3, and Pi4, to assimilate it with the Greek letter  $\pi$ , which is applied in projection planes identification. GeoGebra presented a limitation in inserting Greek letters in the objects' identification. Besides that, we adopted different colors for the modified planes from the original ones.

Regarding to the models' erection, we integrated 2D and 3D interfaces to associate the slider controls to the 3D elements.

#### 4.1 Line Changing Planes Dynamic Model

The line Changing plane model was available to respondents at the beginning of the questionnaire for testing. After testing, the participants answered questions related to the dynamic model displayed in the line Changing planes section. Likewise, this model is illustrated in Figure 1.

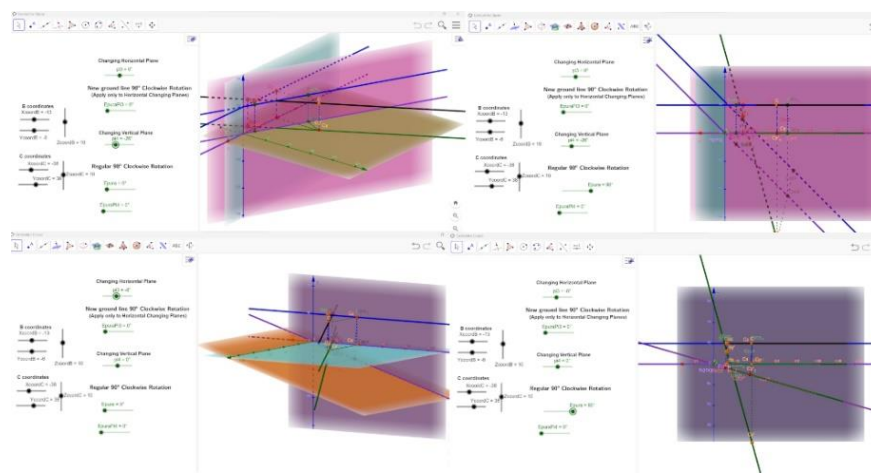


Figure 1. Line Changing planes dynamic model. Source: Authors.

According to the answers, 100% of the participants answered assertively about the model's comprehension. Beyond that, all realized that the Line changing planes model had the objective that gave its title. 89.5% perceived it was possible to compose the adjacent view rotation. The same percentual understood how to read it, in the original system. However, when the participants tested the rotation, to obtain the adjacent view in the modified systems, some did not identify that it was possible to generate the adjacent view and presented interpretation difficulties (15.8%). Besides that, in the system Pi2Pi3, 68.4% perceived where the line projections were in the adjacent view. In this sense, in the system Pi1Pi4, 78.9% identified. Hence, this identification was simpler in comparison to Pi2Pi3. The clearness level of the Line changing planes model, according to the comprehension of the interpretation of the 3D images and the adjacent view composition is demonstrated in Table 2.

Table 2. Comprehension levels of the Line changing planes dynamic model.

Questions	Average Grade (Likert scale)	Std Deviation
Which grade do you attribute to the simplicity of the adjacent view composition in the original system?	3.95	0.911
Which grade do you attribute to the visual comprehension of the 3D space in the original system?	4.47	0.772
Which grade do you attribute to the visual comprehension of the 3D line image in the Pi1Pi2 system (after Changing planes application)?	4.32	0.885
Which grade do you attribute to the visual comprehension of the adjacent view image in the Pi1Pi2 system (after Changing planes application)?	4.05	1.129
Which grade do you attribute to the visual comprehension of the 3D line image in the Pi1Pi4 system (after Changing planes application)?	4.16	0.898
Which grade do you attribute to the visual comprehension of the adjacent view image in the Pi2Pi3 system (after changing plane application)?	3.74	1.485

After testing the dynamic model 63.2% reported its comprehension without consulting the video tutorial. 84.2% intended to implement this 3D dynamic model in their classes. It is noticeable that the answers presented low variety, which conveys the reliability of the average grades. The adjacent view comprehension in the Pi2Pi3 system presented the lowest average and the highest standard deviation. Consequently, it means the comprehension level of this topic is not satisfactory because the average grade is inferior to 2.5 in the minimum boundary.

#### 4.2 Plane Revolution Dynamic Model

Similar to the model above, the Plane rotation dynamic model was available to the participants at the beginning of the questionnaire. Furthermore, the video tutorial was kept in the end. In comparison, the Plane rotation model was more challenging to understand than the first. Besides that, the answers demonstrated the first question that 89.5% answered positively about how the sliders work. It is important to clarify that the slider controls related to points in this model aimed to change the position of the represented plane. The points commanded by the sliders were hidden. We opted to hide the points to keep the 3D image lean. Moreover, they generated the represented plane, identified by the pink color, as shown in Figure 2.

