Bench for observing factors that influence shimmy phenomenon

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ABSTRACT

Shimmy, also known as wobble, is a self-excited oscillation (usually with a frequency between 4 and 10 Hz) that occurs in wheels, especially the front wheels of motorcycles, aircraft landing gear, and automobiles. This effect is influenced by physical quantities such as motorcycle fork stiffness, trail, caster angle, system speed, wheel bearing clearance, tire characteristics, the center of mass height, rolling inertia, and the distance between the center of mass and the rear wheel axle. The energy required to sustain self-excitation is provided by the tire-ground reaction forces and moments exerted on the tire. This work aims to study the phenomenon of shimmy, presents its behavior, demonstrate it in practical examples, and describes the characteristics of the effect and the conditions for attenuating or aggravating the phenomenon. For this purpose, a small-scale experimental bench with 5 degrees of freedom is used, in which physical quantities are altered and their effects on shimmy are observed. As a result of this paper, based on a matrix of experiments, sensitivity analyses provide guidelines for the design of a passive control of the shimmy effect.

INTRODUCTION

The growing flow of people worldwide, whether by car, aircraft, train, or boat, together with regulations aimed at decarbonization and increased safety, has driven research into the development of new technologies to improve the performance and safety of these transportation modes [1,2]. Thus, in the case of vehicles that have contact with the ground, whether aircraft with contact with the runway during taxiing or cars and motorcycles that have constant contact, tire-ground interaction is extremely important for understanding the acting forces and defining stability regions. This analysis is necessary to guarantee the structural integrity of the road system and increase occupant safety [3,4]. One of the phenomena that directly affects stability is the shimmy effect, which generally occurs within the front wheels of an automobile and with aircraft landing gears.

The phenomenon known as shimmy is a self-excited oscillation that occurs in the landing gear of aircraft during ground maneuvers, such as taxiing, take-off and landing, and on motorcycles and automobiles [5]. The effect is characterized by lateral and torsional oscillations of the front wheel axle. This effect can be induced by various factors, including geometric characteristics of the tire support, tire properties, and road conditions [6]. These oscillations can compromise operational stability, reduce the useful life of components, cause structural failures, and lead to accidents [7].

The study of the shimmy effect is essential for the development of efficient damping systems and for the optimization of tire support design. Thus, several studies have explored the dynamic characteristics of the tire with the ground, describing the shimmy effect [8,9]. In the works presented in [10] present a model that describes the contact forces acting on the tire, known as the Magic Formula. By studying the contact interface, Pacejka also presents an approach to the shimmy effect, where the tire is considered massless, and the contact line is approximated by a tangent line to the actual contact line at the leading edge [11]. The author also proposes an approach that only considers a single point. In addition, he presents approaches that consider or do not consider the effect of tread width. A model for studying the shimmy phenomenon is also presented in [12], based on a structure in which the tire is mounted on a fork, and the fork rotates around its own axle. Thus, these models make it possible to simulate and predict the behavior of the landing gear under various operating conditions, making it easier to identify critical parameters and evaluate the effectiveness of control solutions.

In order to validate and improve these models, it is necessary to develop experimental benches that reproduce real operating conditions. These benches allow direct observation of the phenomena, accurate measurement of dynamic variables and the carrying out of controlled tests by varying specific parameters [13]. These benches, therefore, contribute to the calibration of mathematical models, the development of control systems and the testing of different technologies to mitigate the shimmy phenomenon.

In this context, this study proposes the development of a small-scale bench to represent the shimmy phenomenon. The bench is designed to simulate dynamic operating conditions, allowing experimental analysis of the phenomenon and comparison of existing mathematical models. To do this, the bench is equipped with sensors and actuators. A stability curve is obtained by analyzing the variations in trail and tire speed while observing the oscillation cycle limits.

SHIMMY PHENOMENON

Shimmy has its own natural frequency and typically manifests when the wheel's rotational speed is close to this frequency, or when there is a vertical force excitation with a frequency near to twice the natural shimmy frequency. Since the natural shimmy frequency is low, vehicle wheels often pass through this frequency during motion. The frequency of the vertical force, on the other hand, depends on road surface irregularities, wheel imbalance, or wheel eccentricity.

Trail is an important parameter for characterizing shimmy. It corresponds to the distance between two points: the intersection point of the fork axis with the ground plane and the point where the wheel center is projected onto the ground.

According to [12], shimmy stability can be determined through the eigenvalues of the matrix B_{4x4}, by:

$$B_{11} = 0 \quad (01)$$

$$B_{12} = -V_0 \quad (02)$$

$$B_{13} = -V_0 \cos \upsilon \quad (03)$$

$$B_{14} = -T \cos \upsilon \quad (04)$$

$$B_{21} = (CK_1)/C_1 \quad (05)$$

$$B_{22} = -(1 + CC_L V_0)/C_1 \quad (06)$$

$$B_{23} = -(CC_L V_0 \cos \upsilon - C_2 \sin \upsilon)/C_1 \quad (07)$$

$$B_{24} = -(CC_L T \cos \upsilon)/C_1 \quad (08)$$

$$B_{31} = 0 \quad (09)$$

$$B_{32} = 0 \quad (10)$$

$$B_{33} = 0 \quad (11)$$

$$B_{34} = 1 \quad (12)$$

$$B_{41} = (N_0 \sin \upsilon + K_1 T \cos \upsilon)/I_\psi \quad (13)$$

$$B_{42} = (\mu_1 \cos \upsilon - V_0 C_L T \cos \upsilon)/I_\psi \quad (14)$$

$$B_{43} = (N_0 \sin \upsilon \cos \upsilon \, T - V_0 C_L \cos^2 \upsilon)/I_\psi \quad (15)$$

$$B_{44} = -(C_D + C_L T^2 \cos^2 \upsilon)/I_\psi \quad (16)$$

Where:

C, C_1, C_2, C_L, K_1, μ_1	Tire constants
C_D	Rotational viscous damping constant
N_0	Normal load
T	Trail
V_0	Initial sprung mass velocity
K_1	Tire constant
υ	Caster angle
I_ψ	Fork rotation angle (shimmy angle)

If the eigenvalues of B_{4x4} are real and negative, the system is stable with respect to shimmy (shimmy is not observable). However, if the eigenvalues have a negative real part and a complex part, there are conditions for the occurrence of shimmy, but it remains stable and does not grow exponentially. The critical situation arises when the eigenvalues have a positive real part; in this case, shimmy grows exponentially. It should be noted that the occurrence of shimmy, whether stable or unstable, poses a structural risk and affects vehicle handling.

EXPERIMENTAL BENCH

Experimental test benches are essential for the validation, investigation, and monitoring of projects and products. However, building them at full scale involves high technical complexity and significantly higher costs, which can make their application unfeasible in academic settings or early-stage research. On the other hand, reduced-scale benches are capable of proportionally representing the desired behaviors, allowing for the investigation of phenomena with greater accessibility, lower production costs, and still achieving satisfactory results.

The shimmy observation test bench features adjustable geometrical configurations and a compact, small-scale design, which facilitates transport to different environments. As a result, it is fully suited for didactic applications in academic contexts, while still allowing its use in experiments and studies involving factors relevant to industrial applications.

Another relevant feature of the shimmy observation bench is its compact size. Being relatively small and lightweight, it can be easily transported by a single person. This gives it strong academic potential, as it can be conveniently moved between classrooms or laboratories. Furthermore, it facilitates the demonstration of the parameters that influence the onset of the phenomenon.

The main objective of this bench is to investigate the influence of the geometry with which the fork is positioned, forming the caster angle, as well as the influence of the load applied by the tire on the surface, the motion speed of the mechanism, and the displacement of the axis. The combined analysis of these parameters allows the observation of the shimmy phenomenon in stable and unstable regions. The experimental bench is designed to simulate a system similar to the front wheel of a motorcycle, a passenger vehicle, or even an aircraft landing gear.