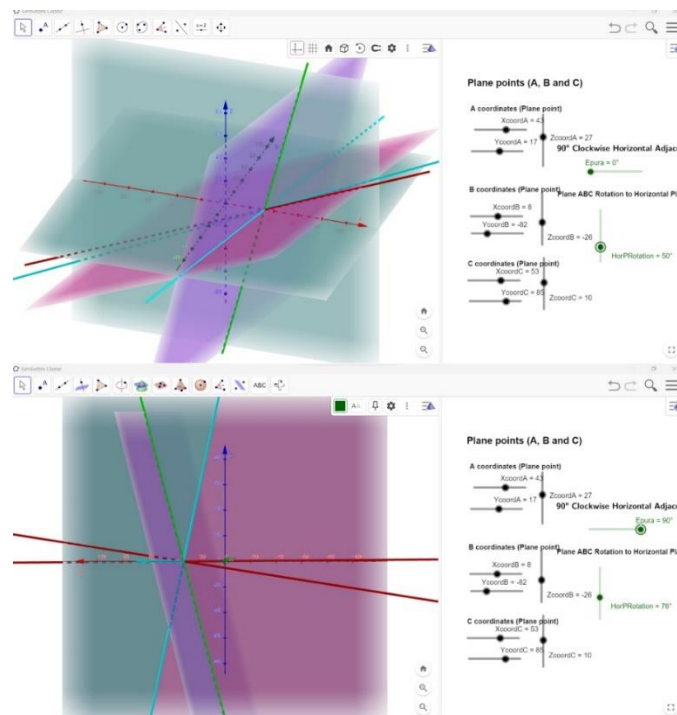


Figure 2. Plane revolution model. Source: Authors.

Table 3. Comprehension level of the Plane revolution model

Questions	Average Grade (Likert scale)	Std Deviation
Which grade do you attribute to the simplicity of the adjacent view composition?	3.79	1.182
Which grade do you attribute to the visual comprehension of the 3D space?	3.68	1.157
Which grade do you attribute to the visual comprehension of the adjacent view of the plane revolution under Pi1?	3.95	1.223
Which grade do you attribute to the visual comprehension of the plane revolution representation under Pi1 in the 3D space?	3.68	1.335

It is noticeable by the average grades, presented in Table 3 that the comprehension level of the Plane rotation dynamic model was undefined. Besides that, the standard deviation was significant in all the topics evaluated by grades. Thus, the minimum boundary would be smaller than 2.5, except in the adjacent view after the revolution execution. Some aspects such as the intersections of the represented plane require improvements. In this context, less than 70% of the participants identified them, before and after applying the plane rotation.

Therefore, the first model demonstrated to be assertive in most of the evaluated aspects. Conversely, the second one did not display similar results. This evidence demonstrates the importance of testing the models with specialists, in the sense of conveying the information properly, before applying them in the teaching-learning process. In the following section, we pointed out the conclusions of this paper.

## 5 Conclusions

A contribution of this paper is offering a new tool developed in GeoGebra to support advanced content in descriptive geometry teaching. According to the results, it needs the association between the 3D space representation and the adjacent view, as well as to understand descriptive methods concepts.

Another contribution of this paper is the evaluation system of the dynamic models. The questionnaire included yes/no and Likert scale questions. Likewise, the yes/no questions preceded the Likert scale to relate the comprehension level with the yes/no answer. In this context, there were



negative answers that the participants attributed grade 3 to his/her comprehension level of the dynamic model.

In the sense of the received answers, the line Changing planes proves simple to understand. However, the comprehension of the Plane revolution model received an average grade above 3, which is essential to remember the answers' variability was expressive. Hence, the Plane revolution model needs to incorporate the suggestions given by the participants, to enhance the clearness of the 3D representations.

Finally, the dynamic models elaborated in GeoGebra allow flexibility for professors and students. The authors did not ask the participants which is the format of their classes, which could be considered a limitation of this study. As similar developing this study in other countries, in order to analyze distinguish educational contexts. Nevertheless, the models can be utilized in online, hybrid, and in-person classes. They do not require special software for installation due to the possibility of use in the internet browser. Therefore, it permits online and in-person teaching application.

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