The shimmy observation bench consists of three main structural modules. The first and second parts include a lower base and an upper structure, which together form a portal-type frame. The third part is a support platform for the motor and other electrical and electronic components responsible for controlling the bench, which is positioned laterally to the main frame.

Figure 1 shows a front view of the bench, indicating the essential components for its operation. The lower section, which supports two rollers and the belt that surrounds them (item 7), provides a setup that simulates tire-road contact as closely as possible to the characteristics of a flat pavement.

To generate belt movement, a 12V permanent magnet DC motor with a maximum electric power of approximately 35W was used, directly coupled to the shaft of one of the rollers in the belt system, as indicated in item 8.

The second section, located at the top, forms a portal frame. This configuration was specifically chosen to ensure greater robustness and stability when the system is subjected to forces generated by the shimmy effect. Inside the frame enclosure, the fork and wheel support mechanism are housed.

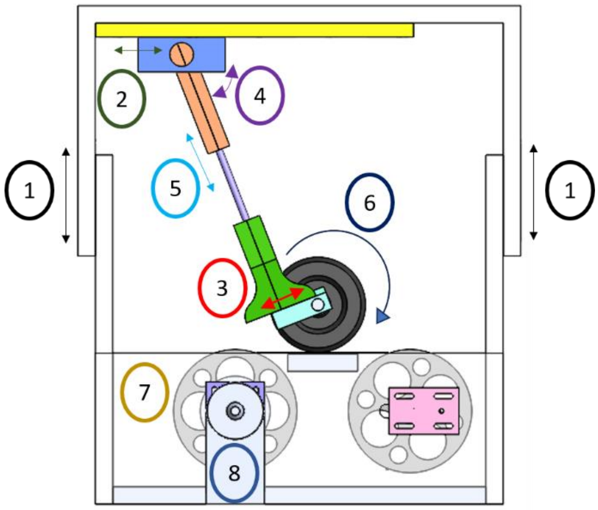

Figure 1. Positioning adjustments and degrees of freedom of the test bench.

The upper section contains adjustable elements that allow for variation of the caster angle. For example, the combination of the portal’s vertical position adjustment (item 1) and the longitudinal guide (item 2), which positions the wheel at any desired point along the belt, defines the caster angle (item 4).

Another essential element of the bench is the fork shaft (item 5), which provides translational and rotational degrees of freedom for the fork around its own axis. The sudden oscillation of this shaft is the primary output parameter for observing the shimmy effect.

At the base of the fork (item 3), adjustment elements are located to control the shaft displacement, which corresponds to the distance between the wheel’s rotation point and the fork shaft. Item 6 represents the degree of freedom associated with the wheel’s rotation about its own axis.

The construction of the shimmy observation bench consists mostly of components manufactured through polymer additive manufacturing, specifically using the Fused Filament Fabrication (FFF) process. The only exception is the support frame, which was built using Medium Density Fiberboard (MDF). This material provided good stiffness and structural stability, allowing the bench to remain steady under the excitation forces generated by the oscillatory motion characteristic of the shimmy effect.

In this way, the support frame configuration ensures the desired stabilizing effect while also facilitating handling and adjustment of the parameters that favor the occurrence of shimmy or the operation outside the system’s self-excitation zone.

To ensure proper operation, a control system was developed for the bench using actuators and sensors. These components, which are considered low-cost, proved effective in fulfilling the intended function.

The control algorithm was developed in both the Arduino® and MATLAB® environments. The Arduino was used as input and output board acquisition, while the MATLAB was responsible for data acquisition, data processing, generating the user interface, and ultimately presenting the results.

Regarding the bench actuators, the main component is the motor, which is used to simulate the wheel movement. This setup can represent either the front wheel of a motorcycle or an aircraft landing gear. The sensors used were LM393 optical sensors, applied for speed measurement.

To verify the compatibility between the speed range operable by the bench and the reading capacity of the speed sensing modules, a sensor configuration capable of reading up to 1750 rpm was used. For this purpose, a disc with 60 holes was constructed for sensor reading, and the speed was measured using a TDV100 laser digital tachometer manufactured by Vonder.

In this way, the user can select a PWM (Pulse Width Modulation) value within a range from 0 to 255. The motor will receive an amount of power consistent with the defined signal. As a result, the speed developed by the system is influenced by inertia and frictional resistance. Therefore, it is essential to measure the actual speed value, which is recorded by the sensors and easily visualized through the response curves provided to the user.

Another important sensor on the shimmy observation bench is the accelerometer and gyroscope module, model MPU6050. From this sensor, the angular speed reading from the gyroscope is used, considering the degree of freedom corresponding to the steering motion of the wheel.

In this way, when the fork is under sufficient excitation for the shimmy effect to occur, the amplitude and frequency of the angular speed curve of the shaft, as displayed graphically, present values that allow the user to visually perceive and confirm the phenomenon. In addition, when the shimmy effect manifests, the user can also perceive its physical impacts on the bench, such as lateral movements caused by excitation forces, as well as the reduction in tangential wheel speed due to energy dissipation.

EXPERIMENTAL PROCEDURE

Several factors contribute to the occurrence of the shimmy effect in front wheels. This platform was developed with the purpose of observing these effects through the controlled variation of selected parameters.

For the experimental procedure in this study, a caster angle (ε) of 35º was defined and used as a reference for demonstrating the results. Another goal of the experiment is to explore regions of the graph that represent the variation in axle displacement (d), that is, the distance between the fork axis and the wheel's rotation point. This variation directly affects the trail distance (t).

In this way, the graph of the tangential speed of the device as a function of the trail distance allows the identification of stability zones, where the likelihood of shimmy occurrence is lower. Conversely, in unstable zones, the system is more prone to enter a self-excited mode, which enables the observation of the phenomenon.

In Figure 2, the parameters that can be adjusted to produce variations in the trail distance can be observed.

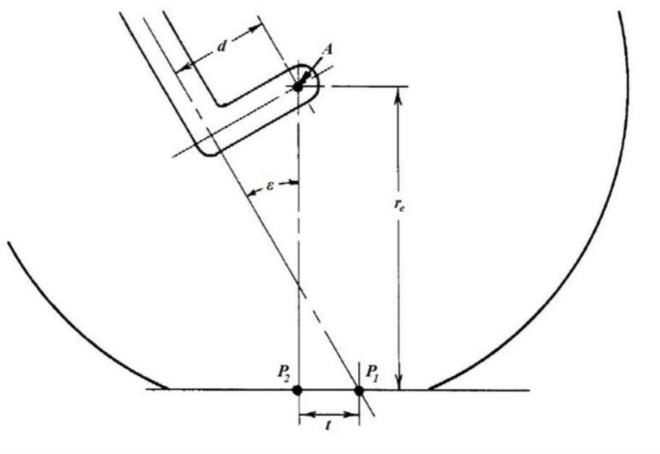

Figure 2. Trail ([12]).

To perform the shimmy effect observation tests, the vertical and horizontal positions of the guide supporting the fork are adjusted so that the wheel is positioned on the flat portion of the belt. The fork forms a fixed angle in 35º.

Once the vertical and horizontal adjustments are set, the trail distance is determined exclusively by the axle displacement adjustment device, as shown in item 3 of Figure 1. This, in turn, changes the trail distance.

The observation of the shimmy effect within stable and unstable zones provides a transition boundary between these conditions. This boundary depends on the previously mentioned geometric characteristics, as well as on the speed and the load applied by the system on the wheel and by the wheel on the surface.

The methodology used to investigate the stability and instability zones of the bench, concerning the occurrence of the shimmy effect, is based on the relationship between the linear speed and the trail distance, following the criteria below:

Currently, the variation of the axle distance in the bench is carried out manually. As a result, the user must manually adjust the axle position to change the trail distance. To facilitate result collection, a fixed trail value is defined, and different speeds are tested by varying the PWM signal configured in the user interface developed in MATLAB.

The linear speed of the bench ranges from 0 to 5 m/s. The same procedure is then repeated, decreasing the trail distance by 1 cm at each step, starting from 7 cm down to 0 cm.

The study by [12] presents a curve representing the stability boundary obtained in an experiment with specific dimensions and loads, aimed at analyzing the landing gear of aircraft, as illustrated in Figure 3. While the present study also seeks to observe the shimmy stability regions, the geometry, load, and dimensions of the test bench used here differ from those of the referenced configuration.

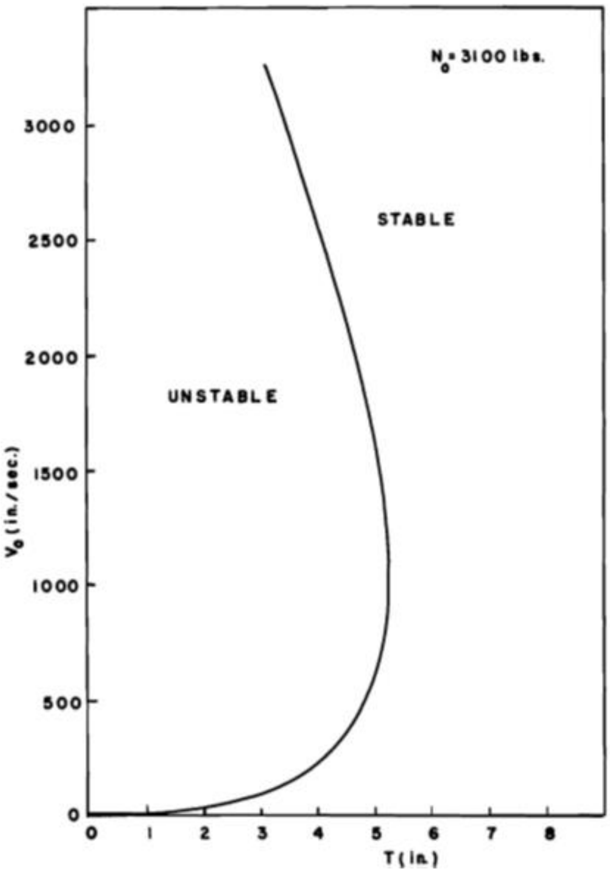

Figure 3. Diagram of the stability. ([12]).

The verification of stability and instability regions related to the occurrence of the shimmy effect is carried out in each experimental section, using a predefined trail value. For each experiment, the displacement speed is gradually increased by the user through the increment of the PWM signal in the control interface.

The bench motor is activated with an initial PWM signal value of 30/255, which is sufficient to overcome the inertia and static friction of the components, resulting in an initial speed of approximately 0.6 m/s. From that point on, the PWM value is increased in steps of 10 until reaching a maximum limit of 180/255, which is compatible with the speed sensor's reading capacity.

It was observed that, for PWM signals above 130/255 (approximately 3 m/s), the shimmy effect does not manifest in any of the trail distance variations tested. Therefore, this condition establishes a practical limit for the speed sweep in the experimental analysis.

This made it possible to build a matrix of points that maps the boundaries between stable and unstable zones, indicating the conditions under which the occurrence of shimmy is more predictable. These results will be discussed in detail in the Results and Discussion section.

Figure 4 presents a perspective view of the shimmy observation bench, showing its main components: the lower structure, the upper structure, the fork, the wheel, the motor control drive, the motor, and the microcontroller.

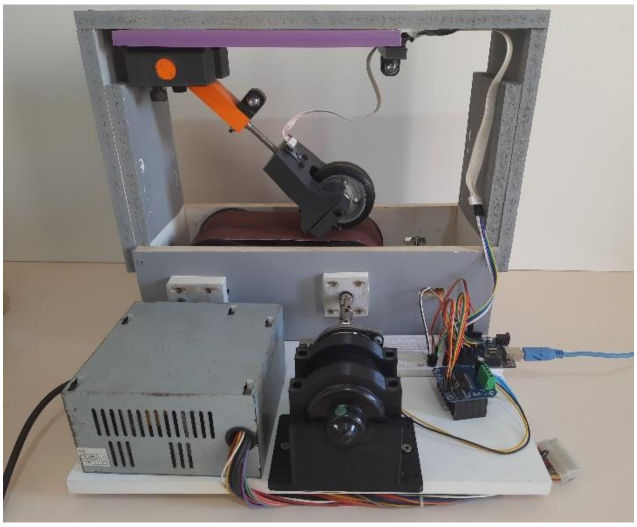

Figure 4. Shimmy phenomenon experiment bench.

Figure 5 shows the experimental bench from a top-down perspective to better visualize the speed sensor, which is mounted on the same shaft as the motor but on the opposite side from where the motor is fixed.

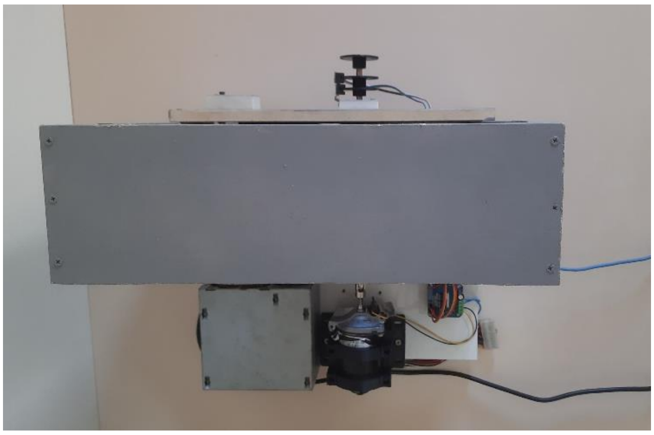

Figure 5. Top view of the experimental bench.

RESULTS AND DISCUSSIONS

The set of practical experiments resulted in a matrix of points, as shown in Figure 6. These data were obtained from eight experiments, each performed with a fixed trail value.

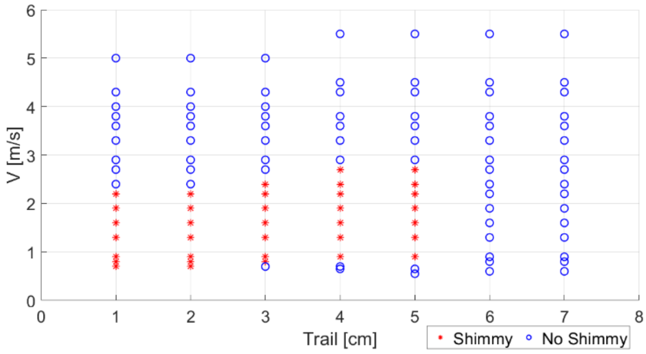

Figure 6. Diagram of stability.

In this graph, it is possible to observe a region defined by red data points. These points were identified based on the gyroscope readings on the fork shaft, considering two criteria: the amplitude of the angular speed (expressed in °/s) and the frequency of the event within a given time interval. Points whose amplitude exceeded the threshold of $2x10^4$ °/s were classified as indicative of the occurrence of the shimmy effect.

The same graph allows for some important observations. The lower portion, corresponding to the points associated with low speeds, as well as the region of intermediate speeds, both show behavior similar to the events reported in the literature [12].

On the other hand, in regions with higher speeds, above 4 m/s, it is not possible to establish the same correlation with the results found in the literature. This indicates that, although the bench presents well-tuned characteristics to represent the shimmy effect, it diverges from the published data for higher speed ranges. This discrepancy reinforces the importance of investigating the possible factors that cause it. An important fact to be observed is that the reference's parameters correspond to an airplane, while the wheel diameter applied in the present test is just 3.5".

Among the experiments conducted, one case was selected for a more detailed analysis of specific characteristics, such as the trail value of 4 cm, shown in Figure 7. This experimental segment presents a stable regime at low displacement speeds. As the speed is gradually increased, the occurrence of the shimmy phenomenon is observed within an intermediate speed range, between 0.9 and 2.3 m/s.

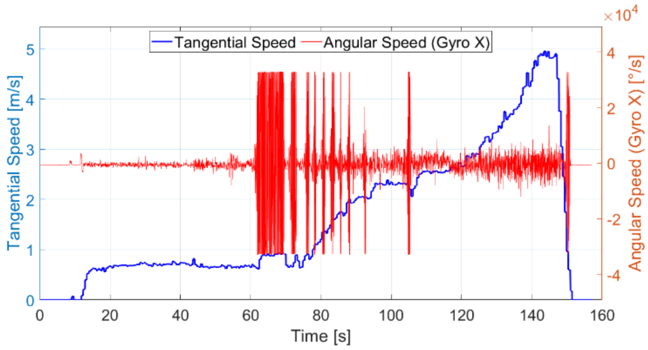

Figure 7. Experimental curve for a trail value of 4 cm.

Another relevant feature observed in this experiment is that the occurrence of shimmy causes an immediate reduction in the system's speed. This happens because part of the kinetic energy is dissipated both by the tire's friction with the surface (belt) and by the self-excited forces generated in the system during the phenomenon. In Figure 8, this significant speed reduction can be observed at time instants 87, 91, and 104 seconds.

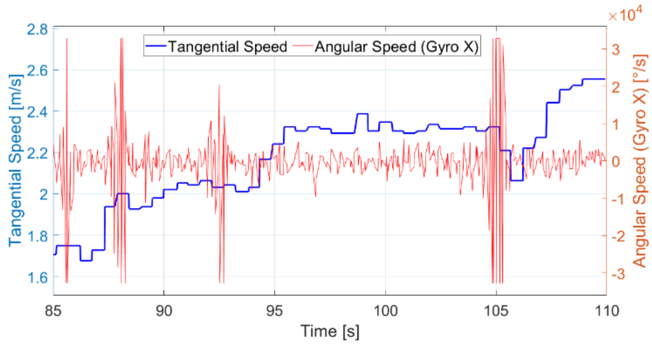

Figure 8. Experimental curve for a trail value of 4 cm, in the time range between 85 and 110 seconds.

It is worth noting that for speeds above 2.5 m/s, the bench enters the stability region for any trail distance, up to the upper speed limit of 5 m/s. After reaching this maximum speed, the user gradually reduces the bench speed to zero.

A relevant observation concerns the occurrence of the shimmy effect during the deceleration phase of the bench, when the speed was reduced to end the experiment. In vehicles, aircraft landing gear, and the front fork of motorcycles, as well as in this experimental bench that represents them, it was observed that at the end of the test cycles, during sharp deceleration, the device entered a shimmy condition, even while still within a stability region. This behavior was evidenced in this experiment during the time interval from 144 to 154 seconds, as shown in the zoomed section of the curve presented in Figure 9.

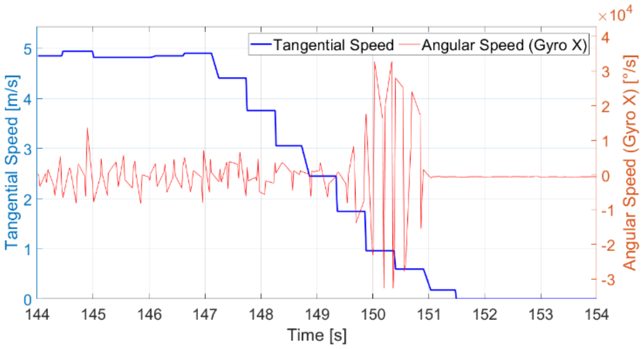

Figure 9. Experimental curve for a trail value of 4 cm, in the time range between 144 and 154 seconds.

During each experiment, the user monitors in real time the motor rotation curves, the belt speed, the angular speed curve of the fork, and the dynamic behavior of the bench caused by the forces generated during the occurrence of the shimmy phenomenon. The shimmy signaling criteria, presented graphically, are simultaneously observed by the user, especially when the effect is less intense and requires greater attention for confirmation.

The secondary effects on the bench during the occurrence of the shimmy phenomenon, as previously mentioned, are perceived by the user as lateral movements of the structure caused by self-excited forces, along with an immediate reduction in belt speed.

Figure 10 presents the curve of the experiment with a fixed trail of 2 cm, which confirms some characteristics like those observed in the previous experiment. However, there are also distinct features that deserve attention. It is observed that at low speeds, the belt does not maintain a consistent speed pattern imposed by the PWM signal, mainly because shimmy occurs right at system startup. In this condition, the available kinetic energy is low, and the instability caused by the shimmy effect results in a significant loss of that energy, making the speed highly variable. On the other hand, it is observed that the system tends toward stability at speeds above 2.3 m/s.

In addition, a lower-intensity shimmy is observed during the final deceleration of the system until it comes to a complete stop.

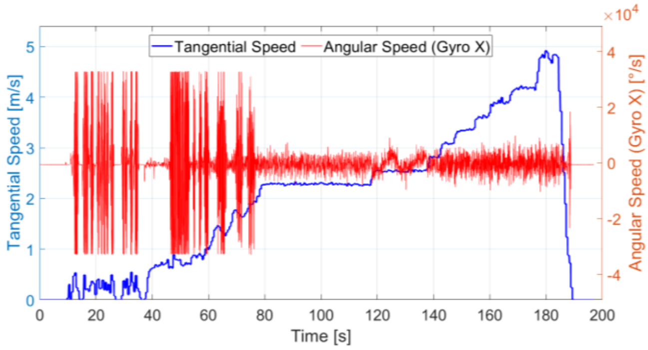

Figure 10. Experimental curve for a trail value of 2 cm.

During operation within the instability range associated with the shimmy effect, a speed reduction was observed at least four distinct moments when the user increased the speed, within the time interval from 60 to 80 seconds. This effect was more evident than in the previous experiment. In this interval, the curve was magnified to allow a clearer visualization of the system's behavior during shimmy, as shown in Figure 11.

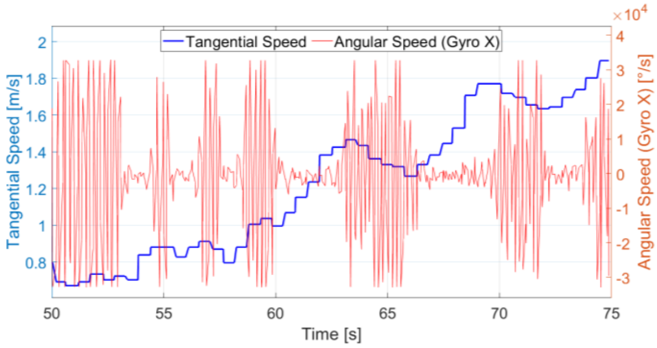

Figure 11. Experimental curve for a trail value of 2 cm, in the time range between 50 and 75 seconds.

CONCLUSIONS

The development and instrumentation of a small-scale bench for the study of the shimmy phenomenon was presented. The results shown indicate the importance of conducting a more rigorous investigation into geometric characteristics, tire load, and material-related effects, particularly those involving the wheel and the surface. The combination of these elements influences vehicle dynamics, especially because it was not addressed as part of this study.

The data collected in each experiment and presented in this work provide an opportunity for more in-depth investigations into the occurrence of shimmy. Moreover, they enable future studies on the development of devices capable of mitigating this phenomenon and its undesirable effects, which can range from minor issues, such as premature wear of mechanical components, to serious accidents.

One consequence of shimmy, observed in the bench tests, is that the speed tends to decrease rapidly, which can affect vehicle handling. Additionally, improving structural geometric characteristics can move operation away from the instability region. All of this reinforces that the use of the test bench, both in academic and industrial applications, provides conditions for predicting risk situations in real vehicle operation.

Another aspect that presents a perspective for future studies is the implementation of a closed-loop speed control system for the bench. Currently, the PWM signal sent to the motor is set by the user and remains constant, while the speed sensor operates only as an open-loop reading.

To address the speed reduction caused by friction generated during tire slip, future work proposes the implementation of a PID control system. This control will allow the motor's input power to increase automatically when a speed drop is detected. The implementation of a closed-loop system aims to reduce unintentional speed variations, which may intensify the occurrence of shimmy.

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